A new solution of solidification problems in continuous casting based on meshless method

Lei Zhang* and Yiming (Kevin) Rong

Department of Manufacturing Engineering,
Worcester Polytechnic Institute,
Worcester, MA 01609, USA
E-mail: lzhang@wpi.edu
E-mail: rong@wpi.edu
*Corresponding author

Hou-Fa Shen and Tian-You Huang

Department of Mechanical Engineering,
Tsinghua University,
Beijing 100084, PR China
E-mail: shen@tsinghua.edu.cn
E-mail: huangty@tsinghua.edu.cn

Abstract: In this paper, a new solution of solidification problems in continuous casting based on meshless method is presented with multiple strategies to solve the practical problems. An additional term is added to stabilise the computation with Neumann boundary. The enthalpy method is used to calculate the latent heat and the corresponding iterative solution is given. An iteration scheme for non-linear material calculation is also constructed. The model is verified by a 2D FEM solidification example. Finally, it is applied to the simulation of the solid shell growth in the continuous bloom and billet casting moulds. The result is coincided with the measurement.

Keywords: meshless method; Finite Point Method; FPM; solidification; Stefan problem; continuous casting.

Reference to this paper should be made as follows: Zhang, L., Rong, Y., Shen, H-F. and Huang, T-Y. (2008) 'A new solution of solidification problems in continuous casting based on meshless method', *Int. J. Intelligent Systems Technologies and Applications*, Vol. 4, Nos. 1/2, pp.177–187.

Biographical notes: Lei Zhang finished his Dissertation 'Numerical Simulation on Solidification and Thermal Stress of Solid Shell in Continuous Casting Mould based on Meshless Method' and earned his PhD in Material Science and Engineering from Tsinghua University, Beijing, China. Currently, he is working at Worcester Polytechnic Institute as a Research Scientist and in charge of development of a computerised heat treatment planning system for quenching and tempering, CHT-q/t. His research interests are multiscale materials modelling and simulation.

Yiming (Kevin) Rong is the Higgins Professor of Mechanical Engineering and the director of Computer-Aided Manufacturing Laboratory (CAM Lab.) at Worcester Polytechnic Institute (WPI). He received a BS in Mechanical Engineering from Harbin University of Science and Technology, China, in 1981; an MS in Manufacturing Engineering from Tsinghua University, Beijing, China, in 1984; an MS in Industrial Engineering from University of Wisconsin, Madison, WI, in 1987 and a PhD in Mechanical Engineering from University of Kentucky, Lexington, KY, in 1989. He worked as a Faculty Member at Southern Illinois University at Carbondale for eight years before joining WPI in 1998.

Hou-Fa Shen is an Associate Professor in Tsinghua University of China, and currently, his interests are on the modelling of continuous casting and solidification.

Tian-You Huang is a Professor in Tsinghua University of China and currently his interest is in materials processing technology.

1 Introduction

In continuous casting, crack is a big issue due to thin thickness of the solid shell at the mould exit. This will cause fatal accident. The crack is closely related to the solidification process in mould (Gan et al., 2001). Since continuous casting is operated at high temperature, melt point of steel, measurement of the temperature variations in the mould is almost impossible. Laboratory experiments of continuous casting is expensive, so numerical simulation tools, primarily Finite Element Method and Finite Deference Method (FEM and FDM) based, are normally used to mathematically simulate the process (Gan et al., 2001).

However, FEM requires using element and it is not convenient in the treatment of discontinuities that do not coincide with the original mesh lines, for example, the large-scale deformation problem and the crack growth with arbitrary and complex paths. FEM solves these problems by remeshing in each step of the evolution. But remesh sometimes leads to degradation of accuracy and complexity in the computer program, and becomes the burden associated with the tedious adaptive and interactive remeshing. As compared to FEM, meshless method eliminates elements by constructing the approximation entirely in terms of nodes. In result, moving discontinuities can be treated without remeshing with minor costs in accuracy degradation. Then, it becomes possible to solve large classes of problems, such as large-scale deformation and crack growth problems, which are very awkward with mesh-based methods (Belytschko, 1996; Liu, 2002).

