

Synthetic phase holograms for auto-stereoscopic image displays using a modified IFTA

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Abstract: A Fourier-transformed synthetic phase hologram for an auto-stereoscopic image display system is proposed and implemented. The system uses a phase-only spatial light modulator and a simple projection lens module. A modified iterative Fresnel transform algorithm method, for the reconstruction of gray-level quantized stereo images with fast convergence, high diffraction efficiency and large signal-to-noise ratio is also described. Using this method, it is possible to obtain a high diffraction efficiency (~90%), an excellent signal-to-noise ratio (> 9.6dB), and a short calculation time (~3min). Experimentally, the proposed auto-stereoscopic display system was able to generate stereoscopic 3D images very well.

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1. Introduction

There have recently been substantial efforts to develop three-dimensional (3D) display systems. A few of those techniques have focused on real-time full color auto-stereoscopic 3D displays, such as integral imaging [1,2], holographic stereography [3,4], partial pixel structures [5], and micro mirror array architecture [6]. Holography is the only 3D imaging technique that is capable of providing all the depth cues and can produce images with virtually unlimited resolution. However, the size and complexity of holographic fringe patterns for general digital holography often preclude their computation at interactive rates. Typical sampling sizes are smaller than the wavelength of visible light. Therefore, the cost of calculating samples is high if a conventional approach is taken. However, Fourier transform holograms (FTHs) provide an excellent design feasibility and make the most efficient use of the hologram space-bandwidth product (i.e., a minimal size of hologram is used to reproduce the object information). In addition, they have a robustness with respect to damage, such as localized defects and degradation of the hologram. In this paper, we propose, for the first time to our knowledge, and implement a Fourier-transformed synthetic phase hologram for an auto-stereoscopic image display system using a phase-only spatial light modulator (SLM) and a simple projection lens module.

The computer-generated hologram (CGH) or digital holography has a variety of advantages and applications including a diffractive optical element (DOE) [7], optical communication [8], optical information processing [9], and three-dimensional display [10]. CGH is a hologram that is computer-generated and fabricated by a semiconductor process or encoded by programmable real-time SLM [11,12]. Specially, phase holograms have been researched and developed because of their high diffraction efficiency, easy of fabrication, and flexibility for optical information processing. This design of a phase hologram satisfying the specified optical function has a combinatorial optimization problem. A number of optimization methods such as direct binary searching (DBS) [13], iterative Fourier or Fresnel transform algorithm (IFTA) [14-16], simulated annealing (SA) [17], and genetic algorithm (GA) [18] have been reported. However, the DBS and IFTA have the disadvantage of a local optimization problem. In addition, the SA and GA methods have weak points, i.e., they require a large computation memory and a long calculation time. As a result, we propose and introduce a modified IFTA method with a rapid convergence rate, no stagnation effect, and a

quasi-global optimization value. This modified IFTA is used in implementing a Fourier-transformed synthetic phase hologram for an auto-stereoscopic image display system.

2. Principle of the synthetic holography

Here we explain the principle of the Fourier-transformed synthetic phase hologram for 3D auto-stereoscopic image display using a phase-only SLM and a simple projection lens. The key idea is to combine and synthesize both the hologram with a directional function, such as a lenticular lens or an integral lens array and the CGH to permit a stereoscopic image to be shown in the Fourier plane. From the Fourier theorem, translation in the space domain introduces a linear phase shift in the frequency domain. The convolution of the two functions in the space domain is entirely equivalent to the operation of multiplying their individual transforms in the frequency domain [19]. Thus, multiple functions, such as directionality, focusing, and image reconstruction, can be combined and synthesized by each optimized phase hologram in the Fourier domain. The schematic diagram of our proposed auto-stereoscopic 3D display system using synthetic phase holograms is shown in Fig. 1.

