WAVELET BASED IMAGE COMPRESSION BY ADAPTIVE SCANNING OF TRANSFORM COEFFICIENTS

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ABSTRACT

An image compression algorithm exploiting the interband and intraband dependencies in a multiresolution decomposition structure is introduced. The transform coefficients, other than those which belong to the DC band, are adaptively scanned into two different 1-D arrays. These arrays are generated by using the observed statistical behavior of wavelet transform coefficients, in an effort to maximize local stationarity. The arrays are then entropy coded separately. The algorithm is also extended for the compression of wavelet packet transform coefficients. Experimental results indicate that the algorithm has competitive performance.

Key words – image compression, wavelet transform, wavelet packets, interband/intraband correlations, adaptive scanning

1. INTRODUCTION

Discrete wavelet transform (DWT) based coders have been very successful in image compression applications. Many of the state-of-the-art DWT coders like EZW [1], SPIHT [2], and MRWD [3] gain compression efficiency by use of intraband and /or interband correlations. Shapiro defined a *parent-children* link across the bands at successive scales in [1]. He developed EZW by exploiting the interband dependency stated as, children descending from a (in)significant parent are more likely to be (in)significant. Servetto *et al.* observed in [3] that, not only the coefficients descending from a significant parent, but also the neighborhood of these coefficients are more likely to be significant (interband dependency). They proposed prediction of significance in a band by Morphological Representation of Wavelet Data (MRWD). The basics of MRWD algorithm were:

- scan a band in region growing manner,
- start scanning from the coefficients these descend from a significant parent,
- use arithmetic coding [4] for encoding the coefficients.

In [5], we improved their algorithm by introducing three new features:

- scanning with magnitude preference of the parent.
- scanning the predicted-to-be-insignificant pixels in the order of closeness to the predicted-to-be-significant ones.
- using hierarchical enumerative coder [6] for encoding the transform coefficients.

Most of the wavelet transform based compression methods can be extended to wavelet packet transform based compression. Wavelet packet transform (WPT) is a generalization of the dyadic wavelet transform (DWT) that offers a rich set of decomposition structures. WPT was first introduced by Coifman *et al.* [7] for dealing with the nonstationarities of the data better than DWT does. WPT is associated with a *best basis selection* algorithm. The best basis selection algorithm decides a decomposition structure among the library of possible bases, by measuring a data dependent *cost function*. In [7], the number of nonzero coefficients after thresholding was used as the cost function. MSE or entropy has been the other choices, but none of these measures provided a genuine rate-distortion optimization for image compression problem. Ramchandran and Vetterli were the first to propose an image compression algorithm, which used wavelet packet transform by rate-distortion (RD) optimization [8]. Their algorithm was extended in [9] by space-frequency quantization approach, which was a joint application of scalar quantization and zerotree quantization within the WPT framework.

In this work, we include the complete algorithm for wavelet transform based image compression, which was partially introduced in [5]. We define a parent-children link for WPT coefficients, and extend the DWT based encoding algorithm for the encoding of RD optimized WPT coefficients.

The organization of this paper is as follows: in Section 2, we describe our encoding algorithm, after reviewing the statistical behavior of DWT coefficients. We include a brief overview of WPT based coding, e.g. best basis selection and RD optimization algorithms, in Section 3. Section 4 presents a novel extension of the DWT based compression algorithm [5] to WPT based compression. Finally, experimental studies and conclusions follow in Sections 5 and 6.

2. ENCODING ALGORITHM FOR DWT COEFFICIENTS

2.1. Statistical properties of DWT coefficients

Dyadic wavelet decomposition is achieved by iterative two-channel perfect reconstruction filterbank operations over the low frequency band at each stage.

One-stage of a two-dimensional discrete wavelet decomposition is depicted in Figure 1. The decomposition produces low-low (DC), low-high (horizontal detail), high-low (vertical detail), and high-high (diagonal detail) subbands, as shown in Figure 2. Iterative processing yields to a DC band, and *horizontal, vertical*, and *diagonal* detail subbands at successive resolutions.

