An Approach to Recursive Equalization via White Sequence Estimation Techniques

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***Absfrucf*—A state space modeling approach to equalization, which is applicable to both finite, as well as infinite impulse response channels, is presented. The order of the state space model is typically much smaller thati the length of the impulse response. It is shown by analysis and simulation that the proposed approach has considerably less digit mean-square-error as compared to the conventional Kalman filter equalizers for much smaller model orders and, hence, computational efforts.**

I. INTRODUCTION

A SIGNIFICANT step in the advance of automatic equalization techniques has occurred with the introduction of the Kalman filtering technique in the adjustment of tap gains [11-[3]. A method has been proposed for formulating the state variable model for updating tap gains using standard estimation theory results [4]. However, even for the channels exhibiting intersymbol interference (ISI) over small to moderate number of bit intervals, the resulting tapped delay line (TDL) -length may be quite large. This situation arises because the TDL equalizer attempts to approximate the inverse of the channel transfer function. Fine expositions of equalizers are available in a review article by Benedetto and Biglieri [SI. Lawrence and Kaufmann [6] and Mark [7] have proposed a Kalman filter equalizer which requires the estimation of the channei impulse response for its implementation, rather than its inverse. For infinite length impulse response channels, the use of infinite impulse response (IIR) filters comes naturally to mind.

In this paper, a state space modeling approach to equalization is considered,. The method is based on the theory of estimating a white noise input sequence from the noise-corrupted output of a linear system, using the smoothing techniques proposed by Mendel [8]. It is shown by analysis and simulation that the proposed approach yields much superior performance as compared to the Mark's Kalman filter equalizer [7] for much smaller model order and hence computational effort.

11. PROBLEM FORMULATION

A general baseband linear time invariant digital communication channel, after sampling at data ratp, can be characterized by the following input-output relationship.

$$r(k)=\sum_m a(m)h(k-m)+n(k) \tag{1}$$

where $a(k)$'s are binary valued data symbols taking the values ± 1, $h(k-m)$ defined as $h(kT-mT)$ is a sample of the impulse response of the baseband channel, T is a symbol interval, and $n(k)$ is the sampled version of the white Gaussian noise process with zero mean and a standard deviation. Channel impulse response may be represented, in general, by

$$H(z)=\frac{\sum_{m=0}^{N-1}\alpha_m z^{-m}}{\sum_{m=0}^{N}\beta_m z^{-m}}. \tag{2}$$

Some straightforward algebraic manipulations yield the following state space, phase canonical representation of the system

$$X(k)=\phi X(k-1)+\Gamma a(k) \tag{3}$$

$$r(k)=HX(k)+n(k) \tag{4}$$

where

$$\phi=\begin{bmatrix} -\frac{\beta_1}{\beta_0} & -\frac{\beta_2}{\beta_0} & \cdots & -\frac{\beta_{N-1}}{\beta_0} & 0 \\ 1 & 0 & \cdots & 0 & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & \cdots & 1 & 0 \end{bmatrix} \tag{5a}$$

$$\Gamma^T=[1\ 0 \cdots 0] \tag{5b}$$

and

$$H=[\cdots \alpha_{N-1}]. \tag{5c}$$

The problem of estimating the data sequence $\{a(k)\}$ is now seen to be equivalent to that of estimating the random "excitation sequence" in a state equation.

111. WHITK NOISE ESTIMATOR EQUALIZER

The problem stated above, however, is a nontrivial one in estimation theory for the reasons given below and, heace, the Kalman filtering approaches used by Mark [7] and Coda@[11, etc., are not directly applicable for its solution. The difficulties arise due to the following reasons

1) Unlike standard estimation problems, which involve estimation of the state vector $X(k)$ from the noisy observations $r(k)$ where the standard Kalman filter would be directly used subject to the approximation mentioned below, the above formulation requires the estimation of the uncorrelated input sequence $\{a(k)\}$.

2) The observed process $\{r(k)\}$ is not really a Gauss-Markov process, since $\{h(k)\}$ is a binary valued independent sequence, as required for the use of a Kalman filter for the state estimation.

The "equalizer" proposed here is thus based on the theory of estimating a "white-noise input sequence" from the noise corrupted output of a linear system using smoothing techniques as proposed by Mendel [SI. The second problem

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outlined above causes the resulting linear receivers to be "suboptimum" (just as in the case of Mark [7] and others).