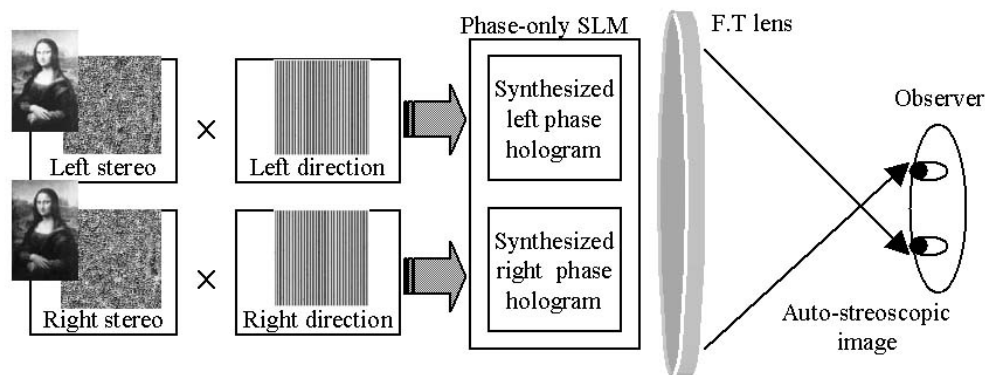


Fig. 1. Schematic diagram of the proposed auto-stereoscopic 3D display system.

In Fig. 1, the left stereo image is encoded by 2D phase-only information using our proposed modified IFTA, and it is synthesized by the CGH with left directionality. The right stereo image is processed in the same manner. Each synthesized phase hologram is then combined on a phase-only SLM. The phase-modulated information is simply Fourier transformed by a lens for an auto-stereoscopic image display to an observer. Due to the symmetry of the lens, synthesized phase holograms should be placed on a symmetric position from the center axis of the lens, or can be interlaced column by column (or row by row) in the SLM plane. For the 3D stereo depth effect, special attention must be paid to satisfy the stereo viewing conditions, such as the viewing distance, the viewer's eye separation, and the viewer's eye pupil size. In this paper, we assume that the stereo viewing conditions are defined as follows: the observer's viewing distance is 300mm, the observer's eye separation is 65mm, and the observer's pupil size is 3mm. We also used an illumination source with a wavelength of 532nm, designed synthetic phase holograms with a size of 256×256, and a phase-only SLM with a minimum pixel size of about 25μm. As a result, the calculated size of the reconstructed image in the image plane is about 16mm. These reconstructed stereo images must then be magnified by a projection lens module to satisfy the predefined auto-stereoscopic viewing conditions.

3. Modified IFTA

IFTA is an efficient and stable numerical design method derived from the scalar diffraction theory. Many practical variants of IFTA have been proposed and investigated [20-23]. All the methods consist of two stages, a stage involving the optical Fresnel or Fourier transform and a

stage where constraints are imposed in both the hologram plane and the image plane. For designing the proposed Fourier-transformed synthetic phase hologram, the IFTA should be considered carefully to impose constraints, such as the minimum pixel size of the phase-only SLM, viewing distance, and eye separation. Hence, we propose a modified IFTA method that can be used to reconstruct stereoscopic gray-level intensity images that satisfy the constraints and provide fast convergence, high diffraction efficiency, and large signal-to-noise ratio.

From the scalar diffraction theory, the light complex amplitude in the hologram plane (SLM plane) $W(u, v) = A(u, v) \exp[i\phi(u, v)]$ is related to the light complex amplitude $\hat{F}(x, y) = \hat{A}(x, y) \exp[i\hat{\phi}(x, y)]$ in the observation plane, in which the intensity distribution $I(x, y)$ is simply formed, via the square of the following Fresnel integral transform

$$\hat{F}(x, y) = \frac{ik}{2\pi z} \exp(ikz) \iint_{-\infty}^{\infty} W(u, v) H(u-x, v-y, z) du dv \quad (1)$$

where, z is the distance between the hologram and the observation plane, H is the transfer function, and $k = 2\pi / \lambda$ is the light wave number for the wavelength λ . In our proposed 3D system, z is replaced by a focal length of 300mm and λ is the applied wavelength of 532nm. Thus, the sampling space of the reconstructed image plane is calculated to be 6.4mm by the equation $N_{sp, min} = \lambda f / \Lambda_{SLM, min}$, where $\Lambda_{SLM, min}$ is the minimum pixel size (25 μ m) of the phase-only SLM. The problem of computing the hologram phase function $\phi(u, v)$ can be solved by the nonlinear integral equation and is calculated by an iterative method. Figure 2 shows the schematic diagram of the proposed IFTA method, which is computationally efficient in calculating the phase patterns.

