

# Intimations Of Relativity Relativity Before Einstein

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## 1. INTRODUCTION

In this paper we try to disentangle the development of two connected strands of thought, associated respectively with Maxwell and Poincaré, so as to throw light on several questions connected with the special theory of relativity. For example, who first used the word 'relativity' in physics in the sense we understand it today, meaning: in all physical experiments, whether mechanical or electromagnetic, only the relative velocity of a body can be measured, i.e. the notion of velocity of a body as such is a concept without physical importance? Which was the earliest experiment undertaken to determine the absolute motion of the earth through the hypothetical ether or space, if such putative motion had a physical consequence at all, and by whom? In short, what exactly were the contributions of various physicists before the publication of Einstein's monumental contribution of 26 September 1905 submitted for publication to *Annalen der Physik* on 30 June 1905?

While we were writing this paper, our attention was drawn to the highly illuminating discussion given by Miller [1981]. We have been able to use Miller's discussion to improve the paper; but his objective is not the same as ours.

## 2. USE OF THE WORD RELATIVITY

Although, according to the OED, the word relativity was first used in 1834 by the poet Samuel Coleridge, this was in a philosophical, not a physical sense. Coleridge was interested in science as a source for imagination, and this seems to have led him to German *Naturphilosophie* [Levere 1981]. Similar remarks apply to a reference to the *principle of relativity* in the

writings of the Scottish philosopher, Sir William Hamilton (died 1856).<sup>1</sup> It was apparently James Clerk-Maxwell who used the words ‘the doctrine of relativity of all physical phenomena’ in the sense in which we understand this doctrine at present, for the first time in his [1877]. But before we go into this, we might trace Maxwell’s intellectual development in so far as the notion of relativity of all physical phenomena is concerned.

### 3. MAXWELL’S EXPERIMENT

In 1864-7 Maxwell performed experiments to detect the effect of the motion of the earth on the sodium  $D_1$  and  $D_2$  Fraunhofer lines of starlight. He examined the effect of the Doppler shift (first order) due to the *relative* motion of the earth and the star, and separately considered the possible effect of the putative motion of the earth relative to the supposed ether. Although his experimental set-up could detect deviations as small as one-twentieth of the difference (in frequencies) between the spectral lines  $D_1$  and  $D_2$ , he concluded, ‘I have tried this experiment at various times of the year since the year 1864, and have never detected the slightest effect due to the earth’s motion.’ Since Fizeau and Ångström had observed a change in the plane of polarization and in the phenomena of diffraction respectively, according as the ray travels in the direction of the earth’s motion or in the contrary direction, Maxwell cautiously asserted ‘but the whole question of the state of luminiferous medium near the earth, and of its connection with gross matter is very far as yet from being settled by experiment.’

The set-up of the experiment and its theory (somewhat simplistic) are described in a letter of Maxwell’s included in an article published by the astronomer Huggins in his [1868]. A brief description with identical conclusions was given by Maxwell in his [1879] article on the ether in the *Encyclopedia Britannica*. Maxwell died in 1879 and it might be assumed that the article was written a few years before 1879, as the dates in the sequel would suggest.

The whole apparatus could be turned round so as to make observations in the direction of orbital motion of the earth and in a direction opposite to that of earth’s motion. A collimator, three prisms of  $60^\circ$  and a spectroscope were used for observation.

### 4. MAXWELL’S ETHER

In the *Brittanica* article Maxwell reflected on the nature of light and the ether. He begins by considering ethers that have been proposed and later rejected (as with Descartes ‘who made extension the sole essential property of matter, and matter a necessary condition of extension’, so that ‘the bare existence of bodies apparently at a distance was a proof of the existence of a

<sup>1</sup> Quoted by an anonymous reviewer in *Engineering*, 11 August 1916.

continuous medium between them'). Maxwell goes on to say, "The only ether that has survived is that which was invented by Huygens to explain the propagation of light.' The phenomenon of interference in which destruction of light takes place in some areas and reinforcement in others, led Maxwell to suppose that light is itself not a substance, but a process going on in a substance and travelling at a characteristic velocity of  $3.004 \times 10^{10}$  cm per sec. This he saw was evidence for the undulatory theory; and in connection with the further evidence on polarisation and its effect on interference he remarks that the rival theories (Fresnel supposed the wave to be a displacement perpendicular to the plane of polarisation, MacCullagh and Neumann in the plane) would be reconciled if the process were an electromagnetic one.