It may be noted that the state space model, (3)-(4), contains *a(/?),* rather than *a(k — 1)* on the right-hand side, so that the estimate *li(k/k)* is no longer zero, since

$$\hat{a}(k/k)\ 2\ E\{a(k)/r(l : k)\} \tag{6}$$

where

$$r(1 :I;) = \{r(1)\ r(2)\ * r(k)\} \tag{7}$$

depends on *r(k),* which in this case is not independent of *a(k).* Thus, it is possible in this case to obtain a filtered estimate of the input sequence *{a(k)}*. This, however, is rather a deceptive simplification since the estimate *d(k/k)* still requires both the filteted and smoothed solutions for the state vectors *Z(k/k)* and *X(k — l/k),* respectively, as outlined by Mendel [SI. In our case, the variance of the data is unity, and therefore using the procedures of [SI, we get

$$\hat{a}(k/k)=\frac{\alpha_0}{\Delta}\,(HP(k/k-1)H^T+\sigma^2)^{-1}(r(k)-H\hat{X}(k/k-1)). \tag{8}$$

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The resulting receiver structure is shown in Fig. 1.

It may be noted that the receiver shown has the flavor of an "estimate-feedback" equalizer discussed in [9]. Unlike these and the decision-feedback equalizers discussed in the literature, however, the quantities being linearly combined in the feedback loop are not necessarily either the past symbols or their quantized estimates, but an estimate *X(k/k — 1)* of the present state vector of the channel model based on past data. In any case, the net effect is similar, viz. subtraction from the present received sample *r(k),* the contribution *P(k/k — 1)* of the past symbols introducing the intersymbol interference. The difference signal *(r(k) — P(k/k — 1))* is obviously proportional to the present symbol value. This is further normalized by the factor $br_ol\sim_o(HP(k/k\ —)H^T + a^2)^{\prime 1}$ to reduce the variance due to the effects of the additive noise and the uncertainty in the estimation of the state vector, before feeding it into the quantizer to execute the binary decisions.

The variance of the estimator [see (S)] can be derived following the procedure of [8] to be given by

$$\psi(k/k) = 1-\frac{\alpha^2}{1}\ (HP(k/k- l)H^T+a^2)^{\sim l} \tag{9}$$

IV. MULTISTAGE SMOOTHED ESTIMATORS

Without going into the details, the following result is reproduced here from [SI for an I-stage smoothed estimator

$$\hat{a}(k/k+l-1)=\hat{a}(k/k+l-2) + N(k/k+l-l)P(k + l-1/k+l-2) \tag{10}$$

where I = 1, 2, • . .. The resulting equalizer is shown in Fig. 2. This equalizer has a tapped delay line associated with it, in addition to the recursive filter of Fig. 1. The filtering part of the system removes all the IS1 after the first postcursor, and the smoothing part will sum up weighted samples of the innovations,. This is analogous to the combinations of feed-forward linear equalizer and "estimate" or "decision" feedback equalizers to effectively compensate for IS1 due to precursors and postcursors, respectively.

V. SIMULATION RESULTS

Comparison of the multistage smoothing systems has been studied in detail in [SI. It is of interest here to consider the steady-state performance of the equalizer proposed in Section

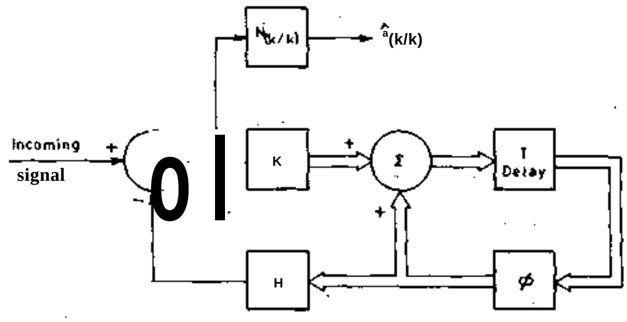

Fig. 1. A schematic diagram for equalization using a white noise estimator.

