MLSE for Correlated Diversity Sources and Unknown Time-Varying, Frequency-Selective, Rayleigh Fading Channels

Brian D. Hart, Member, IEEE
Telecommunications Engineering, Research School of Information Sciences and Engineering,
Australian National University, Canberra, Australia
Brian.Hart@anu.edu.au

Desmond P. Taylor, Fellow, IEEE
Department of Electrical and Electronic Engineering,
University of Canterbury, New Zealand
taylor@elec.canterbury.ac.nz

Abstract

In [1], an MLSE receiver structure was derived for unknown, time-varying, frequency-selective, Rayleigh fading channels and uncorrelated diversity sources. In this letter, the receiver design is extended to the case of correlated diversity sources. Correlated diversity sources typically arise with space diversity, where constraints on antenna volume require that diversity antennae be placed too closely together. Analytic and simulated BER curves are presented for receivers which exploit and ignore the correlation. In the former case, we find a small BER improvement that reduces with decreasing correlation. However, for a fixed receiver complexity, superior performance is achieved when the correlation is ignored.

I. Introduction

In [1], an MLSE diversity receiver structure was derived for uncorrelated diversity sources, based on the Innovation process, or equivalently, on linear predictors. A branch metric is computed as the sum of the diversity sources' branch metrics, which are each calculated separately. However, when the diversity sources are correlated, the correlation can be exploited to improve prediction. This is unlike the case of the known channel, where correlation between diversity sources has no influence [2]. Correlated diversity sources arise for instance in space diversity when antennae are not located at the zeroes of the other antennae's spatial correlation functions. This letter is organised as follows. In section II, the signal model is described. In section III, the MLSE receiver for correlated diversity sources is derived. In section IV, the receiver's performance is characterised.

II. Signal Model

As shown in figure 1, a linearly modulated signal,

$$a(t) = \sum_i \beta_i h(t - iT), \quad (1)$$

is distorted by the dth channel, $d \in \{0, ..., D\text{-}1\}$, as

$$y^d(t) = \int_{-\infty}^{\infty} a(t - \xi) z^d(t, \xi) d\xi + n^d(t) \quad (2)$$

where $a(t)$ is the transmitted signal in complex baseband; $\{\beta_i\}$ is a sequence of differentially-encoded complex phasors taken from an M-ary constellation; $h(t)$ is the transmitted pulse shape, and is only non-zero over $[0; HT)$; $0 < H < \infty$ is the transmitted pulse length in symbol periods; T is the symbol period; $y^d(t)$ is the received signal from the dth diversity source in complex baseband; $z^d(t,\xi)$ is the time-varying, frequency-selective, Rayleigh fading channel experienced by the dth diversity source, and is only non-zero over $[0; \tau]$ in ξ; τ is the maximum delay spread; and $n^d(t)$ is AWGN observed with the dth diversity source. The dth channel can be visualised as a tapped-delay line, where $z^d(t,\xi)$ is the time-varying tap weight at delay ξ [3]. The D received signals are filtered by noise-limiting filters with transfer function $\text{rect}(fT_r)$, then sampled at $t = lT_r$, where $T_r = T/r$, $r \geq 1$, is the sampling period. The number of samples per symbol, r, is chosen large enough that the noise-limiting filters introduce negligible signal distortion. The sampled signal is a linearly modulated signal also, as[1]

$$y_l^d = \sum_{i=-L+1+\lfloor l/r \rfloor}^{\lfloor l/r \rfloor} \beta_i c_{l-ir,i}^d + n_l^d \tag{3}$$

where the received pulse shape, $c_{l-ir,i}^d$, is different each symbol period, i, due to the time-varying channel, as

$$c_{l-ir,i}^d = \int_{-\infty}^{\infty} h\big((l-ir)T_r - \xi\big) z^d\big(lT_r, \xi\big) d\xi \,. \tag{4}$$

It is only non-zero over $l\text{-}ir \in \{0,..,Lr\text{-}1\}$, where $L = H + \lceil \tau/T \rceil$ is its length in symbol periods.