The meshless method, also called meshfree or element-free method was developed about 20 years ago. It starts with the establishment of the Smooth Particle (SPH) method (Lucy, 1977), which is used for modelling astrophysical phenomena without boundaries. Later, Belytschko et al. (1994) developed the Element-Free Galerkin (EFG) method, and the successful application of EFG also triggered the great research effort devoted to the meshless method (Zhang and Liu, 2004). So far, more than ten meshless methods or schemes have been developed, and a few books on them are available (see Atluri and Shen, 2002; Griebel and Schweitzer, 2002; Liu, 2002; Zhang and Liu, 2004).

Finite Point Method (FPM) is one of the easiest to implement (Onate et al., 1996a,b) and it has been successfully applied in solving fluent flow problem. Compared with other meshless methods that need integration on the 'background cell', FPM is really a meshless or meshfree method, which only needs nodes and dose not need integration.

That makes it more efficient, and also is advantageous in solving large-scale deformation and crack growth problems, because it can totally avoid from remeshing by free adding or deleting nodes or points whenever or wherever needed.

Good understanding of the solidification is the fundamental of solving crack problem in continuous casting. Therefore, in this paper, FPM is applied to numerical simulation of continuous casting where a solidification model is constructed based on FPM. The model is verified first and then employed to simulate the solid shell growth in the continuous bloom and billet casting moulds. The results predicted by the FPM model coincidence with the measurement. Observations indicate that meshless method is a potential numerical analysis tool and it is valuable for the analysis of the continuous casting process.

2 Model descriptions

2.1 MLS approximation

The Moving Least-Square (MLS) method (Lancaster and Salkauskas, 1981) is introduced to build the approximation of the unknown temperature function *T*, which is replaced by an interpolating function $T^h(x)$ at all the points, as shown in Figure 1.

$$T(x) \approx T^{\mathrm{h}}(x) = \sum_{I}^{n} \psi_I(x) T_I \tag{1}$$

where ψ_I is the MLS shape function (Lancaster and Salkauskas, 1981).

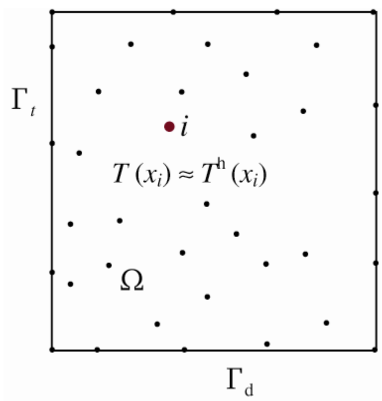


Figure 1 Schematic representation of FPM

2.2 Equilibrium equation and boundary conditions

The two dimensional energy equilibrium equation and boundary conditions of solidification processes are (Gan et al., 2001)

$$\text{Equilibrium equation}: \rho c_{\mathrm{p}} \frac{\partial T}{\partial t} - k\nabla^2 T - q_{\mathrm{v}} = 0 \tag{2.1}$$

Heat flux boundary condition: $-k\dfrac{\partial T}{\partial \mathbf{n}} + q_{\mathbf{n}} = 0$ (2.2)

Initial condition: $T\big|_{t=0} = T_{\text{in}}$ (2.3)

where T is the temperature (K), t is time (sec), c_{p} is the specific heat (J/kg K), ρ is the density (kg/m^3), k is the thermal conductivity (W/m K), q_{v} is the energy source term (W/m^3), $\mathbf{n}$ is the normal to the boundary, $q_{\mathbf{n}}$ is the known heat flux at the boundaries (W/m^2) and T_{in} is the initial temperature (K).

2.3 Neumann boundary stabilisation scheme

For solving convection and diffusion problem by FPM, Neumann boundary needs to be stabilised first. The related work can be seen in Onate's papers (Onate et al., 1996a,b), and a stabilisation item is added to Neumann boundary

$$-k\frac{\partial T}{\partial \mathbf{n}} + q_{\mathbf{n}} - \frac{h}{2}r = 0 \tag{3}$$

where $r = k\nabla^2 T + q_V$ and h is the characteristic length (Onate et al., 1996a,b).