It is not clear why the discoverer of the electromagnetic wave nature of light did not consider the possibility that light was nothing but the oscillating electric and magnetic fields in empty space. However, quoting his rotating table experiments of 1864-7, Maxwell unequivocally stated in the article on ether, 'The experiment was tried at different times of the year, but only negative results were obtained', while the apparatus was turned round so that the direction of light received by the instrument was reversed for observations at any given time. Thus, whatever his theoretical ideas might have been, he was sure that his experiments of others showing positive results, saying, 'The writer was not aware that either of these very difficult experiments (Fizeau's and Ångström's) have been verified by repetition.'

But even his theoretical ideas gradually moved towards acceptance of relativity of all physical phenomena, and to these ideas we now turn.

## 5. MAXWELL'S TRANSFORMATION

It is not widely known that in the *Treatise*, [1873], Maxwell devoted two articles (no. 600 and 601) to the equations of 'electromotive intensity' when referred to moving axes. (Though Miller [1981] says (p.144), 'Maxwell referred to Faraday's discovery again and again, stressing that the direction and magnitude of the induced current depended upon only the relative motion of magnet and conductor. To substantiate this statement Maxwell extended Faraday's law to moving conductors and proved its Galilean covariance', adding in a footnote, 'Needless to say, this result was not exact.')

Using a method of transformation of coordinates and other quantities he yet came to the conclusion that 'the electromotive intensity (in the moving system) is expressed by a formula of the same type'. In view of the Lorentz invariance of Maxwell's equations, it is desirable to look more closely at what he does in these paragraphs, the more so since he appears to prove that 'in all phenomena....relating to closed circuits and the currents in them, it is indifferent whether the axes to which we refer the system are at

rest or in motion' in the general case of rectangular axes moving in any fashion (*i.e.* displacement of origin and rotation) relative to fixed axes.

We can disentangle Maxwell's argument better if we first consider the special case of translation alone, with velocity  $\mathbf{v}$  (say), and if we convert his equations into more modern notation. He argues in article 600 that, if  $(t, \mathbf{r})$ ,  $(t', \mathbf{r}')$  are the two reference frames, so that

$$t = t',$$

and

$$\frac{d\mathbf{r}}{dt} = \mathbf{v} + \frac{d\mathbf{r}'}{dt'}$$

and if we consider (as we may without loss of generality) the case when the two reference frames are initially coincident, then the time-derivatives of the magnetic potential  $\mathbf{A}$  (*assuming that  $\mathbf{A}$  has the same value in each system*) are related by

$$\begin{aligned} \frac{\partial \mathbf{A}}{\partial t'} &= \frac{\partial \mathbf{A}}{\partial t} + \frac{d\mathbf{r}}{dt} \cdot \nabla \mathbf{A} \\ &= \frac{\partial \mathbf{A}}{\partial t} + \mathbf{v} \cdot \nabla \mathbf{A} \end{aligned}$$

since  $\frac{d\mathbf{r}'}{dt'} = 0$ . But for any two vectors  $\mathbf{a}, \mathbf{b}$ ,

$$\nabla(\mathbf{a} \cdot \mathbf{b}) = \mathbf{a} \cdot \nabla \mathbf{b} + \mathbf{b} \cdot \nabla \mathbf{a} + \mathbf{a}_\lambda (\nabla_\lambda \mathbf{b}) + \mathbf{b}_\lambda (\nabla_\lambda \mathbf{a}),$$