The samples from each diversity source are interleaved, $y_l^d \rightarrow y_{lD+d}$, into a vector for receiver processing, as proposed in [12]. Three such vectors are defined: $\mathbf{Y}_{lD+d}$ = $\left[y_0^0, y_0^1, \ldots y_0^{D-1}, y_1^0, \ldots, y_l^d\right]^T = \left[y_0, \ldots, y_{lD+d}\right]^T$, $\mathbf{Y} = \mathbf{Y}_\infty$, and $\mathbf{y}_{lD+d} = \left[y_{lD+d-DB}, \ldots, y_{lD+d-1}\right]^T$, where B is an positive integer constant defined subsequently. The received signal autocovariance satisfies

$$R_{yy,l,m}^{d,e} = \tfrac{1}{2} E\left(y_l^d y_m^{e,*} \middle| \beta\right) = \sum_{\substack{i=-L+1+\\ \lfloor l/r \rfloor}}^{\lfloor l/r \rfloor} \sum_{\substack{k=-L+1+\\ \lfloor m/r \rfloor}}^{\lfloor m/r \rfloor} \beta_i \beta_k^* \tfrac{1}{2} E\left(c_{l-ir,i}^d c_{m-kr,k}^{e,*}\right) + \tfrac{1}{2} E\left(n_l^d n_m^{e,*}\right) \tag{5}$$

$$\tfrac{1}{2} E\left(c_{l-ir,i}^d \; c_{m-kr,k}^{e,*}\right) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} h\big((l-ir)T_r - \xi_1\big) h^*\big((m-kr)T_r - \xi_2\big) \tfrac{1}{2} E\left(z^d\left(lT_r, \xi_1\right) z^{e,*}\left(mT_r, \xi_2\right)\right) d\xi_1 d\xi_2 \tag{6}$$

$$\tfrac{1}{2} E\left(z^d\left(lT_r, \xi_1\right) z^{e,*}\left(mT_r, \xi_2\right)\right) = P_{zz}^{d,e}\big((l-m)T_r, \xi_1, \xi_2\big) \tag{7}$$

$$\tfrac{1}{2} E\left(n_l^d n_m^{e,*}\right) = N_0 / T_r \delta_{de}\, \delta_{lm} \tag{8}$$

where $\tfrac{1}{2} E\left(c_{l-ir,i}^d \; c_{m-kr,k}^{e,*}\right)$ is the received pulse autocovariance; $P_{zz}^{d,e}\big((l-m)T_r, \xi_1, \xi_2\big)$ is the channel autocovariance and is assumed to be wide sense stationary; and $\tfrac{1}{2} E\left(n_l^d n_m^{e,*}\right)$ is the noise autocovariance, and assumed independent between diversity sources and samples. We define $\mathbf{R}_{\mathbf{yy},l}(\beta) = \tfrac{1}{2} E\left(\mathbf{y}_l \mathbf{y}_l^H \middle| \beta\right)$ and $\mathbf{r}_{\mathbf{yy},l}(\beta) = \tfrac{1}{2} E\left(\mathbf{y}_l y_l^* \middle| \beta\right)$. The autocovariance functions are assumed to be known in the subsequent sections, but in practice, they must be estimated, possibly during a training

---

[1] The following notation is used: $\lfloor x \rfloor$ and $\lceil x \rceil$ are the floor and ceiling functions respectively; an asterisk, x^*, denotes complex conjugation; x^H is the Hermitian transpose of x; and x mod y denotes the remainder of x/y.

sequence. Several methods appear in the literature [5,6,7].

In producing this letter's results, we assume space diversity and a WSSUS channel with N discrete fading paths with equal power, so the channel's autocovariance has the form,

$$P_{zz}^{d,e}\left((l-m)T_r,\xi_1,\xi_2\right)=\delta\left(\xi_1-\xi_2\right)\sum_{i=0}^{N-1}\tfrac{1}{N}\delta\left(\xi_1-i\tau/(N-1)\right)R_{ttaa}^{d,e}\left((l-m)T_r\right) \tag{9}$$

