Managerial Insights into the Effects of Interactions on Replacing Members of a Team

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March 7, 2001

Abstract

A mathematical model is presented for studying the effects of interactions among team members on the process of replacing members of a team in an organization. This model either includes or has the capability of including many, but not all, factors related to this problem. A main feature of the model is the ability to control the number of members that interact with each individual on the team. Through the use of mathematical analysis and computer simulations, it is shown how the amount of interaction affects the tradeoff between the expected performance and the number of replacements and interviews needed to find a good team using various replacement policies. New managerial insights into this process—such as the fact that it is not necessarily optimal to replace the team member with the worst performance—are provided.

1 Introduction

This paper provides managerial insights into the process of deciding how to replace members of a team. While the literature has focused primarily on other methods for improving team performance, such as training and feedback, managers can resort to intentional replacement of team members when other methods for improving performance do not achieve the desired results. Research has shown that the impact of hiring new members can be substantial—hiring the wrong person for a job can result in wasting between 40 and 70 percent of the salary for the position [see Cook (1998) and Hunter et al. (1990)]. While most selection research focuses on methods for choosing among candidates for a particular job [McDaniel et al. (1994)], little guidance is provided regarding how to identify the incumbent who needs to be replaced so as to optimize team performance. The ana lyses presented in this paper help to fill this gap.

The process of replacing employees is complex and there are limits on a manager's ability to construct the optimal team, for several reasons. There are time and dollar costs associated with both bringing in and interviewing potential new. Each new hire requires training time from the manager and the rest of the team. Qualified individuals may not be abundantly available or the cost of hiring them may be excessive in the current labor market. Biases can arise in assessing and comparing the performance of a current employee with the potential performance of a job candidate. Furthermore, teams are a particularly challenging setting in which to make replacement decisions because of the interdependence among different members of the team. The replacement of a team member affects morale for other team members and the new member can actually change the performance levels of those who remain because the remaining members cannot always rely on the new member for information or support that was provided by their former teammate.

Obtaining empirical data about the effects of different replacement policies on team performance can require a prohibitive time commitment. As Wittenbaum et al. (1998) point out, each membership change in a team creates a vulnerability, as prior modes of tacit coordination among team members may be disrupted. For this reason, a few months might be required for a new team member to settle in before team performance stabilizes and the need for an additional replacement can be assessed. As a result, tracking even one team through the process of sequentially replacing several different members could require years. Even greater research challenges arise if several members of a real-life team are replaced simultaneously—while such cases might arise for pragmatic reasons, this occurrence would make it very difficult to investigate the impact of each replacement separately. To obtain comparable data from a large number of teams would require surmounting challenges in measuring the performance of teams with varying responsibilities along some common metric and in controlling for the impact of exogenous factors on those teams during the time frame of the replacements. In naturally occurring teams, the effects of voluntary turnover on the team's performance might be confounded with the effects of intentional replacements. For all the foregoing reasons, the ability to simulate team performance is attractive. Therefore, this paper provides insights into the process of replacing team members through mathematical and computer analysis of simulated teams.

In this work, a combinatorial optimization model is proposed for studying how the amount of interaction among team members affects the tradeoff between team performance and the number of interviews and replacements needed to arrive at a good team, when using various replacement policies. Here, a replacement policy is a rule for determining the order in which to consider replacing individual team members. Major results from this work include the following:

1. The average performance of the final team obtained is independent of the class of policies tested here that each involves replacing one member at a time.

2. The average performance of the final team benefits from small amounts of interaction among team members but the benefit decreases as the amount of interaction increases, resulting in what is referred to here as the **interaction catastrophe**.

3. The obvious policy that replaces the team member whose individual performance is the least is not the best for teams with high levels of interdependence among members.

4. Policies that are designed to reduce the number of interviews and replacements needed to find a good team must take into account not only each individual's contribution to performance, but also the total contribution of all those affected by the individual

The remainder of Section 1 is devoted to a discussion of related literature on team performance measurement and ways to enhance team performance, including replacement policies for teams with varying levels of interdependence. Section 2 describes the proposed combinatorial optimization model. In Section 3, replacement policies are presented with an explanation of the need for those heuristics. Section 4 presents managerial implications of the simulation results regarding the various heuristics and their impact on performance in teams of varying size and amounts of interaction. A summary and directions for future research complete this article.

1.1 1.1 Team Performance Measurement and Improvement

As noted in the introduction, the group development literature has focused on methods other than replacements for improving team performance, such as training and feedback. Baker and Salas (1997) go so far as to assert that "effective team performance results from team members monitoring their own and other's performance, communicating with other team members, and providing feedback and back-up when needed." This research has valuable implications for team managers, including lessons about the appropriateness of different types of performance measures.

Research on team training and feedback emphasizes the importance of process-oriented measures of team performance, which assess characteristics of the team's dialogue and interaction along task and interpersonal dimensions. There are a variety of multidimensional conceptualizations of team process in the literature, including Cannon-Bowers et al. (1995), and Dickinson and McIntyre (1997). In the context of training and feedback interventions, process-oriented measures are more valuable than outcome-oriented measures because they permit more insights into the problems encountered by the team and possible ways of fixing them [Brannick and Prince (1997)].

However, there are drawbacks to existing process-oriented measures of performance. As Baker and Salas (1997) point out, theory development of process models has outpaced empirical validation of those theoretical frameworks. Brannick and Prince (1997) acknowledge that the literature has not yet reached a state of consensus regarding the appropriate dimensions of process to measure.

Furthermore, when managers encounter the limits of training and feedback interventions to improve performance, a process-oriented measure becomes less useful. Even when managers are able to avoid attribution errors [Jones and Harris (1967)] and can correctly identify the elements of team process that are causing undesirable performance in a team, they may find that the interventions at their disposal are not working in a particular case. If they have exhausted their range of possible team development interventions, then more detailed process measures to give them insights into the source of the problem are not particularly valuable. One logical alternative to further team development efforts is to adjust the composition of the team by replacing one or more members.

In such situations, obtaining an accurate output-oriented assessment of an individual team member's potential contribution to team, or of the potential performance of a team as a whole, can be challenging. As Brannick and Prince (1997) point out, outcome-oriented measures may contain variance attributable to other factors. Furthermore, if team members have specialized knowledge or abilities, the transaction cost for the manager to assess a team member's performance may be prohibitively high (see Eisenhardt (1989). The difficulty of assessing team members' prospective contributions may contribute to an inability to achieve optimal team performance.

The body of research that is most closely related to an examination of replacement policies is the research on group composition [for reviews, see Levine and Moreland (1998), and Moreland et al. (1996)]. In such studies, issues of team membership are central, although few studies of team composition examine the impact of changes in team membership over time. This research is more likely to assess the impact of variables such as team size or team diversity on outcome-oriented measures of team productivity (although process-oriented measures are sometimes considered in order to understand how those variables have their effects).

1.2 A Simple Replacement Policy and its Potential Drawbacks in Interdependent Teams

Even if a manager is able to assess prospective team performance accurately, finding the best team is a challenging task. It is unlikely that an efficient approach can be developed for finding the best team in general, especially as the number of team members increases. It becomes computationally infeasible to consider all the different possible configurations of team members from the available candidates and assess their potential performance. This conclusion is supported by the NP-completeness results derived in Solow et al. (2000) for the model described in Section 2. However, it is possible to develop polynomial algorithms for finding the globally optimal team in certain special cases, as presented in Section 2.3.

Outside of those special cases, the only practical alternative is to use policies or heuristics for replacing members of a team that, though not necessarily optimal, are expected to perform relatively well. In Section 3, a number of such policies are examined, both analytically and computationally. Assessments are made of their impact on performance and of the number of interviews and replacements needed to obtain a good team.

The simplest approach in practice is to replace one member of the current team at a time. In the case where the contribution of each team member is independent of the other team members, it is straightforward to replace the team member whose individual performance can be improved the most. This approach should also improve the performance of the team the most. A mathematical justification of this policy is provided in Section 4.