Let W^{DC} denote the DWT coefficients in DC band, and $W^{(d,l)}$ denote the DWT coefficients in detail subbands. d refers to the orientation (horizontal, vertical, diagonal) and $l=1,2,\ldots L$, refers to stage, where increasing l correspond to coarser resolution level, and L is the maximum decomposition depth. Shapiro has formed a parent-children link between the elements of $W^{(d,l)}$ and $W^{(d,l+1)}$ such that, a pixel in $W^{(d,l+1)}$ is the parent of four children in $W^{(d,l)}$ (Figure 3). According to this link, the statistical behavior of wavelet transform coefficients can be listed as follows:

- Within a band; significant coefficients are generally located around the edges (intraband correlation).
- Children descending from a significant parent are more likely to be significant (interband correlation).
    - The higher the parent magnitude, the higher the probability that children are significant.

2.2. The encoding algorithm

Given a wavelet transform decomposition of the input image, the algorithm starts by applying a uniform scalar quantizer. The DC band and the detail bands are treated separately. The coefficients are scanned into three one-dimensional arrays, in such a way to exploit their statistical behaviors.

- DPCM is applied to the DC band and the resulting coefficients are fed into *DC*-array.

- The predicted-to-be significant coefficients are fed into *pred*-array.
- The rest are fed into *res*-array.

Absolute magnitudes of these arrays are encoded separately by hierarchical enumerative coder [6], after bit plane decomposition. The sign bits of nonzero magnitudes are transmitted without entropy coding. Let us now introduce the algorithms for generating *pred* and *res* arrays.

2.3 Generating *pred* and *res*

Since the *L*'th subbands do not descend from any parent, all the coefficients in these bands are scanned into *pred* by the following region growing algorithm, adopted from [3]:

grow(i,j){

For all (k,l) in the neighborhood of (i,j), which have not been visited,

- *if (k,l) is insignificant, output a 0.*
- *else,*
    - *output the magnitude of (k,l),*
    - *grow(k,l)*

}

Scanning starts with raster order, and *grow(i,j)* is initiated for each significant coefficient encountered.

For the rest of the bands, all coefficients descending from a significant parent are scanned into *pred*, in the order of descending parent magnitudes, and *grow(i,j)* is initiated for each significant coefficient encountered. The rest of the coefficients are marked as predicted-to-be-insignificant, and they are scanned into *res*, in the order of closeness to the predicted-to-be significant ones.

These arrays are expected to be locally stationary, because the scanning order aims to collect the similar magnitude values together within the array. They are fed into hierarchical enumerative coder, after bit plane decomposition. In [5-6], it is experimentally shown that hierarchical enumerative coder performs better than arithmetic coder.

3. AN OVERVIEW OF WPT BASED CODING

Wavelet packet decomposition is achieved when the filterbank operation is iterated over all frequency bands at each stage, rather than over the low-low frequency band only. This leads to a full tree decomposition as depicted in Figure 4. The final decomposition structure will be a subset of that full tree, chosen by a best basis selection algorithm.

3.1. Best Basis Selection

The best basis selection algorithm starts with constructing the full subband tree grown to a certain depth. Each node of the tree is assigned to a cost value, by computing the cost function at the associated subband. Then, the tree is pruned by starting from the bottom most nodes to the root. Pruning works as follows for each node:

Let J denote the cost value.

- Compare the cost value of the current node, *J_cur,* to that of the sum of the children nodes, *J_children*.
- If *J_cur* > *J_children,* then *J_cur* = *J_children.*
- Else, prune the children nodes.

The WPT coefficients of the best wavelet packet basis will be obtained according to the subtree left after pruning.