The time autocovariance of [8] generalises to a time-antenna autocovariance [10],

$$R_{ttaa}^{d,e}\left((l-m)T_r\right)=J_0\left(2\pi\sqrt{((d-e)A)^2+\left((l-m)f_DT_r\right)^2+2(d-e)A(l-m)f_DT_r\cos\varphi}\right) \tag{10}$$

for multiple antennae in a linear array, spaced every A wavelengths at angle φ to the receiver's velocity. f_D is the maximum Doppler frequency. (10) is difficult to simulate, so we use the following autocovariance instead. For each of the N fading paths, D white Gaussian noise sequences are independently filtered by a 192-tap FIR filter, with impulse response [11], $f_l = J_{\frac{1}{4}}\left(2\pi f_DT_r|l|\right)|l|^{-\frac{1}{4}}\times Hanning\left(lT_r\right)$, thus creating a time autocovariance that approximates $J_0\left(2\pi f_DT_r|l-m|\right)$. A $D\times D$ matrix, computed as the Cholesky decomposition of the inter-antenna correlation, multiplies these D sequences, creating D correlated sequences. Thus the time-antenna covariance is approximately

$$R_{ttaa}^{d,e}\left((l-m)T_r\right)\approx J_0\left(2\pi|d-e|A\right)J_0\left(2\pi|l-m|f_DT_r\right) \tag{11}$$

which is a good approximation to (10) for $\varphi = \pi/2$, and small values of A or f_DT/r.

III. MLSE for Correlated Diversity Threads

The maximum likelihood detector chooses the sequence which maximises

$$\{\beta\}_{ML}=arg\,\frac{max}{\{\beta\}}\,p\left(\mathbf{Y}|\{\beta\}\right) \tag{12}$$

The pdf, $p\left(\mathbf{Y}|\{\beta\}\right)$, of the correlated process $\mathbf{Y}$ is a multivariate, zero-mean Gaussian density involving the inverse received signal autocovariance matrix, $\mathbf{R}_{\mathbf{yy},l}^{-1}(\beta)$. It is Cholesky decomposed into the matrix product $\mathbf{B}^H\Sigma^{-1}\mathbf{B}$, where $\mathbf{B}$ is a lower triangular matrix containing the coefficients of prediction error filters of increasing order. Its determinant is unity. Σ is a diagonal matrix containing the prediction error filters' mean square prediction errors. In this way, a sequence metric equivalent to the negative logarithm of the pdf may be calculated recursively, as the following path plus branch metric [1,4,10],

$$\Lambda\left(\mathbf{Y}_{ir}\middle|\beta\right)+\lambda\left(\mathbf{Y}_{ir}\middle|\beta\right)=$$
$$\sum_{l=0}^{Dir-1}\left(\frac{\left|y_l-y_{l|l-1}(\beta)\right|^2}{\sigma^2_{l|l-1}(\beta)}+\ln\left(\sigma^2_{l|l-1}(\beta)\right)\right)+\sum_{l=Dir}^{D(i+1)r-1}\left(\frac{\left|y_l-y_{l|l-1}(\beta)\right|^2}{\sigma^2_{l|l-1}(\beta)}+\ln\left(\sigma^2_{l|l-1}(\beta)\right)\right) \tag{13}$$

where $y_{l|l-1}(\beta)$ is the MMSE estimate of y_l, calculated as the linear prediction $y_{l|l-1}(\beta)=\sum_{k=1}^{l} b^{ML}_{l,k}(\beta)y_{l-k}$; $b^{ML}_{l,k}(\beta)$ is the kth tap for predicting y_l, assuming the transmitted sequence $\{\beta\}$; and $\sigma^2_{l|l-1}(\beta)$ is the prediction's variance. By interleaving the received samples as **Y**, the prediction, $y_{l|l-1}(\beta)$, exploits all diversity sources.