However, more complex replacement policies may be required for teams at other levels of task interdependence. Interdependence is a key concept in organizational studies. Tindale and Anderson (1967) argue that it is a central feature of naturally occurring groups. Thompson (1967) introduced three categories of task interdependence. *Pooled interdependence* involves the performance of tasks by each team member independent of any other team member. *Sequential interdependence* involves the performance of tasks by each team member, dependent only on the input of one other team member in a sequence of tasks. *Reciprocal interdependence* is characteristic of the case in which the contribution of each person depends not only on themselves but also on preceding and succeeding team members. Subsequently, Van de Ven et al. (1976) introduced a fourth category of interdependence they called *team arrangement*, in which the contribution of each person depends on every other team member. These categories can be sorted ordinally, from pooled interdependence at the lowest level to team arrangement representing the highest level. Research on these three or four rough types of task interdependence has shown that different managerial coordination modes and reward systems are optimal for teams of different types [see for example, Mitchell and Silver (1990); Saavedra et al. (1993); Shea and Guzzo (1989); and Van de Ven et al. (1976)].

This rough sorting may not capture all of the potential complexities that arise in real teams. Wageman (1999) recently called for a more nuanced method of rating a team's task interdependence. Similarly, Perlow (1999) presented descriptive evidence of multiple alternative forms of reciprocal interdependence, including the possibility that an individual team member's level of interdependence with others may vary within a team. The model described in Section 2 addresses complex interdependencies by allowing an individual's contribution to the performance of a team of N individuals to depend on K other team members, for any value of K between 0 and $N-1$. This parameter K enables us to model teams with varying levels of task interdependence. For example, $K = 0$ corresponds to pooled interdependence, $K - 1$ corresponds to sequential interdependence, and $K = N - 1$ corresponds to the team arrangement. The cases where K varies between 1 and $N - 1$ represent possibilities for related but distinct forms of reciprocal interdependence that have not yet been explored rigorously in research on team performance, but are examined in this paper. Furthermore, the model presented here also allows the exploration of cases where K varies for different team members. Such cases have only recently been acknowledged on interdependence in teams (for example, by Moreland et al. (1996), and have not been examined systematically.

In teams with higher levels of interdependence, the individual contributions to team performance may not be additive. Tziner and Eden (1967) documented this possibility in their study of tank crews. They assembled crews representing all possible patterns of ability levels and showed that there were interactive effects of members' individual ability levels on the performance of the team as a whole. Similarly, a series of studies in which leaders were rotated across groups showed that the leader's contribution to the group's performance varied depending on the composition of the group; this stream of research is reviewed in Moreland et al. (1996).

In such teams, a policy for replacing a team member that does not take into account the interactions may not be the most effective policy. For example, the member whose individual performance can be improved the most may have little impact on the performance of other members of the team. That is, replacing that team member might not improve the performance of the team the most. Similarly, the team member who seems to make the biggest individual contribution may actually be hampering the performance of other team members. Thus, replacing that team member might lead to improved performance. The computational results presented in Section 5 show that different replacement policies are desirable, depending on the level of interdependence in the team, the size of the team, and the relative costs of interviews and new hires.

2 A Model for Studying the Replacement of Team Members

In this section, a model is proposed for studying the process of replacing members of a team. In this section, a model is proposed for studying the process of replacing team members. Because the primary focus of this paper is to study the effects of interactions on the replacement process, a simplified model is first presented in which it is possible to control the number of members who interact with each individual on the team. It is then shown how several complicating real-world factors are included in the model, either explicitly or implicitly, or can be included by extending the model in various ways.

The simplified model is based on the following approach, in which the team consists of N job positions. Initially, suppose that only two qualified individuals are available for each position. Thus, each possible team is represented mathematically as a binary N-vector, $\mathbf{x} = (x_1, \ldots, x_N)$, in which $x_i = 0$ means that one of the two individuals is chosen for position i and $x_i = 1$ means that the other individual is chosen for that position (see Figure 1). Geometrically, each of the 2^N teams corresponds to an extreme point of the N-dimensional unit cube.

INSERT FIGURE 1 HERE

Each fixed choice of individuals for the N positions results in a team, $\mathbf{x}$, whose performance is modeled as a real number, $p(\mathbf{x})$, between 0 and 1. A value close to 1 indicates a team with relatively good performance and a value close to 0 indicates a team with relatively poor performance. The objective is to determine which person to choose for each position so that the resulting team has the best performance.

As discussed in Section 1.1, obtaining an actual performance value for a team in practice is challenging. The approach used here is that of the NK model—initially presented in Weinberger and Kauffman (1989) and developed more fully in Kauffman (1993)—because of the ability to control the amount of interaction among the team members. This model was developed for studying the biological evolution of chromosomes and is described now in the context of the process of replacing team members.

2.1 Kauffman's NK and Related Models

With the approach of the NK model, the person chosen for position i is assumed to contribute an amount, $p_i(\mathbf{x})$, to the overall performance of the team $\mathbf{x}$. The performance, $p(\mathbf{x})$, of the whole team $\mathbf{x}$ is then taken to be the average of these individual performance contributions:

$$p(\mathbf{x}) = \sum_{i=1}^{N} p_i(\mathbf{x})/N. \tag{1}$$

(Solow et al. (1999a) generalized the performance measure in (1) to a *weighted* average of the individual contributions.)

The remaining issue is how the performance contribution, $p_i(\mathbf{x})$, of the person chosen for position i is determined. In that regard, the NK model is designed to incorporate interaction among the individuals on the team through the use of an integer parameter K. The value of K $(0 \leq K \leq N-1)$ represents the number of other individuals that affect the performance contribution of the person chosen for position i. Thus, $K = 0$ indicates that the contribution of each person depends on no one else and $K = N - 1$ indicates that this contribution depends on all remaining $N - 1$ people on the team. More specifically, Kauffman assumes that the contribution, $p_i(\mathbf{x})$, of the person chosen for position i to the overall performance of the team $\mathbf{x}$ depends on that person and on K other people on the team—for example, the $K/2$ people on either side of the person in position i, wrapping around, when necessary. For a given value of K, there are 2^{K+1} possible choices for people in these $K + 1$ positions. Consequently, Kauffman defines the value of $p_i(\mathbf{x})$ to be one of 2^{K+1} uniform $0 - 1$ random numbers—the one that corresponds to the person in position i and the $K/2$ people on either side of the person in position i.

Kauffman's *NK model* consists of given values for N, K, the functions $p_i(\mathbf{x})$, for $i = 1, \ldots, N$, and the collection of all 2^N binary vectors—denoted as B^N here—together with their performance values defined by (1). The objective is to find a team $\mathbf{x}^*$ in B^N that has the best performance.

Numerous extensions, variations, and related models have been proposed in the literature. For example, Levinthal (1997) was among the first to recognize the ability to use the NK paradigm in a business environment to model the process of organizational change. In that article, an *organization* (the "team") is represented by a vector of N *attributes* (the "job positions"). These attributes might include the organizational structure, corporate strategy, personnel policies, and so on. For each attribute, he considered two alternative *versions* (the "candidates"). The contribution of each attribute to the overall fitness of the organization is assumed to depend on K other attributes. Additional uses of the NK model for studying business organizations can be found in the special issue of *Organization Science* (**10(3)**, 1999).

2.2 Assumptions of the Process of Replacing Team Members and Real-World Considerations

The objective of the work presented in this article is to provide general managerial insights into how the amount of interaction among team members affects replacement policies rather than to propose a model for use in a specific organization. To that end, the following assumptions are made for the version of the NK model used from here on.

1. There are two qualified individuals available for each position.

2. For a given value of the integer K with $0 \leq K \leq N-1$, the contribution to performance of the individual selected for position i depends on the team members in the K subsequent positions, wrapping around when necessary. Notationally, for each $i = 1, \ldots, N$, let

$$\mathbf{x}^i = (x_{w(i)}, \ldots, x_{w(i+K)}), \quad \text{where} \quad w(j) = \begin{cases} j, & \text{if } 1 \leq j \leq N \\ j - N, & \text{if } j > N \end{cases}$$

   So, p_i is a function of $\mathbf{x}^i$, that is, $p_i(\mathbf{x}^i)$.

3. For numerical computations, the specific values for $p_i(\mathbf{x}^i)$ are drawn independently for each $\mathbf{x}^i$ from a uniform $0 - 1$ probability distribution.