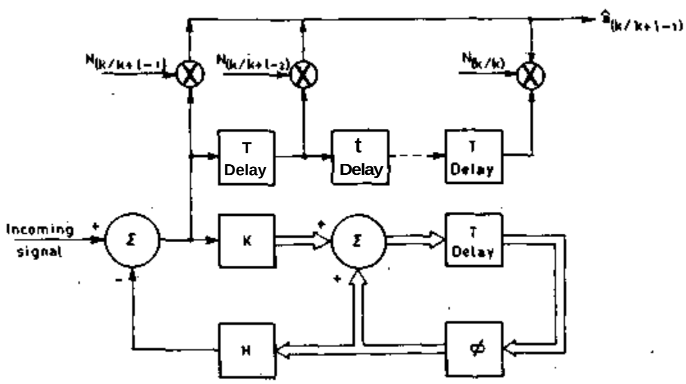

Fig. 2. Equalization using I-stage smoothed estimators.

111. In the steady state, it can be assumed that

$$\lim_{k\to\infty} P(k/k-1) = \lim_{k\to\infty} P(k/k) = P, \tag{11a}$$

which, in turn, gives

$$Ps = <i>[Ps-PsH^T(HP_sH^T+a^2)^{\prime l}HP_s]4>^T+TT^T. \tag{lib}$$

The steady-state mean-square error variance from (9) is then given by

$$\psi_s = 1-\frac{\alpha_0^2}{\beta_0^2}\,(HP_sH^T+\sigma^2)^{-1}. \tag{12}$$

For simplicity of calculations, a first-order low-pass channel is considered whose transfer function in the *z* domain is obtained by using impulse-invariance technique as

$$H(z)=\frac{1}{1-e^{-w_\beta}z^{-1}} \tag{13}$$

where *Wp = 2irfg,fg* being the 3 dB cutoff frequency in hertz. From (13), (2), and (lib), some algebraic manipulations yield the following result for the steady-state error covariance matrix

$$P_s=\begin{bmatrix} p_{11} & e^{-w_\beta}p_{11}\sigma^2(p_{11}+\sigma^2)^{-1} \\ e^{-w_\beta}p_{11}\sigma^2(p_{11}+\sigma^2)^{-1} & p_{11}\sigma^2(p_{11}+\sigma^2)^{-1}\end{bmatrix} \tag{14}$$

with

$$p_{11}=-\frac{\tau}{2}T(1.'+\overline{dl+4a^2/r^2}) \tag{15}$$

where

$$\tau=1-\sigma^2(1-e^{-2w_\beta}). \tag{16}$$

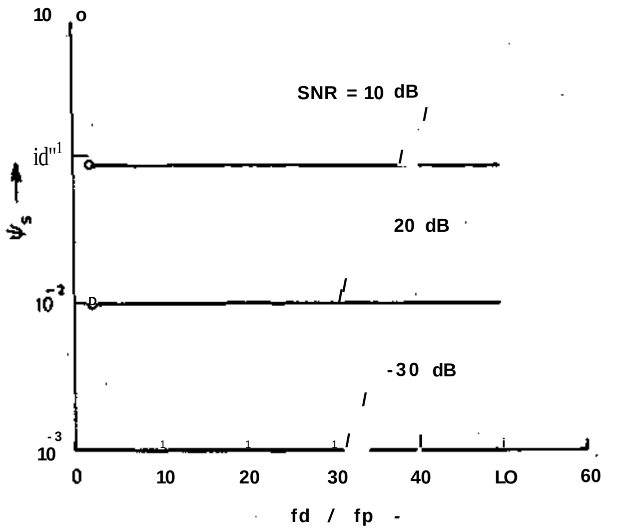

Fig. 3. Steady-state mean-square error for various data rates (analytical results).