2.4 Discretisation in space

By using MLS (Lancaster and Salkauskas, 1981), the approximation function of the unknown temperature T of node i at time t can be expressed as:

$$T_i\big|_t \approx T_i^{\text{h}}\big|_t = \sum_I^n \psi_I T_I\big|_t \tag{4}$$

Then, solving the derivatives of T with respect to space becomes solving derivatives of the shape function ψ with respect to space

$$\nabla^2 \tilde{T}\,\big|_t = \sum_I^n \nabla^2 \psi_I T_I^{\text{h}}\big|_t \tag{5.1}$$

$$\frac{\partial \tilde{T}\,\big|_t}{\partial \mathbf{n}} = \sum_I^n \frac{\partial \psi_I}{\partial \mathbf{n}} T_I^{\text{h}}\big|_t \tag{5.2}$$

2.5 Discretisation in time

Explicit format of T with respect to time t (Kong, 1998) is substituted into the energy equilibrium Equation (2.1), then it becomes

$$\rho c_{\text{p}} \frac{\tilde{T}\big|_{t+\Delta t} - \tilde{T}_i\big|_t}{\Delta t} - r_i\big|_{t+\Delta t} = 0 \tag{6}$$

2.6 Latent heat treatment

Enthalpy method (Eyres and Hartree, 1946) is used to calculate latent heat of solidification, q_{v} is equal to zero, and then the energy equilibrium Equation (2.1) becomes

$$k\nabla^2 T = \rho \frac{\partial h}{\partial T}\frac{\partial T}{\partial t} \tag{7}$$

Replacing $\partial h/\partial T$ with a different form, using the implicit form of discretisation in time, Equation (7) becomes

$$\rho \frac{\left(h\left(\tilde{T}_i\big|_{t+\Delta t}\right) - h\left(\tilde{T}_i\big|_t\right)\right)}{\Delta T}\frac{\left(\tilde{T}_i\big|_{t+\Delta t} - \tilde{T}_i\big|_t\right)}{\Delta t} - r_i\big|_{t+\Delta t} = 0 \tag{8}$$

where $r_i\,|_{t+\Delta t} = k\nabla^2 \tilde{T}_i\,|_{t+\Delta t}$.

Equation (8) cannot be solved directly, so the corresponding iterative solution is given

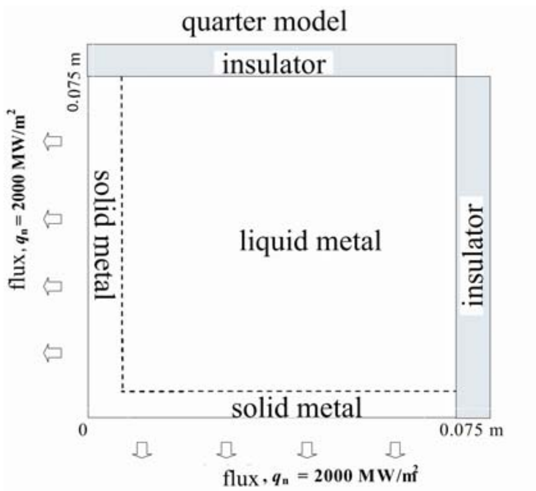
$$\rho \frac{\left(h\left(\tilde{T}_i\big|_{t+\Delta t}^{m}\right) - h\left(\tilde{T}_i\big|_t\right)\right)}{\Delta T}\frac{\left(\tilde{T}_i\big|_{t+\Delta t}^{m+1} - \tilde{T}_i\big|_t\right)}{\Delta t} - r_i\big|_{t+\Delta t}^{m+1} = 0 \tag{9}$$

where $r_i\,|_{t+\Delta t}^{m+1} = k\nabla^2 \tilde{T}_i\,|_{t+\Delta t}^{m+1}$, and m is the number of iteration.

3 Case studies

3.1 Solidification problem

An FEM example of solidification is employed here to verify the FPM model in 2D case. Figure 2 is the quarter model of the problem. Liquid metal in a square mould is cooled by the constant flux (equal to 2000 MW/m^2) at boundary and then solidified.