4. The performance of the team is the average of the individual contributions.

These assumptions provide the ability to obtain managerial insights from theoretical analysis and computer simulations. Regarding assumption (1), the same results hold when more than two individuals are available for each position. This situation is a special case of the NK model in which each position i on the team is represented by n_i components in the vector $\mathbf{x}$—one for each of the n_i candidates for position i on the team. For each of these n_i components in $\mathbf{x}$, a value of 1 indicates that the corresponding individual is chosen for position i on the team and 0's for the remaining $n_i - 1$ components indicate that those candidates are not chosen for position i on the team. However, the more the number of candidates, the harder it becomes to determine the effect of each one, in comparison to the other candidates, on the team as a whole. For this reason, in this study, only two candidates are considered for each position so that the results obtained are general and applicable to all organizational environments.

With regard to assumption (2), the model developed here is equivalent to one in which the contribution of the team member in position i depends on K_i others. Simply let $K = \max_{i=1,\ldots,N} K_i$ while disregarding the impact of those team members in the NK model that do not affect the contribution of the person in position i in the NK_i model. Furthermore, in the proposed model, it is assumed that the contribution of each person depends on the team members in the K subsequent positions. However, the case where these K positions are arbitrary can be viewed as a special case of $K = N - 1$, in which the effect of the team members that do not affect the contribution of a particular position are ignored.

Assumption (3) might not be appropriate for certain real-world situations. For instance, following an interview, different interviewers may attribute performance values that come from different distributions altogether. The assessment of performance values by an interviewer is further complicated by personality issues. For instance, compatible personalities between the interviewer and the candidate can result in a bias toward higher assessed performance contribution values. The attribution of such values can also be affected by multiple hires taking place at the same time and/or multiple candidates interviewed at the same time. The resulting correlations are difficult to quantify in such scenarios and is beyond the scope of this study. In the absence of empirical evidence that supports a specific distribution for determining the contributions of the individual positions, the uniform distribution is used to study the effect of interactions on replacement policies because the uniform distribution provides no prior bias on a candidate's effect on team performance.

According to assumption (4), equal weight is given to the contribution of each team member. This form of modeling the performance of the team as a whole is consistent with empirical outcome measures of productivity (or average output per team member) and is not intended to model the relationship between individual judgments and a team judgment on a decision-making task. The weighting of member contributions to such decision-making teams is beyond the scope of this paper and is thoroughly explored in Einhornetal (1997).

There are numerous situations where the group dynamics are modeled better with a weighted average of the performance contributions of the individual team members [such a variation of the NK model is discussed in Solow et al. (1999a)]. For instance, a particular team member's performance contribution can be significantly more important than others on the team and should therefore be given a correspondingly higher weight. As another example, the performance contribution of a new hire might be attributed a small weight because that team member lacks the experience of the other members, does not have a solid understanding of the group dynamics, his/her role and breadth of responsibilities in the group, and so on. Recent hires not yet adjusted when the next hire is made offer such examples. In other applications, contacts between a candidate and current team members can induce prior biases, both on the performance value attributed to the interactions of the candidate and his/her contacts as well as the weights attributed to these contributions.

The foregoing observations underscore the importance of using an appropriate weighting scheme for the contributions of the team members. Such schemes are group specific and can be developed in a consensus manner. In the absence of such a universally acceptable scheme, from here on, equal weighting of the individual contributions is assumed. This assumption facilitates the evaluation of the impact of the amount of interaction among team members, in isolation of the "noise" induced by a group-specific weighting scheme.

3 Policies for Replacing Team Members

Given the NK model presented in Section 2.1, the objective is to find a team with the best performance. In mathematical terms, the problem is the following:

> NK: Given integers $N > 0$ and K with $0 \leq K \leq N-1$ and polynomially computable functions $p_i(\mathbf{x}^i)$, for $i = 1, \ldots, N$, find a binary N-vector $\mathbf{x}$ in B^N so as to maximize $\sum_{i=1}^{N} p_i(\mathbf{x}^i)/N$.

As shown in Solow et al. (2000), for general N and K, the foregoing NK problem is NP-hard. This means it is unlikely that a polynomial algorithm can be developed for finding the best team. One viable alternative is to develop cost-and-time effective replacement policies that are expected to result in a team whose performance, though not necessarily optimal, is relatively good. Several such policies are described in Sections 3.1 and 3.2. To determine the quality of the team produced by those policies, it is necessary to find the performance of the best team. Theorems 1 and 2 that follow provide conditions under which it is possible to do so in polynomial time. These polynomial methods are used in the subsequent simulations to validate the policies. Proofs for all theorems in this article are given in the Appendix.

When $K = 0$, the performance contribution of each person i on the team $\mathbf{x}$ is determined only by that person, that is, $p_i(\mathbf{x}^i) = p_i(x_i)$. Finding a team with the best overall performance normally involves considering all 2^N possible teams. Theorem 1 reduces this to a polynomial amount of work by establishing that, when $K = 0$, the best team is found by identifying, for each job position i, the person, x_i, whose individual performance contribution, $p_i(x_i)$, is the best.

Theorem 1 *The NK problem is polynomially solvable when $K = 0$ because the team with the best possible performance is $\mathbf{x}^*$ in which:*

$$p_i(x_i^*) = \max_{x_i \in \{0,1\}} p_i(x_i), \quad \textit{for each } i = 1, \ldots, N.$$

Turning to the case when $K \geq 1$, the performance contribution of position i, namely, p_i, depends on the people in position i and the subsequent K positions of the team $\mathbf{x}$, wrapping around when necessary, that is, on $\mathbf{x}^i = (x_{w(i)}, \ldots, x_{w(i+K)})$. Theorem 2 provides a polynomial algorithm for finding the best team when K is fixed and independent of N.

Theorem 2 *For a fixed value of $K \geq 1$, the NK problem is polynomially solvable for any value of $N > K$ by solving 2^K shortest-path problems in an appropriate directed network.*

From Theorems 1 and 2, it is possible to determine the best team only when N is small—by explicitly checking all 2^N possible teams—or when K is small—by finding the shortest of 2^K shortest-path problems, as described in the proof of Theorem 2 in the Appendix. When N or K exceeds these limits, heuristics are needed to find an effective team with a reasonable amount of computational effort. Several such heuristic policies are proposed here, based on the following one used in Kauffman (1993).

3.1 Kauffman's Replacement Process

In the biological setting of genome evolution, Kauffman suggested a heuristic based on mutation. In the context of replacing team members, that heuristic proceeds as follows. Starting with an initial team, $\mathbf{x}$, a new team, $\mathbf{x}'$, is created by considering what happens if the person in position i is replaced with the other available person for that position—the *candidate*— resulting in what is called here a **one-replacement neighbor of $\mathbf{x}$**. The new team is retained only if $\mathbf{x}'$ has better performance than $\mathbf{x}$, that is, if $p(\mathbf{x}') > p(\mathbf{x})$. This **replacement process** results in a sequence of teams, each with better performance than its predecessor team, until obtaining a **local maximum**—that is, a team whose performance is greater than or equal to that of all its one-replacement neighbors.

Kauffman studied a specific version of this heuristic in which the new team $\mathbf{x}'$ is chosen randomly from among all the one-replacement neighbors of the current team $\mathbf{x}$ that result in better performance than $\mathbf{x}$. Because managers are concerned with the amount of time and money needed to build an effective team as well as the uncertainty created for team members who may be replaced, Kauffman's approach for choosing the new team $\mathbf{x}'$ is not practical for an organization. This is because his approach requires trying the replacement person for each of the N positions on the team, which is both expensive and time consuming since each such *trial*, in practice, requires an interview to determine if choosing that replacement will result in a team with better performance than the current team. Several alternatives for determining the order in which to consider replacing team members are proposed and analyzed now.

3.2 Myopic Policies for Determining the Order for Conducting Interviews

In general, when replacing a team member, it is necessary to determine who to consider replacing so as to achieve the best long-term results at the least cost. Doing so appears difficult because of the long-term considerations. A first step in that direction is to seek a myopic strategy that attempts both (1) to maximize the expected performance of the team with the selected candidate and (2) to minimize the expected number of interviews needed to find a single replacement that results in a better team. With these considerations in mind, several policies for determining the order of interviews and who to select are proposed now.

3.2.1 Myopic Policies that Do Not Take Into Account Interactions Among Team Members

If the manager's objective in replacing a single member of the current team is to obtain a new team with the greatest improvement in performance, then this goal is achieved by using the following policy:

Policy 1: Optimal Performance Policy (OPP)—Interview the candidate for each position of the current team $\mathbf{x}$ to identify the team $\mathbf{x}'$ with the best performance. If $p(\mathbf{x}') > p(\mathbf{x})$, then create the new team $\mathbf{x}'$. Otherwise, stop, the current team $\mathbf{x}$ is a local maximum.