3.2. Rate-distortion optimization

Rate-distortion optimization is controlled by the following Lagrangian function,

$$J(\lambda) = D + \lambda R, \tag{1}$$

where D refers to distortion, R refers to rate, and λ is the Lagrange multiplier controlling the trade-off between the distortion and the rate. In this work, D is measured as the MSE between the quantized and the unquantized transform coefficients, and R is measured as the number of nonzero coefficients. This choice of measure for R leads to a subop-timal solution, since the true bit rate we would like to minimize is the output of the entropy coder. However, it is not far beyond the optimal one, since the number of nonzero coefficients has a direct impact on the bit rate, and it is com-putationally much more efficient. Using the true bit rate in the best basis selection routine requires the execution of the encoding algorithm several times. The reader is referred to [8-9] for a detailed description of the rate-distortion op-timization.

4. A NOVEL EXTENSION OF DWT BASED CODING TO WPT BASED CODING

4.1. Analysis of WPT coefficients

Although the layout of WPT subbands is rather different from that of DWT, WPT coefficients exhibit similar intra-band and interband correlation as in the case of DWT [10]. Inside a subband, significant WPT coefficients are clustered around the edges and details. The locations of these clusters in a subband is more or less the same as the locations of these in another band with the same orientation. Hence, they can be predicted if the locations of nonzero band at a coarser scale is known. For this purpose, we will define a parent-children link between the WPT bands at consecutive depths.

4.2. A way to establish parent-children link for WPT coefficients

Consider a wavelet packet decomposition structure as in Figure 5.a. In that case, it is easy to depict a parent-children link across bands. However, for a structure as in Figure 5.b., there may be more than one way to do this, one of which is presented in [10].

To avoid defining a different link for different cases, we propose the following rule:

Let the sizes of the input image and the DC band be $M \times M$ and $N \times N$. Define $depth\_max = log(M/N)$. The *parent-children* link is established between the horizontal detail blocks is:

$(0 : iN\text{-}1 , iN : 2iN\text{-}1) \;–\; (0 : 2iN\text{-}1 , 2iN : 4iN\text{-}1)$,

for the vertical detail blocks:

$(iN : 2iN\text{-}1 , 0 : iN\text{-}1) \;–\; (2iN : 4iN\text{-}1, 0 : 2iN\text{-}1)$,

and for the diagonal detail blocks:

$(iN : 2iN\text{-}1 , iN : 2iN\text{-}1) \;–\; (2iN : 4iN\text{-}1 , 2iN : 4iN\text{-}1)$,

for $i = 1,\ldots, depth\_max\text{-}1$. Inverse WPT is applied to the parent block until the full resolution parent block is retained (See Figure 6). Then, forward WPT is applied to the retained block by using the decomposition basis of the descendant block. The layouts of the parent and the child are now the same. So, the parent-children link can be established in the same way as in the dyadic wavelet decomposition case, which is depicted in Figure 6.

4.3. The encoding algorithm for WPT coefficients

The encoding algorithm described in Section 2 can be used for encoding of WPT coefficients, once the best decomposition basis is chosen, and the parent-children link is established. The selected decomposition structure is encoded by quadtree algorithm.

5. EXPERIMENTAL RESULTS

Two sets of experiments were made. In all of the experiments, four 512*x* 512 images, *Lena, Barbara, Goldhill, Harbour,* were used for testing our algorithm. Wavelet decomposition is obtained by 7/9 biorthogonal filters [11]. In the first set, wavelet transform coding by our algorithm, EZW [1], SPIHT [2], and MRWD[3] are tested at various bit rates. EZW results are only available for *Lenna* and *Barbara* images. The results show the improvement obtained by the proposed algorithm over MRWD, *0.15dB* at *0.125bpp*, *0.06dB* at *0.25bpp*, *0.20dB* at *0.5bpp*, *0.22dB* at *1.0bpp*, on the average. Performance of the proposed algorithm is very close to that of SPIHT on the average, overperforming SPIHT *0.12dB* at *0.125bpp*, *0.05dB* at *0.25bpp*, *0.12dB* at *0.5bpp*, *0.02dB* at *1.0bpp*, on the average.