so that

$$\frac{\partial \mathbf{A}}{\partial t'} = \frac{\partial \mathbf{A}}{\partial t} - \nabla \psi' - \mathbf{v}_\lambda \mathbf{B}_\lambda,$$

where  $\mathbf{B} = \nabla_\lambda \mathbf{A}$  and  $\psi' = -\mathbf{v} \cdot \mathbf{A}$ . But since the force on a current is  $\int \mathbf{F} \cdot d\mathbf{r}$ , where

$$\mathbf{F} = -\frac{\partial \mathbf{A}}{\partial t} - \nabla \psi' + \frac{d\mathbf{r}}{dt} \wedge \mathbf{B},$$

and for the moving axes

$$\mathbf{F}' = -\frac{\partial \mathbf{A}}{\partial t'} - \nabla(\psi + \psi') + \frac{d\mathbf{r}'}{dt'} \wedge \mathbf{B},$$

*subject to correct transformations for  $\mathbf{F}$ ,  $\psi$* , Maxwell's contention is established by noting that the gradient terms contribute nothing to the line- integral.

But what of the two assumptions in italics? It is obvious that not all vectors will have the same value in each system, since  $\frac{d\mathbf{r}}{dt}$  does not. In fact,

with hindsight we can see that Maxwell is really dealing with the first order form of the Lorentz Transformation. The coordinate transformation is the Galilean, to first order,

$$t' = t, \mathbf{r}' = \mathbf{r} - \mathbf{v}t,$$

but for a covariant vector like the 4-potential  $(-\psi, \mathbf{A})$  one easily calculates the first-order transformation

$$\psi' = \psi - \mathbf{v} \cdot \mathbf{A}, \mathbf{A}' = \mathbf{A}.$$

This justifies the first assumption. Now the second depends on the first order transformation

$$\mathbf{E}' = \mathbf{E} + \mathbf{v} \wedge \mathbf{B},$$

(a transformation with which Maxwell was implicitly familiar because of the force on a moving current).

Hence,

$$\begin{aligned} \mathbf{E}' &= -\frac{\partial \mathbf{A}}{\partial t} - \nabla \psi + \mathbf{v} \wedge (\nabla \cdot \mathbf{A}) \\ &= -\frac{\partial \mathbf{A}}{\partial t} - \nabla \psi + \nabla (\mathbf{v} \cdot \mathbf{A}) - \mathbf{v} \cdot \nabla \mathbf{A} \\ &= -\left[ \frac{\partial \mathbf{A}}{\partial t} + \mathbf{v} \cdot \nabla \mathbf{A} \right] - \nabla (\psi - \mathbf{v} \cdot \mathbf{A}) \\ &= -\frac{\partial \mathbf{A}}{\partial t} - \nabla (\psi + \psi') \end{aligned}$$

so that Maxwell's conclusion is correct and he has, in effect, deduced  $\psi \rightarrow \psi - \mathbf{v} \cdot \mathbf{A}$  from the expression for the Lorentz force. One may, however, wonder whether Maxwell was entirely clear about this last piece of argument, or whether he was convinced that the answer must be independent of the motion of the axes.

It remains to take up Maxwell's argument in the generality which he assumes, i.e., to take

$$\frac{d\mathbf{r}'}{dt'} = \frac{d\mathbf{r}}{dt} + \mathbf{v} + \boldsymbol{\omega} \wedge \mathbf{r}.$$

As Miller remarks [1981] footnote to p. 178), 'It is good to remember that most of Faraday's key experiments on electromagnetic induction concerned relative rotatory motion between magnet and conductor.'

The familiar Lorentz formulae are no guide here but, to the first order, it is easy to verify that the results for the transformations of  $\psi$ ,  $\mathbf{A}$  and  $\mathbf{E}$  are as before with  $\mathbf{v}$  replaced by  $\mathbf{v} + \boldsymbol{\omega} \wedge \mathbf{r}$ , so here again Maxwell's assumptions are justified in retrospect. There are two important qualifications in this case. Firstly, the transformations for constant  $\mathbf{v}$  are simply the linearised forms of

the corresponding (Lorentz) transformations between inertial frames; but the rotating case is not the linearised forms of such a known transformation since only one of the two frames can be inertial. Secondly, Maxwell's argument is couched in terms of the forces on a current; in the rotating frame this force, if it is defined by (mass)  $\times$  (acceleration) implies a suitably generalized concept of acceleration, incorporating the Coriolis and centripetal accelerations.