As mentioned previously, the disadvantage is that OPP requires interviewing candidates in each of the N positions—a time consuming and expensive task, especially when N is large, as well as potentially demoralizing for the current team members. To address this problem, the manager might consider the following policy of selecting the first candidate interviewed who improves team performance:

Policy 2: First Come First Serve (FCFS)— Starting from the first position, sequentially interview the candidate for each position of the current team $\mathbf{x}$ in an attempt to find a team $\mathbf{x}'$ with performance $p(\mathbf{x}') > p(\mathbf{x})$. If successful, then create the new team $\mathbf{x}'$. Otherwise, stop, the current team $\mathbf{x}$ is a local maximum.

The FCFS policy has the benefit of requiring fewer interviews on average than OPP but will do so at the expense of team performance for each replacement. However, neither OPP nor FCFS takes into account the individual contributions of the members in the current team. When doing so, it is natural to consider replacing the person whose contribution to performance is the smallest. Indeed, in their recent article that models changes in organizational units, Morel and Ramanujam (1999) state that "In the organizational context, such rules are typically implemented by initiating changes in, or shutting down, the worst performing units." This strategy gives rise to the following policy:

Policy 3: Sorted First Come First Serve (S/FCFS)— Sort the people of the current team $\mathbf{x}$ in increasing order of their contributions to $p(\mathbf{x})$. Then, sequentially interview candidates based on this ordering in an attempt to find a team $\mathbf{x}'$ with performance $p(\mathbf{x}') > p(\mathbf{x})$. If successful, then create the new team $\mathbf{x}'$. Otherwise, stop, the current team $\mathbf{x}$ is a local maximum.

Observe that OPP, FCFS, and S/FCFS do not take into account interactions among team members. A myopic policy that does so is developed next.

3.2.2 A Myopic Policy that Takes Into Account Interactions Among Team Members

To incorporate interactions among team members, as reflected by the parameter K, note that the person in position i of the current team $\mathbf{x}$ affects the contribution of the K other team members in positions $w(i-1), \ldots, w(i-K)$ (where $w(j) = j+N$ if $j < 1$). To analyze this effect, let

$$
\begin{aligned}
q_i^K(\mathbf{x}) &= \text{the probability that replacing the person in position } i \text{ of the current team } \mathbf{x} \text{ results in a team with better performance}\\
&= \text{the probability that the sum of } K+1 \text{ i.i.d. uniform } 0-1 \text{ random variables, corresponding to the new contributions of the team members in positions } w(i), w(i-1), \ldots, w(i-K) \text{ after the replacement, exceeds the sum of the current contributions of the team members presently in those positions.}
\end{aligned}
$$

It is now possible to derive the probability distribution of obtaining a better team in exactly t interviews when interviewing candidates for each successive position.

Theorem 3 *Let $\mathbf{x}$ be the current team and suppose candidates are interviewed successively beginning with the first position. Then, for $t = 1, 2, \ldots, N$, the probability of needing exactly t interviews to find a replacement that results in a better team than $\mathbf{x}$ is*

$$\text{Prob}\{\text{requiring exactly } t \text{ interviews to find a replacement}\} = \prod_{i=1}^{t-1}(1 - q_i^K(\mathbf{x}))q_t^K(\mathbf{x}). \quad (2)$$

From the probability distribution in Theorem 3, it is possible to determine the order in which to interview candidates so as to minimize the expected number of interviews needed to find a replacement that results in a better team, as shown in the following theorem.

Theorem 4 *If the positions of the current team $\mathbf{x}$ are ordered so that $q_1^K(\mathbf{x}) \geq \cdots \geq q_N^K(\mathbf{x})$ then, for any other ordering of the N positions, say, $j_1, \ldots, j_N$,*

$$\sum_{t=1}^{N} t \left[\prod_{i=1}^{t-1}(1 - q_i^K(\mathbf{x}))q_t^K(\mathbf{x})\right] \cdot \sum_{t=1}^{N} t \left[\prod_{i=1}^{t-1}(1 - q_{j_i}^K(\mathbf{x}))q_{j_t}^K(\mathbf{x})\right].$$

The consequences of Theorem 4 for $K = 0$, $0 < K < N-1$, and $K = N-1$ are examined in more detail now.

The Stochastic Optimal One-Replacement Policy When $K = 0$. In the case when $K = 0$, the contribution of each person to the team depends only on that person. Thus, the probability $q_i^K(\mathbf{x})$ that the candidate for position i will result in a better team is simply the probability that the contribution of that candidate—which is a uniform $0-1$ random variable—exceeds the contribution, $p_i(x_i)$, of the person currently on the team, so,

$$q_i^K(\mathbf{x}) = 1 - p_i(x_i).$$

Then, according to Theorem 4, to minimize the expected number of interviews needed to obtain a replacement, the positions should be ordered so that $p_1(x_1) \cdot \ p_2(x_2) \cdot \ \cdots \cdot \ p_N(x_N)$. In other words, when $K = 0$, the myopic policy that minimizes the expected number of interviews needed to obtain a replacement is to consider replacing team members in increasing order of their contribution to performance—the S/FCFS policy already presented in Section 3.2.1. The S/FCFS policy has the added benefit of maximizing the expected amount of improvement in team performance for a single replacement because, for this ordering, the expected amount of improvement in team performance for each of the candidates is

$$\frac{(1-p_1(x_1))^2}{2} \geq \frac{(1-p_2(x_2))^2}{2} \geq \cdots \geq \frac{(1-p_N(x_N))^2}{2}.$$

The Stochastic Optimal One-Replacement Policy When $0 < K < N-1$. In this case, according to Theorem 4, the order that minimizes the expected number of interviews needed to obtain a replacement is in decreasing value of $q_i^K(\mathbf{x})$. It is unfortunately not easy to compute the values $q_i^K(\mathbf{x})$ because there is no closed-form expression for the sum of $K+1$ i.i.d. uniform random variables. However, there is no need to do so, as shown in the following theorem.

Theorem 5 *For each position $i = 1, \ldots, N$ of a given team $\mathbf{x}$, let*

$$\begin{aligned} \bar{p}_i &= \text{the total contribution of the person in position } i \text{ and all those} \\ &\quad\ \text{whose contributions are affected by that person} \\ &= \sum_{k=w(i-K)}^{w(i)} p_k(\mathbf{x}^k). \end{aligned}$$

If the positions are ordered so that $\bar{p}_1 \cdot \cdots\cdot\ \bar{p}_N$, then $q_1^K(\mathbf{x}) \geq \cdots \geq q_N^K(\mathbf{x})$.

Theorem 5 provides one of the most important contributions of this work, namely, that when $0 < K < N-1$, the optimal myopic policy must take into account the fact that the person in position i affects the contributions of the individuals in positions $w(i)$, $w(i-1), \ldots, w(i-K)$ by ordering the positions so that $\bar{p}_1 \cdot \cdots\cdot\ \bar{p}_N$. This is accomplished by the following policy:

Policy 4: Sorted First Come First Serve Based on K (SK/FCFS)—Sort the people on the current team $\mathbf{x}$ in increasing order of the total contribution of the person in position i and all those whose contributions are affected by that person. Then, sequentially interview candidates based on this ordering in an attempt to find a team $\mathbf{x}'$ with performance $p(\mathbf{x}') > p(\mathbf{x})$. If successful, then create the new team $\mathbf{x}'$. Otherwise, stop, the current team $\mathbf{x}$ is a local maximum.