Source: Li et al. (2003).

Figure 2 2D solidification problem

The computation parameters are shown in Table 1.

Table 1 Material property

Parameters	*Value*
Conductivity of solid, k_s (W/m K)	237.65
Conductivity of liquid, k_l (W/m K)	94.14
Specific heat of solid, $c_{p,s}$ (J/kg K)	903.74
Specific heat of liquid, $c_{p,l}$ (J/kg K)	1079.47
Density, ρ (kg/m^3)	2700
Latent heat, L (kJ/kg)	397.48
Melting point temperature, T_l (K)	933
Initial temperature, T_{in} (K)	973

Source: Li et al. (2003).

The solid front location at the diagonal and the *y* symmetrical axial of the mould against time is calculated by the FPM model. The results are displayed in Figure 3 with a comparison to the FEM predicted results.

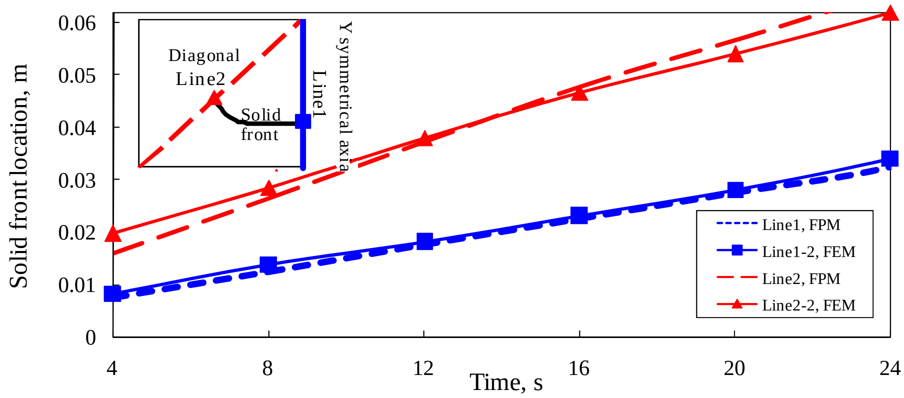


Figure 3 Solid front locations calculated with FPM in comparison to FEM results

In Figure 3, the FPM results represented by lines and the FEM results represented by symbols are well matched.

3.2 *Continuous bloom casting*

Solid shell growth in a continuous bloom casting mould is analysed with the FPM model in this research. The continuous bloom casting process is schematically shown in Figure 4.

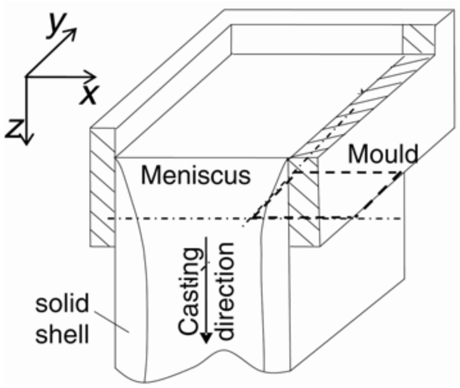


Figure 4 Schematic of continuous bloom casting

Figure 5 is the shape and size of the bloom mould and its quarter node discretisation model for the calculation.