## 6. MATTER AND MOTION

Whether or not Maxwell was clear about relativity in the *Treatise*, four years later there appeared his [1877] *Matter and Motion*. The originality of this treatise on mechanics should not obscure the self-conscious following (with improvements) of the *Principia*. After further castigation in section 16 of Descartes, whose ether was called forth by his error in supposing space was the form of substance, he considers time in section 17: 'in its most primitive form... probably the recognition of an order of sequence in our states of consciousness.' He contrasts this in section 18 with Newton's absolute time and space; he seems to be persuaded of Newton's ontology, but he is an epistemological relativist: 'But as there is nothing to distinguish one portion of time from another except the different events that occur in them, so there is nothing to distinguish one part of space from another except its relation to the place of material bodies.' 'All our knowledge, both of time and place, is essentially relative.' 'Any one, however, who will try to imagine the state of a mind conscious of knowing the absolute position of a point will ever after be content with our relative knowledge.'

Indeed, in section 35, even acceleration is described as a relative term, and correspondingly in section 103 so is force: 'We cannot even tell what force may be acting on us; we can only tell the difference between the force acting on one thing and that acting on another.' So it is not surprising that, in the *Treatise*, he is prepared to claim invariance under both translations and rotations. And this gets its most complete expression in section 102, which is headed '*Relativity of dynamical knowledge*. Our whole progress up to this point may be described as a gradual development of the doctrine of relativity of all physical phenomena.' 'There are no landmarks in space; one portion of space is exactly like every other portion, so that we cannot tell where we are. We are, as it were, on an unruffled sea, without stars, compass, soundings, wind, or tide, and we cannot tell in what direction we are going. We have no log which we can cast out to take a dead reckoning by; we may compute our rate of motion with respect to the neighbouring bodies, but we do not know how these bodies may be moving in space.'

## 7. POINCARÉ PAPER

Was Poincaré aware of these earlier investigations of Maxwell? At any rate Poincaré picked up the matter where Maxwell left it. He too gradually

moved to the notion of relativity of all physical phenomena. Poincaré's conjectures of 1895 and 1899 that experiment could reveal nothing but relative motion; his use in 1899, in reviewing Russell's *Essay on the Foundations of Geometry*, of the 'law of relativity' and of the highly descriptive and precise phrase, 'The principle of relative motion', in 1900; his clear enunciation of 'The Law of relativity' (the word *relativité* having, it seems, appeared in French in 1805)<sup>1</sup> and 'The principle of relativity, according to which the laws of physical phenomena should be the same, whether for an observer fixed, or for an observer carried along in uniform movement of translation', his [1902], *Science and hypothesis*; and his assertion that 'there is no absolute time... Not only have we no direct intuition of the equality of two periods, but we have not even direct intuition of the *simultaneity* of two *events* occurring at two different places', in the same book; his method of synchronization of watches at rest within any inertial system and his observation that moving watches go slower, and his postulate that 'an entirely new mechanics (which) would be characterised by the fact that no velocity could surpass that of light', contained in an address given in September 1904; and his paper of July 1905 (published after Einstein's paper) in January 1906, have been well documented (Whittaker [1953] and others).

However, a paper of Poincaré [1905] delivered at the meeting of the Academy of Sciences in Paris on 5 June, has not received the attention it deserves, though it is mentioned by Miller [1981]. Appendix I contains an English translation of this paper, which was in elaboration of Lorentz's well-known paper of 1904 in which the Lorentz transformation equations were first correctly posited for systems moving at velocities less than that of light.