The Stochastic Optimal One-Replacement Policy When $K = N-1$. In the case when $K = N-1$, the order in which one attempts to replace team members is irrelevant with regard to minimizing the expected number of interviews needed to find a replacement—any ordering yields the same expected number of interviews. This is because whenever a person on the team is replaced, the contributions of all team members change. Furthermore, in this case, each $q_i^{N-1}(\mathbf{x})$ has the same value, namely,

$$\begin{aligned} q(\mathbf{x}) &= \text{the probability that the sum of } N \text{ i.i.d. uniform } 0-1 \text{ random} \\ &\quad\ \text{variables (corresponding to the new contributions of all the} \\ &\quad\ N \text{ team members) exceeds } p_1(\mathbf{x})+\cdots+p_N(\mathbf{x}) \text{ (corresponding} \\ &\quad\ \text{to the contributions of all the current team members).} \end{aligned}$$

3.3 Heuristics for Determining the Order for Conducting Interviews When There Are Sequential Replacements

The myopic policies proposed in Section 3.2 for determining the order in which to conduct interviews so as to minimize the expected number of interviews needed to obtain a single replacement is not valid when there are sequential replacements. This is due, in part, to the fact that some of the candidates interviewed for the first replacement need not be re-interviewed for the second replacement, while others must still be interviewed. How this reduction in the total number of interviews is implemented with the various policies in Section 3.2.1 is described next.

3.3.1 Reducing the Number of Interviews When Using OPP

Recall the first policy, OPP, from Section 3.2.1, in which the candidate for each position in the current team $\mathbf{x}$ is interviewed. Suppose that, after these N interviews, the person in position i of $\mathbf{x}$ is replaced, resulting in the new team $\mathbf{x}'$. Of course the candidate for position i in the team $\mathbf{x}'$ was just on the team $\mathbf{x}$ and therefore need not be interviewed again when considering the next replacement. However, because the new person just hired for position i affects the contributions of the members in positions $w(i-1), \ldots, w(i-K)$, it is necessary to re-interview the candidates for those K positions to determine the second replacement, if any. Likewise, because the members in positions $w(i+1), \ldots, w(i+K)$ affect the contribution of the new person in position i, it is also necessary to re-interview the candidates for those K positions. All other positions in the team $\mathbf{x}'$ remain unaffected by the new person in position i and therefore the candidates for those positions need not be interviewed again for the second replacement.

In view of the foregoing discussion, after conducting the first N interviews and replacing the person in position i, the positions that need to be re-interviewed for the next replacement, the number of interviews needed for each subsequent replacement, and the total number of interviews needed to obtain a local maximum after r replacements is based on K, as follows:

K	Set of Positions to be Re-interviewed	interviews per Replacement	Total interviews to a Local Max.
$K = 0$	$\emptyset$	0	N
$0 < K < \frac{N-1}{2}$	$\{w(i-K), \ldots, w(i+K)\} - \{i\}$	$2K$	$N + 2K(r-1)$
$K \geq \frac{N-1}{2}$	$\{1, \ldots, N\} - \{i\}$	$N-1$	$Nr - r + 1$

3.3.2 Reducing the Number of Interviews When Using Sorting Policies

Whatever sorting policy (FCFS, S/FCFS, or SK/FCFS) is used to determine the order of positions in which to interview candidates, suppose that the person in position i of the current team is replaced after j candidates have been interviewed. In view of the discussion in Section 3.3.1 for OPP, of these j positions interviewed, the candidates in positions $\{w(i-K), \ldots, w(i+K)\} - \{i\}$ must be considered for another interview. In addition, all remaining candidates not yet interviewed must also be considered. The specific order in which this collection of candidates is interviewed for the next replacement is determined by the sorting procedure.

To be more specific, for the current team $\mathbf{x}$, let

$\pi \quad = \quad (\pi_1, \ldots, \pi_N)$ be the order in which the candidates are to be interviewed and

$R \quad = \quad \{1 \cdot \ i \cdot \ N$: the candidate for position i must still be interviewed$\}$
(initially, $R = \{1, \ldots, N\}$).

Then, π determines the sequence in which the candidates in R are interviewed, as follows:

$$(\pi_k : \pi_k \in R).$$

Suppose that, as a result of these interviews, the person in position π_j of $\mathbf{x}$ is replaced to form the new team $\mathbf{x}'$ and that, as a result of a sorting policy, candidates for the replacement in $\mathbf{x}'$ are to be interviewed sequentially in positions $\pi' = (\pi'_1, \ldots, \pi'_N)$. Then, the remaining set of positions for which candidates still need to be interviewed is:

$$R' = (R - \{\pi_1, \ldots, \pi_j\}) \cup (\{w(\pi_j - K), \ldots, w(\pi_j + K)\} - \{\pi_j\}) \tag{3}$$

The update formula in (3) takes on the following form for special values of K:

K	**Formula for Computing R'**
$K = 0$	$R - \{\pi_1, \ldots, \pi_j\}$
$0 < K < \frac{N-1}{2}$	$(R - \{\pi_1, \ldots, \pi_j\}) \cup (\{w(\pi_j - K), \ldots, w(\pi_j + K)\} - \{\pi_j\})$
$K \geq \frac{N-1}{2}$	$\{1, \ldots, N\} - \{\pi_j\}$

Many other policies are imaginable. Any such policy will create a new team $\mathbf{x}'$ by replacing a number of members in the current team $\mathbf{x}$. However, replacing many team members simultaneously can have a demoralizing and disruptive effect on the remaining team members. Furthermore, interviewing and finding groups of new team members repeatedly is impractical from a cost and time perspective. Nevertheless, to investigate the effects on team performance, the following policy in which up to two team members are replaced simultaneously is proposed:

Policy 5: The Two-Replacement Policy—Interview all candidates and all pairs of candidates in an attempt to find a team $\mathbf{x}'$ with performance $p(\mathbf{x}') > p(\mathbf{x})$. If successful, then create the new team $\mathbf{x}'$. Otherwise, stop, the current team $\mathbf{x}$ is a local maximum.

4 Results of Computer Simulations

For each policy in Section 3, simulations are used to estimate, for different values of N and K, the quality of a local maximum and the amount of effort needed to obtain that local maximum. Here, the "quality" refers to the performance of the team corresponding to a local maximum. The amount of "effort" is measured not by CPU time, but rather, by such statistics as the average number of interviews needed to find a single replacement and the average number of replacements and interviews needed to obtain a local maximum. In the real world, the number of interviews for each position and, as a result, replacement decisions, are affected by budget limitations, the personnel conducting the interviews, and the pool of available candidates. All of these factors affect the number of interviews preceding a new hire. For this reason, the number of interviews are used as a proxy for the aforementioned factors. Rather than finding the optimal team performance for a prespecified number of interviews, team performance is shown as a function of the number of interviews. This approach allows a manager to assess both the expected team performance for a given number of interviews and also the expected team performance if additional funds were available to interview more candidates. Also of interest to a team manager is the average percentage improvement in performance per interview. One conclusion from the results of these computational experiments is that the team manager should choose a heuristic based on which of the foregoing criteria is considered most important and on the value of K.

4.1 Results on the Performance of a Local Maximum

For each team of size $N = 8, 16, 24, 48$, and 96, and $K = 0, 2, 4, 8, 16, 24, 48$, and $N - 1$, with $K < N$, all policies were used to obtain a local maximum on each of 1000 randomly-generated problems. The results of these experiments are presented in Table 1. Based on these results, when $K = 0$, the global maximum of approximately 0.66 is identified by all heuristics, which matches the known theoretical value of 2/3 [see Kauffman (1993)]. For values of $K > 0$, the average performance of a local maximum obtained by the heuristics OPP, FCFS, S/FCFS, and SK/FCFS is virtually the same and below that of the two-replacement policy. This is expected as the latter heuristic searches through a larger set of replacements than in the former group.

INSERT TABLE 1 HERE

Another observation is that all heuristics exhibit the property that, for large values of N and small positive values of K ($0 < K < 8$), the expected performance of a local maximum exceeds the performance of 2/3 associated with $K = 0$. But then, as K increases, the expected performance of a local maximum decreases toward 1/2 (see Figure 2). This phenomenon of decreasing performance associated with increasing interaction is referred to here as the *interaction catastrophe* and was observed by Kauffman (1993) in what he called the *complexity catastrophe*.

INSERT FIGURE 2 HERE

Intuitively, the interaction catastrophe is due to a tradeoff that arises as K increases. The larger the value of K, the greater the number (2^{K+1}) of possible values for the performance contributions of each chosen individual, thus providing many choices for team members that could result in an individual's performance contribution being close to 1. However, as K increases, there are more potential conflicts between the chosen people in that the person chosen for position i is likely to benefit the performance contributions of some team members while being detrimental to the performance contributions of others. The simulation results obtained from the NK model indicate that for small positive values of K, the benefits of having more choices for the performance contributions of the chosen individuals outweigh the few potential conflicts. However, as K increases, the negative effects of the increasing number of potential conflicts dominate the benefits of having more choices for the performance contributions of the chosen individuals, thus resulting in no more than average overall performance for each person and hence for the team as a whole. A more formal explanation of the causes of the interaction catastrophe, and ways to attenuate it, are discussed in Solow et al. (1999b). The important conclusion, however, is that a small amount of interaction among team members is beneficial to team performance while too much interaction is detrimental.