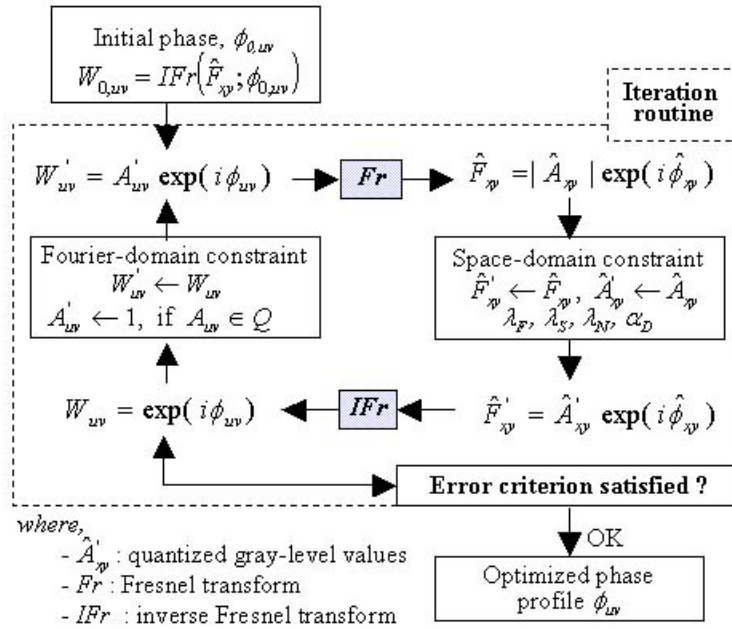


Fig. 2. Schematic diagram of the proposed IFTA method.

The modified IFTA involves the following steps:

- (a) An initial phase estimate $\phi_0(u, v)$ is chosen.

(b) The $A_0(u, v)\exp[i\phi(u, v)]$ function is integrally transformed using Eq. (1). $A_0(u, v)$ is the amplitude of the illuminating beam.

(c) The resulting complex amplitude $\hat{F}(x, y)$ in the plane of image formation is replaced by $\hat{F}'(x, y)$ by the rule

$$\hat{F}'(x, y) = B_0(x, y)\hat{F}(x, y)\left|\hat{F}(x, y)\right|^{-1} \quad (2)$$

where, $B_0(x, y) = \sqrt{|I_0(x, y)|}$ and $I_0(x, y)$ is quantized by predefined gray-level values of 2^5 .

(d) The inverse of the transform in Eq. (1) of the function $\hat{F}'(x, y)$ is taken

$$W(u, v) = \frac{ik}{2\pi z} \exp(-ikz) \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \hat{F}'(x, y) H^*(x-u, y-v, z) dx dy \quad (3)$$

(e) The resultant complex amplitude $W(u, v)$ in the hologram plane is replaced by $W'(u, v)$ according to

$$W'(u, v) = \begin{cases} A_0(u, v)W(u, v)\left|W(u, v)\right|^{-1}, & (u, v) \in Q \\ 0, & (u, v) \notin Q \end{cases} \quad (4)$$

where Q denotes the enclosure of the hologram aperture.

(f) Repeat (b)-(e) until the error between the intensity distribution of the reconstructed image and the target image is within a predetermined tolerance, or a predefined number of iterations has completed, i.e., this procedure is iterated until the error criterion is satisfied. The error criterion function is defined by Eq. (5) to obtain a high diffraction efficiency, a large signal-to-noise ratio, and a low uniformity error distribution between the reconstructed gray-level image and the original image.

$$\begin{aligned} \varepsilon_0 = & \lambda_F \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \left[\left| \hat{F}(x, y) \right| - B_0(x, y) \right]^2 dx dy + \lambda_S \int_S \int_S \left[\left| \hat{F}(x, y) \right| - B_0(x, y) \right]^2 dx dy \\ & + \lambda_N \int_N \int_N \left| \hat{F}(x, y) \right|^2 dx dy + \alpha_D \int_S \int_S \left[\left(\partial_x \left| \hat{F} \right| \right)^2 + \left(\partial_y \left| \hat{F} \right| \right)^2 \right] dx dy \end{aligned} \quad (5)$$