The following aspects of Poincaré's paper deserve special notice:

- (i) His clear realisation that it is impossible to demonstrate the absolute motion of the earth (i.e. relative to the ether); as a general law of nature.
- (ii) His statement that the transformation equations given by Lorentz (which Poincaré called the Lorentz transformation) show the impossibility of measuring absolute motion.
- (iii) His understanding of the fact that these transformations, together with spatial rotations, form a group, and his derivation from this of Lorentz's condition  $l = 1$ .
- (iv) The correct equations for transforming Maxwell's equations to moving coordinates were given for charge-occupied space. Poincaré did not elaborate the proof but the relativistic velocity-addition formula was apparently used implicitly.

1. *Grand Larousse de la Langue française*, Librairie Larousse, Paris [1977], vol.6. Paul Robert, *Dictionnaire Alphanumérique et Analogique de la Langue française*, vol. 5, Société du Nouveau Littré, Paris [1970]. A.Danzat et al., *Nouveau Dictionnaire Etymologique et Historique*, p. 640, Librairie Larousse, Paris (Relativité [1805], *Ann. Chimie.*) Reference to this Journal was not made in the 1938 edition of this same dictionary, in which it was traced to 1823 (Boistre).

- (v) Modification to the law of gravity to conform to Lorentz transformation was examined.
- (vi) It was conjectured that gravitational waves are propagated with the velocity of light.

Detailed notes on it are to be found in the Notes and References.

The authors wish to thank the referees of an earlier version of the paper for comments which have led to its substantial improvement.

## APPENDIX I

Meeting of 5 June 1905 Of Academy Of Sciences, Paris, On The Dynamics Of The Electron

Added: (Perhaps, the first English translation from French by the present authors.)

H.Poincaré

It appears at first sight that the aberration of light and other related optical phenomena would furnish us a means of determining the absolute motion of the earth, that is, its motion relative to ether rather than relative to the stars; there are no such phenomena. The experiments in which one takes account only of the first power of aberration have been unsuccessful, and one knows the reasons for that. But Michelson, having thought of an experiment in which one could measure effects depending on the second power of aberration, was equally unsuccessful. It appears that this impossibility of demonstrating the absolute motion of the earth is a general law of nature.<sup>1</sup>

An explanation has been proposed by Lorentz, who introduced the hypothesis of a contraction of all bodies in the direction of the motion of the earth. This hypothesis would explain the experiment of Michelson and all those which have been realised so far, but there would remain other, still more delicate, experiments, more easy to conceive than to execute, which could measure the absolute motion of earth. But if we regard the impossibility of such a verification as highly improbable, then one may predict that such experiments, if one ever succeeded in realising them, would also give a negative result. Lorentz has tried to modify his hypothesis so as to make it in accord with the hypothesis of *complete impossibility of measuring absolute motion*. He has succeeded in doing this in his article, *Electromagnetic Phenomena in a System Moving with any Velocity Smaller than that of Light* (Proceedings of the Amsterdam Academy, 27 May 1904).

The importance of the problem has made me take up the question again; the results that I have obtained agree on all the important points with those

1 As Miller says ([1981], p. 166): 'Poincaré, on the other hand, believed that the ether was real because it established relationships among sensations and this criterion of physical reality was central to his epistemology. He offered, in *Science and Hypothesis*, several other reasons for the necessity of an ether "...by means of unknown mechanisms of compensation, the ether reacts back upon a moving emitter...". The last reason led Poincaré to suggest a mechanical experiment for detecting effects of the ether; therefore, even though he believed that optical experiments would be unaffected by the earth's motion, he was still able to write that "the ether is all but in our grasp".'



of Lorentz; I have been led only to modify and complete them on some points of detail.