The proposed algorithm is applied to wavelet packet transform coefficients in the second set of experiments. WPT improves the performance for images containing patterns and textures significantly, as expected. In Figure 7 it is seen that some details which are not visible by DWT encoding is retained by WPT encoding, at the same bit rate.

For WPT based coding, the maximum decomposition depth is set to 5. The Lagrangian cost function in Equation (1) is used for the best basis selection. The PSNR results are tabulated in Table 1. Some of the WPT encoding results obtained by SFQ [9] coder are included for comparison. [9] provides two sets of experimental results, one for the optimum implementation, and the other for fast but suboptimal implementation. We include here these of suboptimal implementation, for a fair comparison. It is worth mentioning that, their suboptimal implementation achieves almost the same performance as that of the optimal one. Our results are competitive with SFQ coder, better for *Lena* at all bit rates, and better for *Barbara* at high bit rates.

6. CONCLUSIONS

A wavelet transform based image compression algorithm is introduced. The algorithm exploits the interband and intraband correlations of wavelet transform coefficients. The algorithm is extended for wavelet packet transform based image compression by proposing a way of obtaining a parent-children link across the wavelet packet decomposition bands. The algorithm is tested on different images, and it is seen that both the objective and perceptual results achieved by WPT are consistently better than these obtained by DWT. It is also seen that the algorithm has competitive performance among the most cited wavelet transform based image coders.

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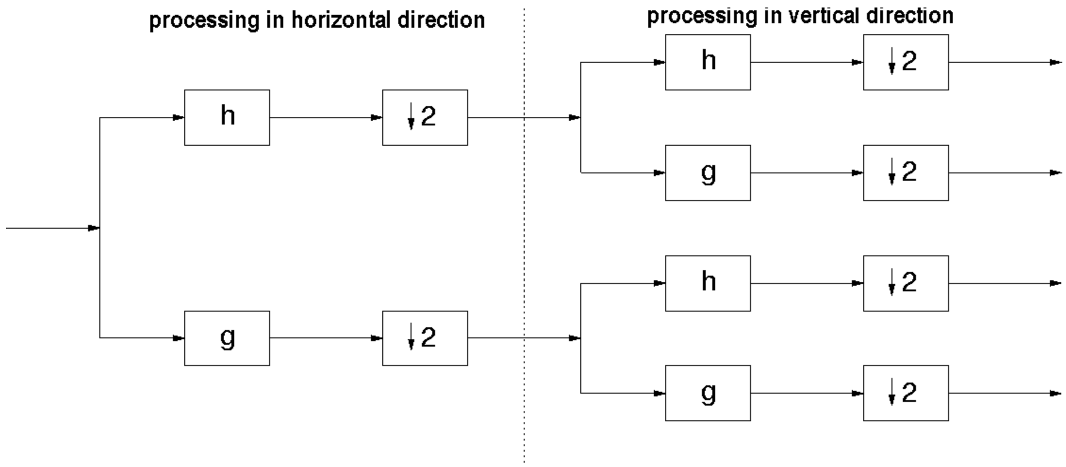

Figure 1. One stage filterbank implementation of a two-dimensional discrete wavelet decomposition.

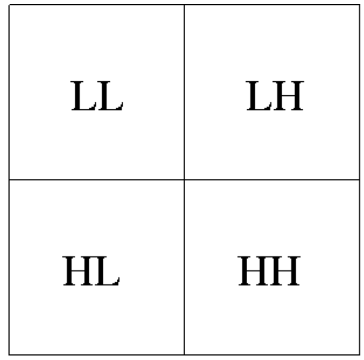

Figure 2. Layout of the one-stage DWT coefficients.