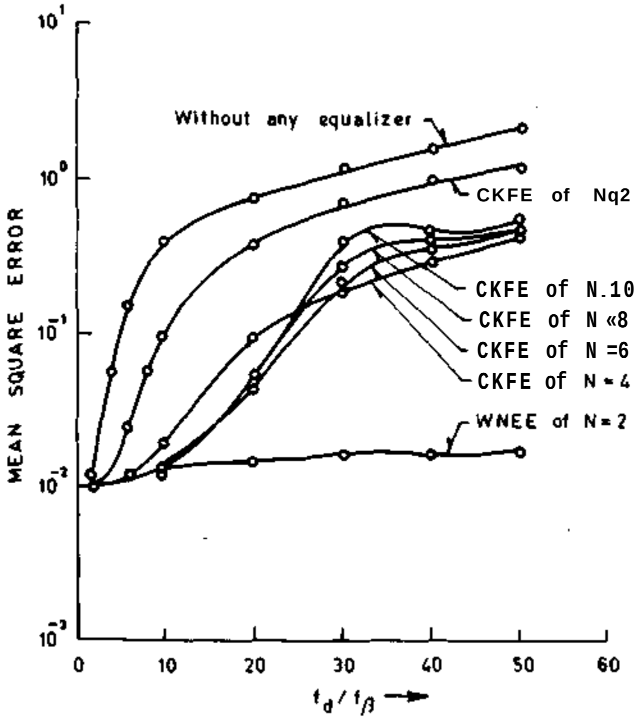

Fig. 4. Performance of WNEE and CKFE for a first-order low-pass channel.

The mean-square error G_S after equalization can be obtained from (12) and is plotted in Fig. 3 as a function of the ratio f_d/f_0, where f_d is the data rate and f_0 is the 3 dB cutoff frequency of the channel. The signal-to-noise ratio (SNR), defined here as $1/a^2$, is taken to be a parameter. It is obvious that the steady-state mean-square error increases with a decrease in the SNR but the effect of data rate is not significant upto a value of data rate 50 times that of the channel bandwidth.

Simulation experiments have also been conducted to verify the performance gain of the white noise estimator equalizer (WNEE) over that of the conventional Kalman filter equalizer (CKFE). The binary data sequence $\{a(k)\}$ was governed by quantization of independently generated sequence of uniform random numbers, and the additive Gaussian noise of the channel was assumed to have a standard deviation of 0.1. The

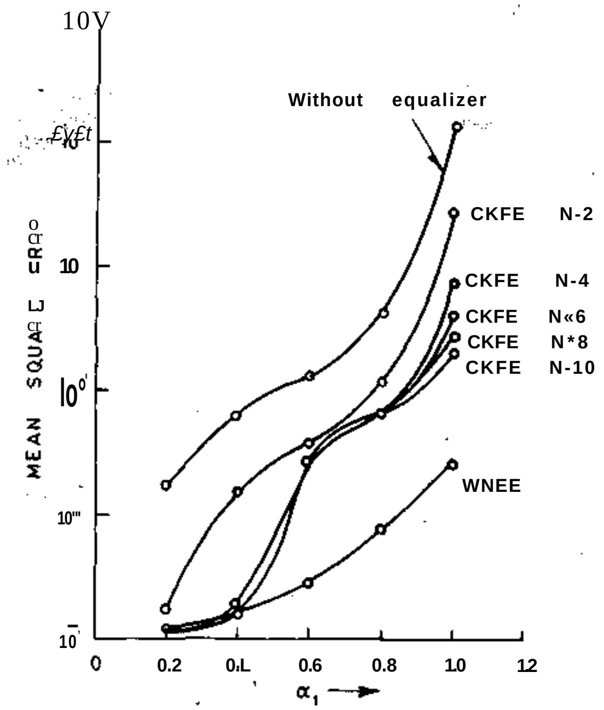

Fig. 5. Performance of WNEE and CKFE for a first-order all-pass channel with a zero at a_i.

mean-square error was calculated by averaging over 2000 values after steady state had been reached. The results have been plotted in Fig. 4.

It should be noted that for the example considered here, the conventional linear equalizer (which is inverse of channel impulse response) also will have only two taps. A more complex model for the channel having both poles and zeros was, therefore, taken up for simulation study. In this model, the channel was assumed to have an all-pass transfer function, with its zero as mirror image of the poles at $z = -a_i$, i.e., with

$$H(z) = \frac{1 - \alpha_1 z}{1 + \alpha_1 z}. \tag{17}$$

The results of the simulation are shown in Fig. 5 for various values of a lying inside or on the unit circle. It is once again demonstrated that the WNEE performs much better than the CKFE's of order up to ten.

VI. CONCLUSION

The performance of the WNEE has been found to agree closely with the theoretical result reported in Section V and presented in Fig. 3. While the CKFE is seen to yield improved performance as its order is increased, eventually converging to a constant value after a certain order, the WNEE with a single stage smoother is found to yield much superior performance with a filter of order two only. This demonstrates the significant computational (complexity) advantage over that of CKFE.

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