The essential point, established by Lorentz, is that the equations of electromagnetic field are not altered by a certain transformation (which I will call by the name of Lorentz) of the form:

$$x' = kl(x + \varepsilon t), y' = ly, z' = lz, t' = kl(t + \varepsilon x) \tag{1}$$

where  $x, y, z$  are the coordinates and  $t$  the time before the transformation and  $x, y, z'$  and  $t'$  after the transformation. Here  $\varepsilon$  is a constant which defines the transformation,

$$k = \frac{1}{\sqrt{l - \varepsilon^2}},$$

and  $l$  is an arbitrary function of  $\varepsilon$ .<sup>1</sup> One sees that in this transformation the  $x$ -axis plays a preferential role, but one can evidently construct a transformation in which this role would be played by any arbitrary line passing through the origin. The ensemble of all these transformations together with all rotations of space, should form a group; but for this it is necessary that  $l = 1$ .<sup>2</sup> One is thus forced to take  $l = 1$ , and this is a conclusion to which Lorentz was led by a different way.<sup>3</sup>

Let  $\rho$  be the electric density of the electrons and  $\xi, \eta, \zeta$  (the components of) its velocity before the transformation and  $\rho', \xi', \eta', \zeta'$  the same quantities after the transformation. Then:

$$\rho' = \frac{k}{l} \rho (1 + \varepsilon \xi), \quad \rho' \xi' = \frac{k}{l^3} \rho (\xi + \varepsilon), \quad \rho' \eta' = \frac{\rho \eta}{l^3}, \quad \rho' \zeta' = \frac{\rho \zeta}{l^3}. \tag{2}$$

These formulae differ somewhat from those found by Lorentz.<sup>4</sup>

Let  $X, Y, Z$  and  $X', Y', Z'$  be the three components of the force before and after the transformation (*the force refers to unit volume*); I find

$$X' = \frac{k}{l^3} (X - \varepsilon \Sigma X \xi), \quad Y' = \frac{Y}{l^3}, \quad Z' = \frac{Z}{l^3}. \tag{3}$$

These formulae also differ a little from those of Lorentz.<sup>4</sup> The complementary term  $\Sigma X \xi$  recalls a result obtained by M.Lienard.

1. Poincaré takes the velocity of light as unity.
2. The argument being, presumably, on these lines: Since rotations include  $x \rightarrow -x$ , it follows that  $l(\varepsilon) = l(-\varepsilon)$ . But since the inverse transformation belongs to the group, it follows that  $l(\varepsilon) l(-\varepsilon) = 1$ , so that  $l(\varepsilon) = \pm 1$  and  $l = 1$  follows from an (implicit) appeal to continuity.
3. Lorentz's different way is, in fact, a very roundabout one in which two different expressions for transformation of mass are compared.
4. Poincaré is, perhaps generous in finding anything comparable to his results for charge, current and force in Lorentz, since he works throughout in somewhat different terms. But certainly the transformations of charge and current seem to differ from the consequences of Lorentz's paper. The question of the force is more complicated, since it is possible to use different definitions of force.

If we now designate by  $X_1, Y_1, Z_1$  and  $X'_1, Y'_1, Z'_1$ , the components of the force related not only to unit volume but also to unit of mass of electron, we will have

$$X'_1 = \frac{k}{l^3 \rho'} \rho (X_1 + \varepsilon \Sigma X_1 \xi), \quad Y'_1 = \frac{\rho}{\rho' l^3} Y_1, \quad Z'_1 = \frac{\rho}{\rho' l^3} Z_1 \quad (4)$$

Lorentz was led to suppose that an electron in motion takes the form of a flat ellipsoid; this is also the hypothesis made by Langevin, but whereas Lorentz supposed that the two other axes of the ellipsoid remain constant, which is in accord with his hypothesis  $l = 1$ , Langevin supposed that it is the volume which remains constant. The two authors have shown that these hypotheses agree with the experiments of Kaufmann, as well as the old hypothesis of Abraham (spherical electron). The hypothesis of Langevin would have the advantage of being self sufficient, for it suffices to regard the electron as deformable and incompressible in order to explain why it takes a deformable shape while in motion. But I show that it is not in agreement with the impossibility of measuring absolute motion. This is connected, as I have said, with the fact that  $l = 1$  is the only hypothesis for which the set of all Lorentz transformation form a group.