From the next-to-last column in Table 1, it is seen that the additional benefit in team performance obtained by the two-replacement policy is between 2 to 5%. These results provide justification for the usual policy of organizations interviewing one candidate at a time. The rationale for this policy is to avoid both the cost of bringing in two candidates simultaneously and the associated scheduling problems. The current research indicates that even in the absence of such difficulties, the benefit to performance of interviewing multiple candidates simultaneously is not worth the additional effort. For these reasons, the two-replacement policy is not considered further.

To estimate the quality of the locally maximum team obtained from these simulations, the average performance of the best possible team for the 1000 problems is found by solving the 2^K shortest-path problems described in the proof of Theorem 2 in the Appendix. Given the amount of computational time and memory needed to solve these shortest-path problems, however, global maxima are obtained only for values of $K = 2, 4$, and 8. A comparison of the performances of locally maximum teams with the best possible team is shown in Figure 3. From these results, for small amounts of interactions, the performance of a locally maximum team is close to that of the best team, but the difference in these values increases as the amount of interaction among team members increases. This increasing difference is a direct consequence of the interaction catastrophe.

INSERT FIGURE 3 HERE

4.2 Results on the Effort Needed to Obtain a Local Maximum

The remaining discussion focuses on the heuristic policies because it is computationally infeasible to obtain the best team. Using the same 1000 problems, for each policy, the average number of replacements needed to obtain a local maximum is reported in Table 2. With this criterion, as seen in Figure 4, for virtually all values of N and K, OPP requires the fewest replacements, SK/FCFS requires the next fewest, S/FCFS requires the next fewest, and FCFS requires the most. These results are explained intuitively by the fact that the larger the amount of improvement per replacement, the fewer replacements should be needed to obtain a locally maximum team. Indeed, OPP, by design, attains the most improvement per replacement (see the discussion following Theorem 4 in Section 3.2.2) and should therefore require the fewest replacements. At the other extreme, the amount of improvement per replacement is the least with FCFS, which should therefore require the most replacements to obtain a locally optimal team.

INSERT TABLE 2 AND FIGURE 4 HERE

Also of interest to a team manager is the average number of interviews needed to obtain a local maximum. That information, for the same 1000 problems, is reported in Table 3 for each policy. With this criterion, for virtually all values of N and K, OPP (by design) and FCFS require the most interviews, S/FCFS requires fewer, and SK/FCFS requires the fewest. The latter is a result of the fact that SK/FCFS is the optimal myopic policy with respect to minimizing the expected number of interviews needed to find a replacement (see Theorems 4 and 5 in Section 3.2.2). It is interesting to note that for $N = 96$ and $K = 95$, the simulation value of approximately 167 interviews needed to reach a local maximum by FCFS agrees closely with the theoretical value of $1.78N = 171$ derived for FCFS by Macken and Perelson (1989) for the special case of large N and $K = N - 1$.

INSERT TABLE 3 AND FIGURE 5 HERE

Furthermore, the fact that for virtually all values of N and K, SK/FCFS dominates all other policies with respect to the number of interviews (see Figure 5) indicates that, with this criterion, a manager should decide the order in which to attempt replacing team members based not solely on each individual's contribution to performance, but rather, on the total contributions of all those affected by an individual.

In practice, due to restrictions on time and costs, team managers can conduct only a limited number of interviews and replacements. Figures 6(a)-(d) present the average percentage of improvement per interview in the first five replacements for each of the four policies. That is, for each replacement after the first one, the percent of improvement in performance from the previous team is computed and then divide by the number of interviews needed to obtain this replacement. The patterns in Figure 6 are representative of all values of K and N tested. Comparing Figures 6(a) and 6(b) ($N = 8$) with Figures 6(c) and 6(d) ($N = 96$), it is seen that smaller teams yield significantly greater initial improvement in performance than larger teams. Comparing Figures 6(a) and 6(c) ($K = 0.25N$) with Figures 6(b) and 6(d) ($K = 0.5N$), it is evident that teams with greater levels of interdependence yield slightly better performance improvements than teams with low levels of interdependence. In all cases, the results show that the SK/FCFS policy results in the greatest percentage of improvement in the first five replacements.

INSERT FIGURE 6 HERE

From these cumulative results, it appears that SK/FCFS offers a favorable tradeoff between team performance and the number of interviews required to reach a local maximum. Therefore, the statistics from the remaining simulations are reported only for SK/FCFS.

Additional observations from these simulation results indicate that, for a fixed value of N, the average number of replacements needed to reach a local maximum decreases exponentially with the number K of interactions [see Figure 7(a)]. This is because, the higher the amount of interaction, the more likely it is that a replacement will not improve team performance. Moreover, for a fixed value of K, the average number of replacements needed to obtain a local maximum increases linearly with N [see Figure 7(b)]. The patterns demonstrated for SK/FCFS in Figure 7 are representative of all heuristics and all values of N and K tested.

INSERT FIGURE 7 HERE

Similarly, Figure 8(a) demonstrates that, for a fixed large value of N, the total number of interviews needed by SK/FCFS increases rapidly as K increases from 0, then decreases gradually through a large range of values for K, and finally starts to increase again as K approaches $N-1$. On the other hand, for fixed K, the average number of interviews increases linearly with N [see Figure 8(b)].

INSERT FIGURE 8 HERE

The final issue addressed here is the number of interviews needed on average to find a replacement that results in a better team. As shown in Figure 9, starting with a team whose performance is relatively poor, only one or two interviews are needed to find a suitable replacement using SK/FCFS. However, as the team performance gets closer to that of a local maximum, the number of interviews needed increases slowly toward the value N, associated with verifying that a team $\mathbf{x}$ is a local maximum. Intuitively, this is because the better a team's performance, the less likely it is that a candidate will be able to improve the performance of the team. The pattern demonstrated in Figure 9 for SK/FCFS and $N = 96$ is representative of other values of N, as well as the other heuristics, except for OPP.

INSERT FIGURE 9 HERE

Conclusions and Future Research

This paper presents a model for studying the process of replacing team members that provides the ability to study the effect of the amount of interaction on various replacement policies. Tradeoffs between performance and the number of interviews and replacements needed to find a locally optimal team are considered. Mathematical analysis and computer simulations show that the interaction catastrophe of decreasing performance associated with increasing amounts of interaction arises for all policies studied.

Similar research on the effects of team size has been conducted, and some of the same tactics that are recommended for remedying the liabilities of large teams may help to mitigate the impact of high levels of interaction. Moreland et al. (1996) review some of these tactics, including planning, process consultation, restructuring, and goal setting. They point out that few researchers have studied the impact of these tactics. Future research in real teams could explore whether the detrimental effects of increasing levels of interaction could be offset by other managerial interventions into team process. Solow et al. (1999b) provide a number of theoretical approaches to overcoming the interaction catastrophe in the NK model, some of which might be applicable to teams.

Another key conclusion from this study is that, to attain desirable levels of performance with fewer interviews and replacements, it is necessary to take into account the performance of all those who are affected by an individual who may be replaced. For example, if interviews are costly (because of transportation costs, for example), and replacing an individual is inexpensive, then the SK/FCFS replacement policy is the best policy. With this policy, a manager should first seek a replacement for the team member who makes the least total contribution to the team, considering both the team member's individual performance and the impact on the performance of other team members. Alternatively, if interviews are inexpensive and replacing a team member is relatively expensive (because of training or other job-transition costs), then the OPP replacement policy should be the preferred strategy for improving team performance. With this policy, a manager should consider interviewing one potential replacement for each of the different members of the team before deciding which current team member to replace.

A third finding of this research is that interviewing for two positions simultaneously yields little additional benefit to team performance in our simulation results. The implication of this finding is that the current practice of replacing positions one at a time is not only easier from an administrative point of view, but also has negligible adverse effects on team performance.