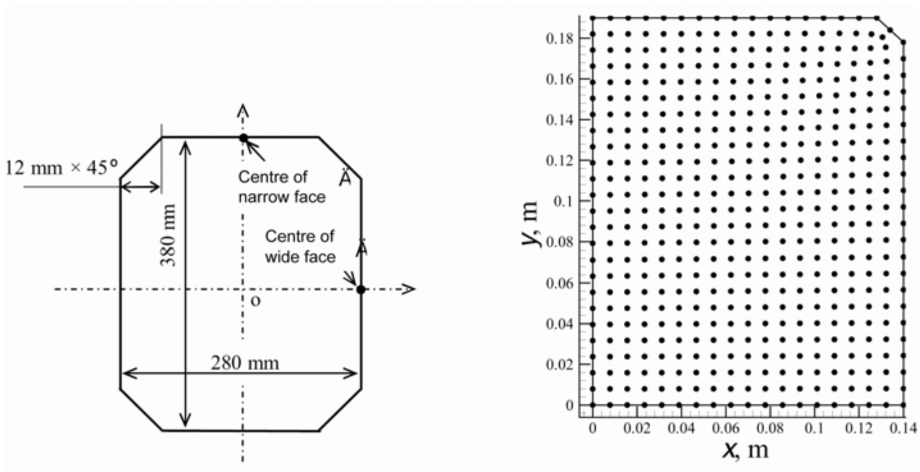
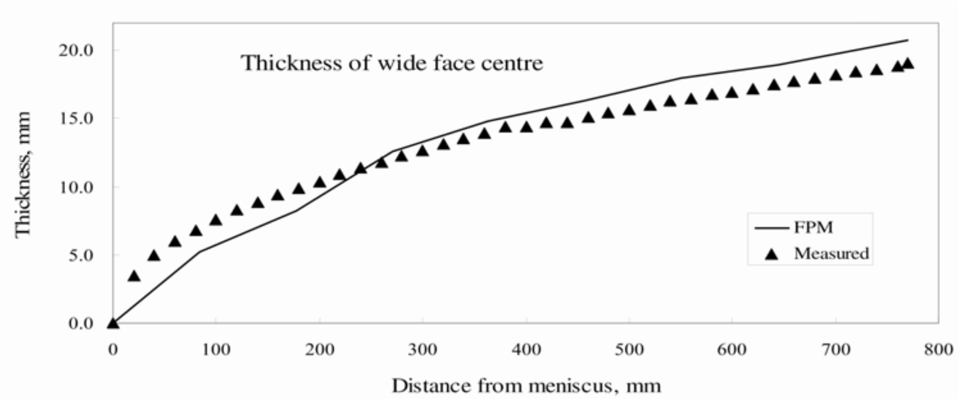


Figure 5 Calculation model of the continuous bloom casting mould: (a) geometry model and (b) discretisation model (quarter)

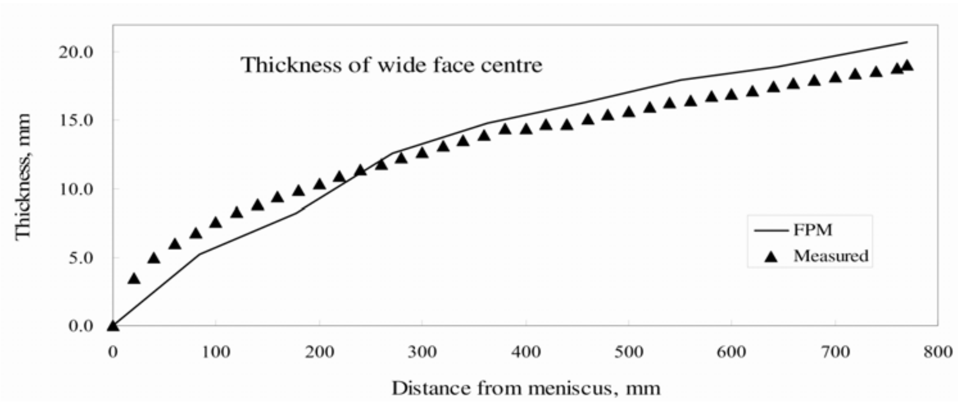
Based on the assumptions on the process conditions, the heat transfer along the casting direction is very small and can be ignored. The heat flux on the surface of the solid shell will react on the change of casting speed and is known. The solidification process in continuous casting can be mathematically described by Equations (2.1)–(2.3), and then it can be simulated by the FPM model.

The height of mould is 800 mm, casting speed is 0.72 m/min, the casting material is 0.7 in C%, and pouring temperature is 1500°C. The calculation results are illustrated below.

The predicted thickness results of the solid shell at the narrow face centre against the distance from meniscus are shown in Figure 6(a) and the thickness of the solid shell at the wide face centre are plotted in Figure 6(b).