But with the Lorentz hypothesis we obtain agreement between the formulas, and at the same time a possible explanation of the contraction of the electron, by supposing that the deformable and compressible electron is under the action of constant external pressure which is proportional to variation of volume.

I show, by application of the principle of least action, that, under these conditions, the compensation is complete if we suppose that inertia is a completely electromagnetic phenomenon, as it is generally considered to be since the experiment of Kaufmann, and also the constant pressure which I just mentioned and which acts on the electron, and all other forces, are of electromagnetic origin. We thus have an explanation of the impossibility of measuring absolute motion and of the contraction of all bodies in the direction of motion.

But this is not all; Lorentz, in the quoted reference, has judged it necessary to complete his hypothesis by supposing that all the forces, whatever their origin, should be affected by a translation, in the same manner as the electromagnetic force, and that, consequently, the effect produced on their components by the Lorentz transformations is still defined by equations (4).

It is important to examine this hypothesis more closely, and in particular to find what modifications it will oblige us to make in the laws of gravitation. This I have decided to do; I was first led to suppose that propagation of gravitation is not instantaneous, but it propagates with the velocity of light. This seems to contradict a law obtained by Laplace, who announced that this propagation is if not instantaneous, at least much more rapid than that

of light.<sup>1</sup> But in reality the question which Laplace considered was considerably different from that which we are taking up here. For Laplace the introduction of a finite velocity of propagation was the *only* modification which he made in Newton's law. Here, on the contrary, this modification is accompanied by several others; it is therefore possible that they produce among themselves a partial compensation.

When we speak of the position or the velocity of an attracting body, it will be the position or the velocity at the instant when the *gravitational wave* just leaves the body; when we talk of the position or the velocity when the wave emitted by the other body has reached this attracted body; it is clear that the first instant is prior to the second.

If then  $x, y, z$  are the projections on the three axes of the vector which joins the two positions, if the velocity of the attracted body is  $\xi, \eta, \zeta$ , and that of the attracting body is  $\xi_1, \eta_1, \zeta_1$  the three components of attraction (which I could again call  $(X_1, Y_1, Z_1)$ ) will be functions of  $x, y, z, \xi, \eta, \zeta, \xi_1, \eta_1, \zeta_1$ . I have asked myself if it would be possible to determine these functions in such a manner that they would be affected by the Lorentz transformations in conformity with equations (4), so that the ordinary law of gravitation is recovered, when the velocities  $\xi, \eta, \zeta, \xi_1, \eta_1, \zeta_1$  are sufficiently small, to be neglected compared to the velocity of light.

The answer seems to be affirmative.<sup>2</sup> We find that the corrected attraction consists of two forces, one parallel to the vector  $x, y, z$  the other to the velocity.

The difference from the ordinary law of gravitation is, as I calculate, of order of  $\xi^2$ ; if we merely suppose, with Laplace, that the velocity of propagation is that of light, this difference would be order  $\xi$ , which is 10,000 times larger. It is therefore not *prima facie* absurd to suppose that astronomical observations are not sufficiently precise to detect such small differences. But this requires a deeper discussion.

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1. Laplace's argument being, of course, that the predictions of celestial mechanics are too accurate (on the hypothesis of infinite velocity of propagation) to allow for transmission at light velocity. But Poincaré could have remarked that the unexplained motion of the orbit of Mercury causes the question to be open again.

2. Poincaré is here referring to the gravitational theory set out in slightly more detail in his paper of the same name in *Rend. Del Circolo Mat. Di Palermo*, **21**, 129-75 [1906]. He poses the problem to determine the components of gravitational force between two bodies in terms of their two velocities and their relative position. He determines 4 Lorentz invariant quantities which are homogeneous and of degree zero in the coordinate differentials. This still leaves a good deal of latitude, even with the two requirements of unit velocity of propagation and approximately inverse square law for slow motions. Still, he is able to find some special forms of the force, which consists of the Newtonian modified by second-order velocity terms.

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