One primary direction for future research is to adapt the model proposed here to specific organizational environments. Doing so necessitates differentiating the contributions of the individuals on the team, allowing different sets of team members to affect the contribution of each individual on the team, and possibly using different approaches to compute team performance. Some of these approaches are under active investigation.

A second key direction for future research is to validate the findings from the simulated teams in empirical settings. While it is possible that the simplifying assumptions in the model may limit the ability to generalize the results presented here, several of the simulation findings are consistent with past research in real teams. Examining the impact of replacing team members on repeated measures of team performance is necessary to further reinforce the validity of these findings.

Acknowledgments

The authors thank anonymous referees and the Associate Editor for encouraging us to write the paper in a manner that focuses less on technical details and is therefore more accessible to managers. We hope the current version has achieved that goal.

Appendix

Proof of Theorem 1. Let $\bar{\mathbf{x}}$ be a team that achieves the best possible performance. By definition, then, the performance of $\bar{\mathbf{x}}$ is at least as good as the performance of the team $\mathbf{x}^*$, that is,

$$\sum_{i=1}^{N} p_i(\bar{x}_i)/N \geq \sum_{i=1}^{N} p_i(x_i^*)/N \tag{4}$$

For the other inequality, note that, by definition of $\mathbf{x}^*$,

$$p_i(\bar{x}_i) \cdot \ p_i(x_i^*), \quad \text{for each } i = 1, \ldots, N.$$

Summing over i and dividing both sides by N yields that

$$\sum_{i=1}^{N} p_i(\bar{x}_i)/N \cdot \ \sum_{i=1}^{N} p_i(x_i^*)/N \tag{5}$$

The desired conclusion follows by combining (4) and (5).

The functions $p_i(x_i)$ are polynomial computable, so finding the team $\mathbf{x}^*$ in which

$$p_i(x_i^*) = \max_{x_i \in \{0,1\}} p_i(x_i), \quad \text{for each } i = 1, \ldots, N$$

can be done in polynomial time simply by evaluating, for each $i = 1, \ldots, N$, both $p_i(0)$ and $p_i(1)$ to determine which value is larger. Thus, the NK problem is polynomially solvable when $K = 0$. □

Proof of Theorem 2. It is now shown that the NK problem can be solved by finding the minimum of 2^K shortest-path problems in an appropriate directed network. To that end, consider a directed network that has $N+1$ columns of 2^K nodes each. Each of the 2^K nodes in one column corresponds to a binary vector $\mathbf{y} \in B^K$. Notationally, the complete set of nodes is

$$V = \left(v_{\mathbf{y}}^i\right)_{\mathbf{y} \in B^K}^{i=1,\ldots,N+1}.$$

The nodes of such a network for $N = 4$ and $K = 2$ are shown in Figure 10. For each $i = 1, \ldots, N$ and for each $\mathbf{y} \in B^K$, the node $v_{\mathbf{y}}^i$ is labeled with $\mathbf{y}$ and corresponds to $x_{w(i)} = y_i, \ldots, x_{w(i+K-1)} = y_K$. The nodes $v_{\mathbf{y}}^{N+1}$ are duplicate copies of $v_{\mathbf{y}}^1$.

INSERT FIGURE 10 HERE

For each $i = 1, \ldots, N$, a node $v_{\mathbf{y}}^i$ is connected by a directed arc to $v_{\mathbf{z}}^{i+1}$ if and only if $y_j = z_{j-1}$, for each $j = 2, \ldots, K$. In this case, define $\mathbf{x}^i = (y_1, \ldots, y_K, z_K) \in B^{K+1}$ and the distance for this arc is then $-p_i(\mathbf{x}^i)/N$. Because K is fixed, each p_i is polynomially computable, the foregoing directed network can be constructed in polynomial time.

Each directed path from $v_{\mathbf{y}}^1$ to $v_{\mathbf{y}}^{N+1}$ in the network gives rise to a team in which $x_1 = y_1, \ldots, x_K = y_K$. This is because the node $v_{\mathbf{y}}^1$ corresponds to $x_1 = y_1, \ldots, x_K = y_K$ and, such a path must go through an appropriate node $v_{\mathbf{z}}^i$, for each $i = 2, \ldots, N-K+1$, thus providing a corresponding value for x_{K-1+i}. The remaining nodes $v_{\mathbf{z}}^i$, for $i = N - K + 2, \ldots, N+1$, serve the purpose of wrapping around. For example, the bolded path in Figure 10 that begins and ends at the node labeled 10 corresponds to the team $\mathbf{x} = 1011$. Likewise, each team $\mathbf{x}$ gives rise to a directed path in the network from $v_{\mathbf{y}}^1$ to $v_{\mathbf{y}}^{N+1}$, where $y_1 = x_1, \ldots, y_K = x_K$. Furthermore, the total distance of any one of these paths is $-\sum_{i=1}^{N} p_i(\mathbf{x}^i)/N$.

In view of the foregoing discussion, a solution to the NK problem for a fixed value of $K > 0$ is found in polynomial time by performing the following steps:

1. Create the directed network consisting of $N+1$ columns, each containing 2^K nodes and the arcs between appropriate nodes in column i and nodes in column $i+1$, for $i = 1, \ldots, N$.

2. For each $\mathbf{y} \in B^K$, find a shortest directed path from $v_{\mathbf{y}}^1$ to $v_{\mathbf{y}}^{N+1}$. (Note that this step can be done in polynomial time using the reaching algorithm described in Ahuja et al. (1993) because K is fixed and the network has no directed cycles.)

3. Use whichever of the 2^K paths in Step 2 is shortest to construct a team $\mathbf{x}^*$ that has the best possible performance and check whether $\sum_{i=1}^{N} p_i(x_i)/N \geq C$.

The proof is now complete. □

Proof of Theorem 3. Consider interviewing the candidate for the first position on the team. By the definition of $q_1^K(\mathbf{x})$,

$$\text{Prob}\{\text{successful replacement after exactly 1 interview}\} = q_1^K(\mathbf{x}).$$

If, however, this candidate is not a successful replacement, which happens with probability $1 - q_1^K(\mathbf{x})$, then the probability of a success with the second candidate is $q_2^K(\mathbf{x})$, so,

$$\text{Prob}\{\text{successful replacement after exactly 2 interviews}\} = (1 - q_1^K(\mathbf{x}))q_2^K(\mathbf{x}).$$

More generally,

$$\begin{aligned}&\text{Prob}\{\text{successful replacement after exactly } t \text{ interviews}\} = \\ &\quad \text{Prob}\{\text{first } t-1 \text{ candidates do not result in a better team than } \mathbf{x}\} * \\ &\quad \text{Prob}\{\text{candidate } t \text{ does result in a better team}\} = \prod_{i=1}^{t-1}(1 - q_i^K(\mathbf{x}))q_t^K(\mathbf{x}).\end{aligned}$$

This completes the proof. □

Proof of Theorem 4. The theorem is clearly true for $N = 1$. Proceeding by induction on N, assume the theorem is true when the team has N positions. To prove the result when there are $N + 1$ positions, note first that by the choice of the ordering, $q_{N+1}^K(\mathbf{x}) \cdot \;\; q_{j_{N+1}}^K(\mathbf{x})$ and so $1 - q_{N+1}^K(\mathbf{x}) \geq 1 - q_{j_{N+1}}^K(\mathbf{x})$, from which it follows that

$$\frac{q_{N+1}^K(\mathbf{x})}{1 - q_{N+1}^K(\mathbf{x})} \cdot \;\; \frac{q_{j_{N+1}}^K(\mathbf{x})}{1 - q_{j_{N+1}}^K(\mathbf{x})} \tag{6}$$