(a)



(b)

Figure 6 (a) Growth of the solid shell at narrow face centre against the distance from meniscus and (b) growth of the solid shell at wide face centre against the distance from meniscus

The above results indicate that the solid shell evenly grows in *x* and *y* directions. At the mould exit, the thickness of the solid shell at narrow face is 19.6 mm and the thickness at the wide face centre is 19.2 mm. The predicted results coincided with the measurement.

3.3 *Effect of air gap in continuous casting billet mould*

In continuous casting mould, due to the shrinkage of the solidified shell, the surface of the shell will take off from the mould and an air gap will be formed. It usually starts at corner zone of the mould, the air gap then decreases the cooling effects on the surface of the shell and makes the shell at corner zone thinner and easy to crack.

The air gap in another continuous casting billet mould is considered to investigate its influence on the shell growth. Figure 7 is the calculation domain with the FPM model.

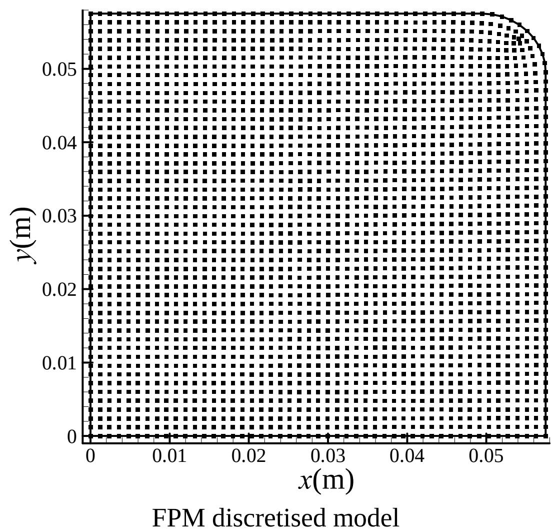


Figure 7 Shape, sizes and nodes discretisation in the solution area (a quarter)

The height of mould is 700 mm, the casting speed is 2.3 m/min, the casting material is 0.16 in C%, and the pouring temperature is 1520°C.

Due to the air gap, it obviously results in the thinner shell at corner zone as shown in Figure 8. The predicted results also coincided with the FEM solutions.

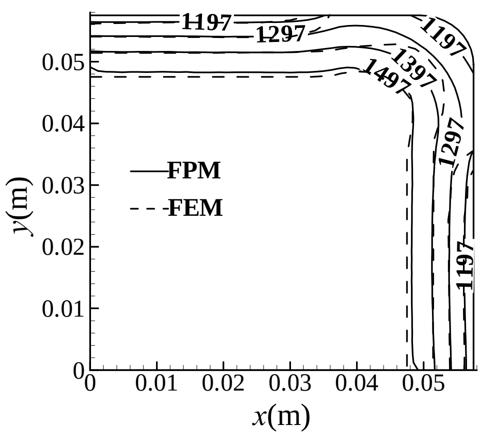


Figure 8 FPM predicted temperature results of the solidified shell at the mould exit

The results in Figure 8 are obtained at casting speed of 2.3 m/min, if the casting speed increases, the shell thinner problem at corner zone will become more serious. Figure 9 shows the predicted solidified shell at the mould exit at different casting speed. At 5 m/min casting speed, the solidified shell at corner zone is even thinner than usual, which is very dangerous if there is no particular control in the practice.

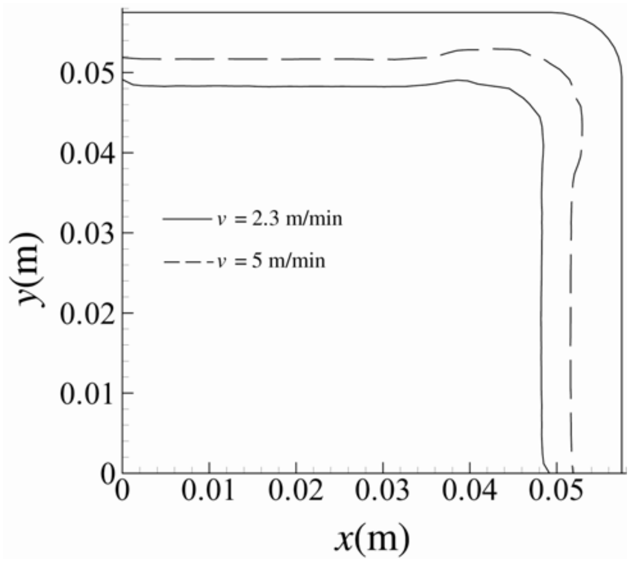


Figure 9 The predicted solidified shell at the mould exit with different casting speeds