Note, however, that the procedure for the conventional iterative algorithm [14] convergence features a stagnation effect: in the course of several initial iterations the error ε_0 decreases rapidly, but subsequent iterations do not result in further significant reduction. In a previously proposed IFTA [15] and a regularized IFTA [21] method the issues of eliminating the stagnation effect and maximizing the diffraction efficiency of some binary-level quantized images have been discussed. In our modified IFTA, four terms are utilized to obtain a high diffraction efficiency, a large signal-to-noise ratio, a small noise figure, and a low uniformity error deviation of the gray-level quantized stereo images, as described in Eq. (5). The first term is the minimizing function of the square deviation of the reconstructed image amplitude from a pre-given value. As the diffraction efficiency approaches the objective, the error function decreases. The second term is also the square of the deviation of the reconstructed image amplitude from the desired amplitude within the signal region S . The third term, which is an integral over the region N (outside the signal region S), denotes the low noise figure and the last term denotes the low uniformity error between the reconstructed gray-level quantized image values as the second derivative of the diffracted image intensity. Through this modified IFTA, an improved performance with respect to gray-level quantized images is achieved, compared to previously proposed IFTA methods. To enhance the convergence rate and avoid the stagnation effect, the relaxation parameters (λ_F , λ_S , and λ_N) must be carefully controlled. The regularization parameter α_D is also optimized for smoothness control without reducing the diffraction efficiency of the reconstructed image. We used the parameters $\lambda_F = 0.5$, $\lambda_S =$

0.5, $\lambda_N = 1$, and $\alpha_D = 0.15$, respectively for this purpose. If the regularization parameter α_D is larger than 2, the modified IFTA is stagnated. The converging processes for the reconstruction error and the diffraction efficiency are shown in Fig. 3, in which our IFTA method and the DBS method are compared. We used a Mona Lisa image with a size of 128×84 and the quantized gray-level values of 2^5 . The size of the designed holograms is 256×256 . In Fig. 3(a) and (b), the left y-axis scale shows the converging process of the diffraction efficiency and the right y-axis scale shows the variations in the root mean square (RMS) as the iteration routines are increased.

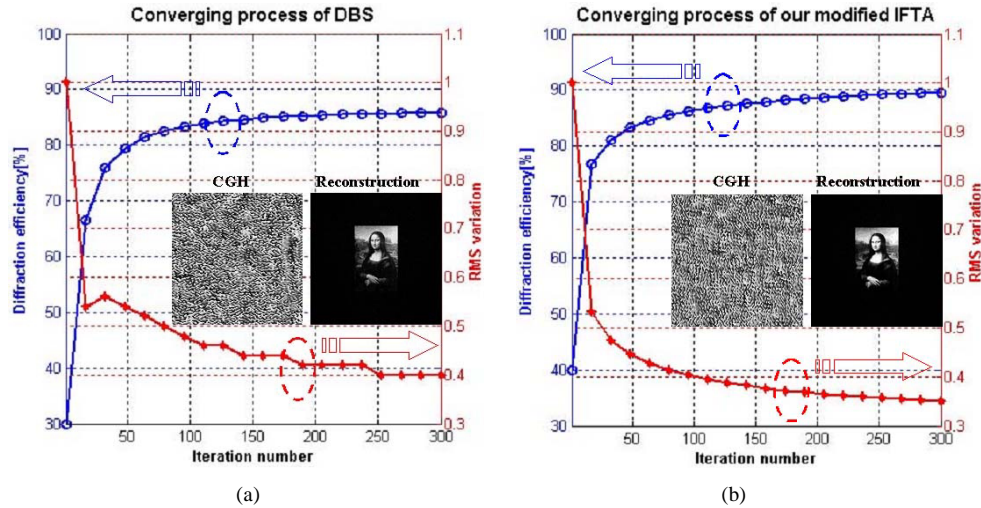


Fig. 3. Converging process for the diffraction efficiency and RMS deviation of the designed phase hologram (a) DBS method and (b) our proposed IFTA method.

As shown in Fig. 3, we can see that the proposed IFTA has not stagnated and has converged well. A high diffraction efficiency of 90.28% and a large signal-to-noise ratio of 9.68dB can also be obtained from the reconstructed image, while the DBS method has a diffraction efficiency of 85.89% and a signal-to-noise ratio of 7.84dB. A quasi-optimized phase hologram can be produced with a fast calculation time of 3 minutes. The detailed performance characteristics of the designed phase holograms are shown in Table 1.