The induction step now follows because

$$\begin{aligned}
&\sum_{t=1}^{N+1} t \left[\prod_{i=1}^{t-1}(1 - q_i^K(\mathbf{x}))q_t^K(\mathbf{x})\right] \\
&= \sum_{t=1}^{N} t \left[\prod_{i=1}^{t-1}(1 - q_i^K(\mathbf{x}))q_t^K(\mathbf{x})\right] + (N+1)\prod_{i=1}^{N}(1 - q_i^K(\mathbf{x}))q_{N+1}^K(\mathbf{x}) \\
&\cdot \sum_{t=1}^{N} t \left[\prod_{i=1}^{t-1}(1 - q_{j_i}^K(\mathbf{x}))q_{j_t}^K(\mathbf{x})\right] + (N+1)\frac{\prod_{i=1}^{N+1}(1-q_i^K(\mathbf{x}))q_{N+1}^K(\mathbf{x})}{1-q_{N+1}^K(\mathbf{x})} && \text{(ind. hyp.)} \\
&\cdot \sum_{t=1}^{N} t \left[\prod_{i=1}^{t-1}(1 - q_{j_i}^K(\mathbf{x}))q_{j_t}^K(\mathbf{x})\right] + (N+1)\frac{\prod_{i=1}^{N+1}(1-q_{j_i}^K(\mathbf{x}))q_{j_{N+1}}^K(\mathbf{x})}{1-q_{j_{N+1}}^K(\mathbf{x})} && \text{[from (6)]} \\
&= \sum_{t=1}^{N} t \left[\prod_{i=1}^{t-1}(1 - q_{j_i}^K(\mathbf{x}))q_{j_t}^K(\mathbf{x})\right] + (N+1)\prod_{i=1}^{N}(1 - q_{j_i}^K(\mathbf{x}))q_{j_{N+1}}^K(\mathbf{x}) \\
&= \sum_{t=1}^{N+1} t \left[\prod_{i=1}^{t-1}(1 - q_{j_i}^K(\mathbf{x}))q_{j_t}^K(\mathbf{x})\right]
\end{aligned}$$

The proof is now complete. □

Proof of Theorem 5. Let S_{K+1} represent the sum of $K + 1$ i.i.d. uniform random variables. It follows that if $\bar{p}_i \cdot \;\; \bar{p}_j$, then

$$q_i^K(\mathbf{x}) = \text{Prob}\{S_{K+1} \geq \bar{p}_i\} \geq \text{Prob}\{S_{K+1} \geq \bar{p}_j\} = q_j^K(\mathbf{x}).$$

In other words, if the positions are ordered so that $\bar{p}_1 \cdot \;\; \cdots \cdot \;\; \bar{p}_N$, then $q_1^K(\mathbf{x}) \geq \cdots \geq q_N^K(\mathbf{x})$, as desired. □

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$N=8$	K	OPP	FCFS	S/FCFS	SK/FCFS	2-Replace	Global Max
	0	0.666	0.666	0.666	0.666	0.666	0.666
	2	0.704	0.696	0.696	0.699	0.728	0.746
	4	0.702	0.692	0.695	0.698	0.728	0.771
	7	0.676	0.663	0.663	0.663	0.727	0.775
$N=16$	K	OPP	FCFS	S/FCFS	SK/FCFS	2-Replace	Global Max
	0	0.655	0.655	0.655	0.655	0.655	0.655
	2	0.708	0.703	0.702	0.704	0.735	0.744
	4	0.703	0.697	0.695	0.698	0.741	0.775
	8	0.688	0.679	0.679	0.681	0.725	0.788
	15	0.652	0.640	0.640	0.640	0.690	
$N=24$	K	OPP	FCFS	S/FCFS	SK/FCFS	2-Replace	Global Max
	0	0.665	0.665	0.665	0.665	0.665	0.665
	2	0.709	0.705	0.702	0.703	0.728	0.746
	4	0.707	0.702	0.700	0.701	0.743	0.775
	8	0.687	0.681	0.680	0.684	0.727	0.792
	16	0.654	0.649	0.649	0.651	0.688	
	23	0.631	0.623	0.622	0.622	0.671	
$N=48$	K	OPP	FCFS	S/FCFS	SK/FCFS	2-Replace	Global Max
	0	0.665	0.665	0.665	0.665	0.665	0.665
	2	0.708	0.705	0.701	0.704	0.732	0.743
	4	0.707	0.704	0.700	0.700	0.743	0.773
	8	0.688	0.688	0.683	0.686	0.730	0.796
	16	0.659	0.658	0.656	0.656	0.698	
	24	0.642	0.638	0.637	0.641	0.677	
	47	0.603	0.596	0.598	0.598	0.638	
$N=96$	K	OPP	FCFS	S/FCFS	SK/FCFS	2-Replace	Global Max
	0	0.664	0.664	0.664	0.664	0.664	0.664
	2	0.708	0.704	0.701	0.702	0.732	0.744
	4	0.705	0.704	0.697	0.700	0.745	0.773
	8	0.687	0.688	0.681	0.683	0.729	0.796
	16	0.660	0.663	0.655	0.658	0.701	
	24	0.643	0.643	0.639	0.642	0.680	
	48	0.613	0.610	0.610	0.613	0.643	
	95	0.580	0.575	0.575	0.575		

Table 1: Average Performance of Locally Maximum Teams.

K	OPP	FCFS	S/FCFS	SK/FCFS
Results for $N = 8$				
0	4.0	4.0	4.0	4.0
2	2.7	3.6	3.4	3.2
4	2.1	3.1	2.9	2.7
7	1.4	2.2	2.2	2.2
Results for $N = 16$				
K	OPP	FCFS	S/FCFS	SK/FCFS
0	7.8	7.8	7.8	7.8
2	5.6	7.8	7.1	6.6
4	4.1	6.6	5.8	5.2
8	2.8	5.0	4.3	3.8
15	1.6	2.9	2.8	2.8
Results for $N = 24$				
K	OPP	FCFS	S/FCFS	SK/FCFS
0	11.9	11.9	11.9	11.9
2	8.2	11.6	10.4	9.5
4	6.1	10.2	8.7	7.7
8	4.0	7.5	6.5	5.6
16	2.3	4.5	4.2	3.5
23	1.6	3.1	3.2	3.2
Results for $N = 48$				
K	OPP	FCFS	S/FCFS	SK/FCFS
0	24.0	24.0	24.0	24.0
2	16.8	23.8	21.0	19.7
4	12.5	21.4	17.8	15.8
8	8.3	16.7	13.4	11.5
16	4.8	11.3	9.1	7.6
24	3.4	8.1	7.0	5.8
47	1.7	4.0	3.9	3.9
Results for $N = 96$				
K	OPP	FCFS	S/FCFS	SK/FCFS
0	48.0	48.0	48.0	48.0
2	33.5	47.8	41.9	39.1
4	24.8	43.0	35.0	31.4
8	16.3	34.9	26.4	23.0
16	9.7	24.0	18.0	15.3
24	7.0	18.5	14.0	11.8
48	3.8	10.3	8.5	6.9
95	1.7	4.7	4.6	4.6

Table 2: Average Number of Replacements to Obtain a Locally Maximum Team.

K	OPP	FCFS	S/FCFS	SK/FCFS
Results for $N = 8$				
0	8.0	8.0	8.0	8.0
2	17.3	13.9	12.2	11.0
4	21.2	15.5	13.5	11.4
7	17.4	12.3	12.3	12.3
Results for $N = 16$				
K	OPP	FCFS	S/FCFS	SK/FCFS
0	16.0	16.0	16.0	16.0
2	35.1	29.8	24.7	22.3
4	45.7	37.4	28.3	23.5
8	55.9	39.0	29.8	23.4
15	39.5	26.4	26.6	26.6
Results for $N = 24$				
K	OPP	FCFS	S/FCFS	SK/FCFS
0	24.0	24.0	24.0	24.0
2	52.2	44.7	36.5	32.9
4	68.6	58.5	42.7	35.4
8	84.8	66.3	47.8	36.1
16	76.3	50.0	44.8	33.1
23	59.0	40.0	40.2	40.2
Results for $N = 48$				
K	OPP	FCFS	S/FCFS	SK/FCFS
0	48.0	48.0	48.0	48.0
2	105.4	91.4	73.3	66.9
4	139.1	124.5	86.1	71.4
8	173.3	159.8	96.8	73.2
16	198.1	170.2	102.5	72.4
24	206.0	146.7	102.7	69.4
47	125.5	82.9	84.5	84.5
Results for $N = 96$				
K	OPP	FCFS	S/FCFS	SK/FCFS
0	96.0	96.0	96.0	96.0
2	210.7	183.7	146.9	133.1
4	276.6	254.0	170.7	142.6
8	342.9	346.0	191.2	146.2
16	397.1	411.4	206.6	144.7
24	427.7	421.3	212.2	141.8
48	455.9	316.8	205.7	132.6
95	254.0	167.1	165.5	165.5

Table 3: Average Number of Interviews to Obtain a Locally Maximum Team.