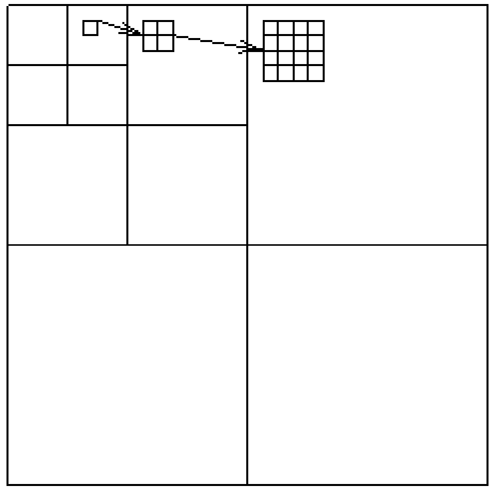

Figure 3. Parent-children link across the dyadic wavelet decomposition bands.

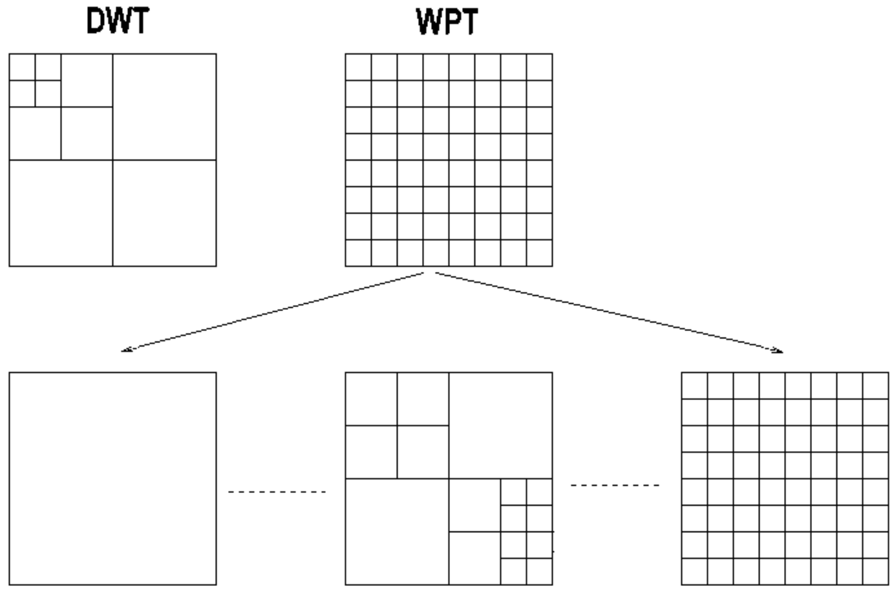

Figure 4. Decomposition bases offered by DWT and WPT.

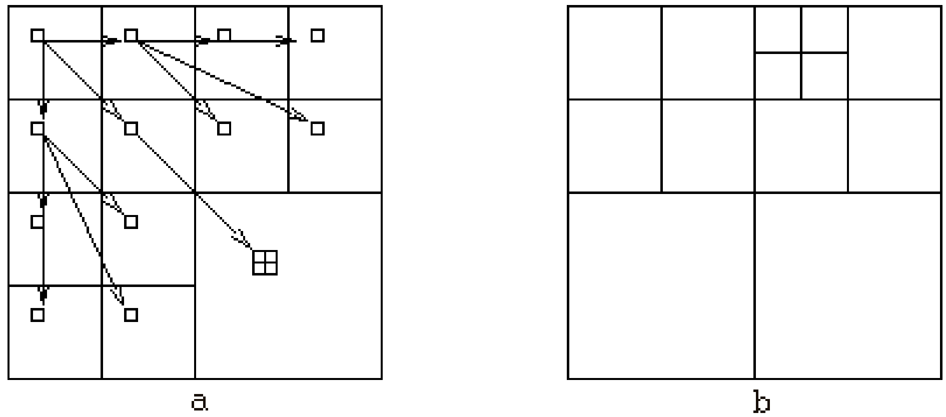

Figure 5. Examples of different decomposition structures.