Table 1. Simulated performance characteristics of the designed phase holograms

Performance characteristics	DBS		Modified IFTA	
	Left	Right	Left	Right
Diffraction efficiency [%]	85.68	85.89	90.60	90.28
RMS error	0.2609	0.4015	0.3123	0.3178
SNR [dB]	7.77	7.84	9.84	9.68
Computation time [sec]	157636	126912	197	190

Table 1 shows simulated performances for each stereoscopic Mona Lisa image and compares the proposed IFTA with the DBS method, such as diffraction efficiency $I_{DE} = 100\% \times \sum_S I_S / (\sum_S I_S + \sum_N I_N)$, finalized RMS reconstruction error, signal-to-noise ratio $SNR = 10 \times \log_{10}(I_S / I_N)$, and computation time. We used a personal computer with a Pentium4 2GHz CPU in this calculation. From the simulation results, we can see that the proposed IFTA has a much better performance than the DBS method in diffraction efficiency, signal-to-noise ratio, as well as computing time.

4. Simulation and experimental results for auto-stereography

For 3D stereoscopic image display, we used two crossed-view stereo images with quantized gray-level values of 2^5 , as shown in Fig. 4(a) and (b).

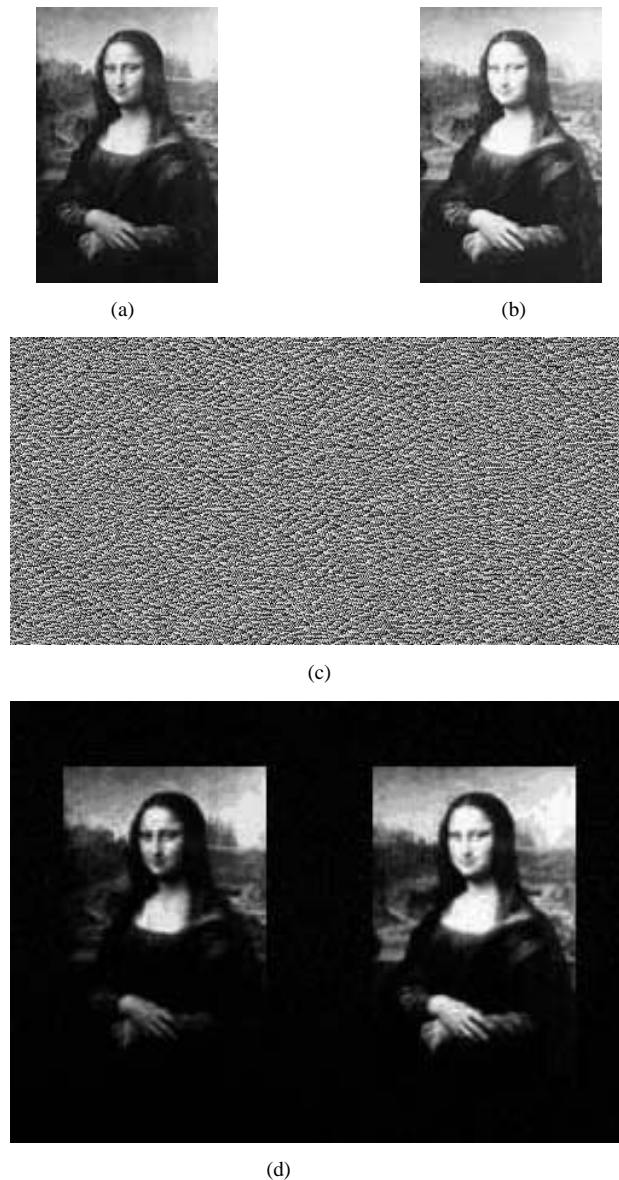


Fig. 4. Mona Lisa input images for (a) left view and (b) right view of the synthetic phase hologram, (c) combined synthetic phase hologram, and (d) reconstructed stereoscopic image (simulation) of the hologram (c).

Each size of the desired Mona Lisa image is 128×84 . The sizes of the designed synthetic phase hologram for stereoscopic image generation and the combined synthetic phase hologram are 256×256 and 256×512 , respectively. In Fig. 4(c), the different gray-levels represent different phase levels ranging from 0 (black) to 2π (white). Figure 4(d) shows the reconstructed stereo images (simulation) from the designed synthetic phase hologram. With the reconstructed stereo images of Fig. 4(d), one has the feeling of a stereo effect. In our

projection system, eye crossing is not needed because the reconstructed images for left and right eyes are immediately redirected to each of the observer's eyes.