4 Conclusions

In this paper, a new solution of solidification problems in continuous casting based on meshless method is presented with multiple strategies to solve the practical problems: the meshless method, FPM, is preliminarily studied for the simulation of solidification in continuous casting. The FPM solidification simulation model has been developed with characteristics of

- Onate stabilisation scheme for Neumann boundary
- non-linear material treatment
- solidification enthalpy treatment.

The model is validated with comparison to a FEM solution of a 2D solidification problem. Then, the solid shell growth in the continuous bloom and billet casting moulds is simulated by using the model. The calculation results of the shell thickness in the mould coincided with the measurement. The air gap in a continuous casting billet mould is considered to investigate its influence on the shell growth. Numerical observations show that meshless method is a potent numerical analysis tool and would be valuable to analyse the continuous casting process even to solve the related crack and large-scale deformation problems in the future.

References

Atluri, S.N. and Shen, S. (2002) *The Meshless Local Petrov-Galerkin (MLPG) Method*, Stuttgart: Tech Science Press.

Belytschko, T., Krongauz, Y., Organ, D., Fleming, M. and Krysl, P. (1996) 'Meshless methods: an overview and recent developments', *Computer Methods in Applied Mechanics and Engineering*, Vol. 139, Nos. 1–4, pp.3–47.

Belytschko, T., Lu, Y.Y. and Gu, L. (1994) 'Element free Galerkin methods', *International Journal for Numerical Methods in Engineering*, Vol. 37, pp.91–100.

Eyres, N.R. and Hartree, D.R. (1946) 'The calculation of variable heat flow in solid', *Philosophical Transactions on Royal Society A*, Vol. 240, pp.1–57.

Gan, Y., Chou, S. and Xiao, Z. (2001) *Numerical Simulation of Continuous Casting Processes*, Beijing: Metallurgy Industry Press.

Griebel, M. and Schweitzer, M.A. (2002) *Meshfree Methods for Partial Differential Equations*, Vol. 26, Berlin: Springer Verlag.

Kong, X.Q. (1998) *Application of FEM in Heat Transfer Process*, Beijing: Science Press.

Lancaster, P. and Salkauskas, K. (1981) 'Surfaces generated by moving least-squares methods', *Mathematics of Computation*, Vol. 37, No. 155, pp.141–158.

Li, C.Y., Garimella, S.V. and Simpson, J.E. (2003) 'Fixed-grid front-tracking algorithm for solidification problems, part I: method and validation', *Numerical Heat Transfer*, Part B, Vol. 43, No. 2, pp.117–141.

Liu, G.R. (2002) *Mesh Free Methods: Moving Beyond the Finite Element Method*, CRC Press.

Lucy, L.B. (1977) 'A numerical approach to the testing of the fission hypothesis', *The Astronomical Journal*, Vol. 8, No. 12, pp.1013–1024.

Onate, E., Idelsohn, S., Zienkiewics, O.C. and Taylor, R.L. (1996a) 'A finite point method in computational mechanics. Applications to convective transport and fluid flow', *International Journal for Numerical Methods Engineering*, Vol. 39, pp.3839–3866.

Onate, E., Idelsohn, S., Zienkiewics, O.C., Taylor, R.L. and Sacco, S. (1996b) 'A stabilized finite point method for analysis of fluid mechanics problems', *Computer Methods in Applied Mechanics and Engineering*, Vol. 139, pp.315–346.

Zhang, X. and Liu, Y. (2004) *Meshless Methods*, Beijing: Tsinghua Press.