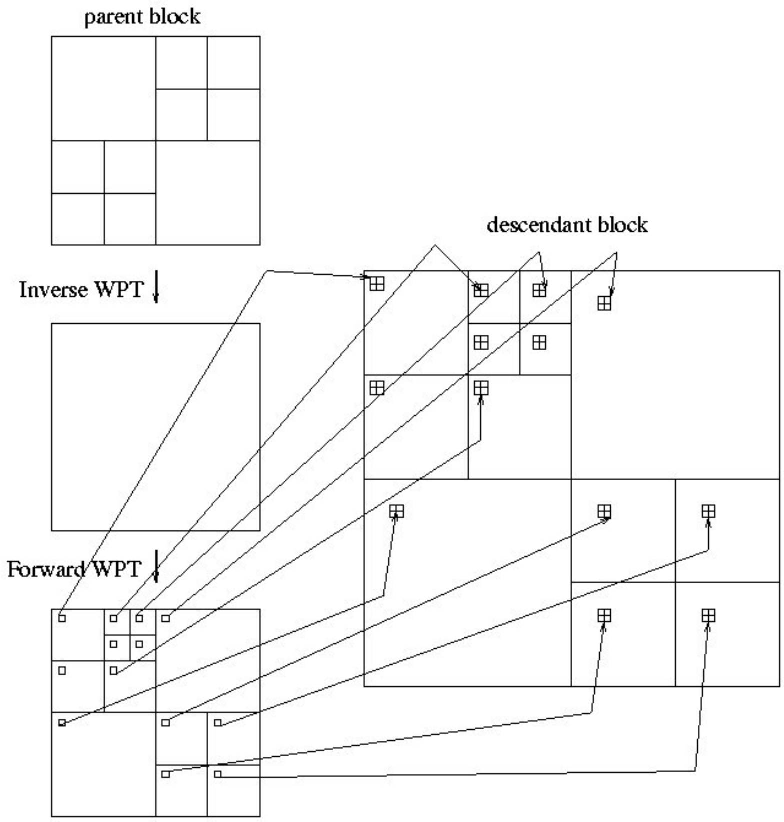

Figure 6. A way of linking two consecutive scales for an arbitrary WP decomposition.

		Lena	Barbara	Goldhill	Harbour
0.125 bpp	**DWT+SPIHT**	**31.10**	**24.86**	**28.48**	**24.61**
	DWT+MRWD	**31.09**	**24.99**	**28.47**	**24.38**
	DWT+proposed	**31.16**	**25.25**	**28.51**	**24.62**
	WPT+proposed	**31.36**	**25.56**	**28.70**	**25.30**
0.25 bpp	**DWT+SPIHT**	**34.11**	**27.58**	**30.56**	**26.80**
	DWT+MRWD	**34.12**	**27.78**	**30.53**	**26.58**
	DWT+proposed	**34.20**	**27.69**	**30.58**	**26.79**
	WPT+proposed	**34.51**	**28.52**	**31.56**	**27.40**
	WPT[9]	**34.35**	**29.25**	**N/A**	**N/A**
0.50 bpp	**DWT+SPIHT**	**37.21**	**31.40**	**33.13**	**30.01**
	DWT+MRWD	**37.17**	**31.38**	**33.15**	**29.71**
	DWT+proposed	**37.25**	**31.81**	**33.25**	**29.91**
	WPT+proposed	**37.54**	**33.05**	**33.71**	**30.47**
	WPT[9]	**37.40**	**33.12**	**N/A**	**N/A**
1.00 bpp	**DWT+SPIHT**	**40.41**	**36.41**	**36.55**	**34.64**
	DWT+MRWD	**40.33**	**36.03**	**36.56**	**34.28**
	DWT+proposed	**40.36**	**36.70**	**36.57**	**34.45**
	WPT+proposed	**40.80**	**38.03**	**37.14**	**35.02**
	WPT[9]	**40.55**	**37.69**	**N/A**	**N/A**

Table 1. PSNR(dB) versus bit rate results for DWT and WPT coded images.

a b

Figure 7. a. DWT based, b. WPT based encoded images, at 0.125bpp

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