For experimental verification of the proposed auto-stereoscopic image display with the designed synthetic phase holograms, we used a simple Fourier optic system using a phase-only SLM. The optical setup is shown in Fig. 5(a) and is first illuminated by a DPSS Nd:YAG laser with a wavelength of 532nm. The plane wave is modulated by the designed synthetic phase holograms through the reflective phase SLM (PPM X8267, Hamamatsu) and the diffracted beams from the reflective SLM produce the output stereo images at a charge coupled device (CCD) plane through a Fourier transform lens with a focal length of 300mm. The output image is photographed by a CCD (DT1100, MegaPlus) with an image capturing software program (IMAQ, NI). From the experimental results, as shown in Fig. 5(b), each stereo image is well defined. The bright spot on the optical axis is due to Fresnel reflections from the cover glass and the undiffracted beam arising from phase mismatch because of the nonlinear modulation transfer characteristics of the phase-only SLM [12].

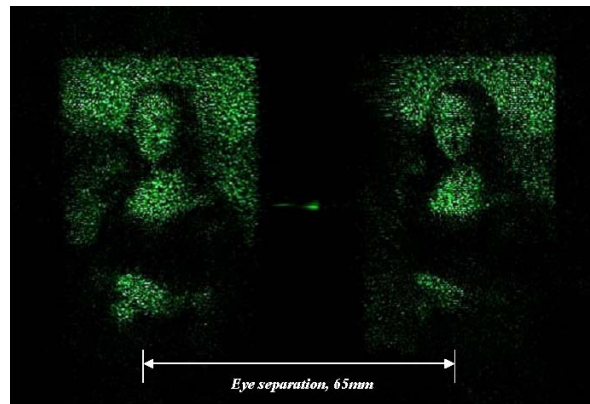
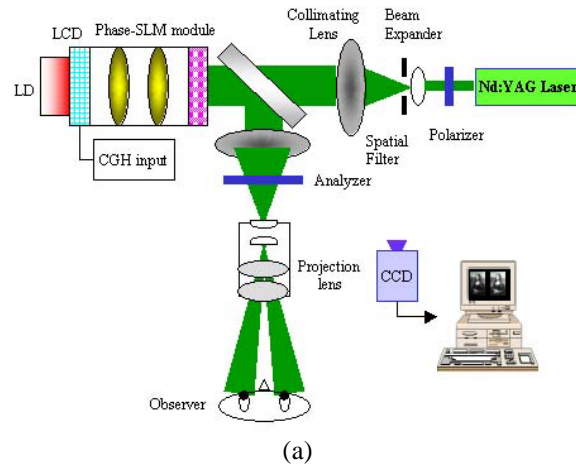


Fig. 5. (a) Experimental setup using a phase-only SLM and (b) reconstructed image.

We also used a projection lens module (EMP-811, Epson) to produce a real auto-stereoscopic 3D image at the observer plane. As a result, we were able to observe a real 3D image with a high depth effect. To reduce the speckle noise effect due to utilizing a high coherence laser as the illumination source, a transmission-type diffusing screen can be placed at the viewer plane or a partially coherent light source can be used. For full-color 3D image generation, the optimization of gray-scaled red, green, and blue components (composing a

full-color image) can be simply acquired by using a grating equation to reduce color dispersion among the holograms [24]. If color-optimized holograms are composed in the Fourier plane, it is possible to obtain superposed full-color stereoscopic images with binocular parallax.

4. Conclusion

In this paper, we proposed and implemented an auto-stereoscopic method that combines and synthesizes phase holograms for the reconstruction of gray-level stereo images. We also proposed and introduced a modified IFTA method with fast convergence without any stagnation effect. In a comparison of our proposed IFTA with the DBS method, we verified that our IFTA has a superior performance from the standpoint of diffraction efficiency, signal-to-noise ratio, and computing time. Using this method, we were able to obtain a high diffraction efficiency (~90%) and an excellent signal-to-noise ratio (> 9.6dB) in a short calculation time (~3min.). Finally, we experimentally demonstrated that our designed synthetic phase hologram was able to define the auto-stereoscopic image regions well through a simple optical setup with a phase-only SLM and a projection lens system. Hence we were able to observe real auto-stereoscopic 3D images with a high depth effect through the proposed synthetic phase holograms.

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