The number of predictor taps must be truncated for a finite complexity implementation [1,4]. There are two approaches: (*a*) joint prediction, where the last *DB* received samples, $y_{l-DB},\ldots,y_{l-1}$, are used (i.e. *B* taps per diversity source); or (*b*) disjoint prediction, where only the last *B* samples, $y^d_{l-B},\ldots,y^d_{l-1}$, of a diversity source are used to predict its next sample, y^d_l. In both cases, the choice of *B* balances complexity against performance. The disjoint approach has already been described in [1], so we describe the joint approach only. The tap weights are arranged in a vector, $\mathbf{b}_l(\beta)=\left[b_{l,DB}(\beta),\ldots,b_{l,1}(\beta)\right]^T$. The prediction, tap weights, and MMSEs, $\sigma^2_{l|l-1}(\beta)$, are computed according to

$$y_{l|l-1}(\beta)=\mathbf{b}_l(\beta)^T\mathbf{y}_l \tag{14}$$

$$\mathbf{R}_{\mathbf{yy},l}(\beta)\mathbf{b}^*_l(\beta)=\mathbf{r}_{\mathbf{yy},l}(\beta), \tag{15}$$

$$\sigma^2_{l|l-1}(\beta)=R_{yy,l,l}(\beta)-\mathbf{b}^H_l(\beta)\mathbf{r}_{\mathbf{yy},l}(\beta) \tag{16}$$

The branch metrics are distinct for each combination of $W=\lceil B/r\rceil+L$ hypothesised symbols, up to the rotational symmetry of the constellation, labelled *P*. Searching for the ML sequence is undertaken efficiently by the Viterbi algorithm, with M^W/P distinct branch metrics, and M^{W-1}/P states. Given precomputed $b_{l,k}(\beta)$, $1/\sigma^2_{l|l-1}(\beta)$, and $ln\,\sigma^2_{l|l-1}(\beta)$, the receiver complexity is dominated by the predictions. When joint or disjoint prediction are performed, prediction requires (#branches) × (#diversity sources) × (#samples) × (#taps) = $M^W/P\times D\times r\times DB$ or $M^W/P\times D\times r\times B$ complex multiply-adds per symbol respectively.

IV. Receiver Performance

The receiver's BER is characterised by simulation and analysis [7,10]. The simulated results are generated from a Monte-Carlo simulation of figure 1. Randomly generated data is transmitted and detected until at least 200 bit errors are observed. The simulation parameters are as follows. The data is differentially encoded BPSK. A root raised cosine pulse is used, with 50% excess bandwidth, and windowed with a Hanning window to H = 2.5 symbol periods. The channel is modelled by N = 3 Rayleigh fading paths, spaced equally over $\tau = 0.5T$ seconds, having equal mean power. D = 2 space diversity sources are available. The channel is fast fading, $f_D T = 0.1$. Analytic results use the union bound technique and residue calculations [7,10]. Approximate lower bounds consider only nearest-neighbour cycle slips, and truncated union bounds consider these and nearest neighbour one symbol error events.

In figure 2, uncorrelated diversity sources provide optimal performance. However, excellent BERs are achieved even with reasonable correlations. The improvement diminishes as the diversity sources' correlation increases. In figure 3, the relative performance of joint and disjoint prediction is compared. Joint prediction provides D-fold longer predictors without increasing the number of states, whereas for disjoint prediction, the number of states increases $M^{\lceil DB/r \rceil - \lceil B/r \rceil}$ times for length-DB predictors. From figure 3 however, the source of the samples is more important. When the diversity sources are nearly uncorrelated, disjoint prediction with B = 6 has essentially the same performance as joint prediction with DB = 12, but with 1/Dth the complexity. Accordingly, joint prediction is unwarranted for lightly correlated diversity sources. Even when the diversity sources are highly correlated, joint prediction does not substantially improve the BER. In fact, the BER from disjoint prediction with B = 9 substantially outperforms the BER from joint prediction with DB = 12.

V. Conclusions

The MLSE diversity receiver for correlated diversity sources has been derived. By interleaving samples from all diversity sources, their correlation can be exploited to improve the prediction. However, the predictor length must be truncated in practice, and the best samples to use in the predictions are those from matching diversity sources. Thus it is best to ignore the correlation between diversity sources.

VI. References

[1] X.Yu and S.Pasupathy, “Innovations-Based MLSE for Rayleigh Fading Channels,” *IEEE Trans. Commun.*, vol. 43, no. 2/3/4, pp1534-1544, Feb./Mar./Apr. 1995

[2] B.D.Hart, D.P.Taylor, “Extended MLSE Diversity Receiver for the Time- and Frequency-Selective Channel,” *IEEE Trans. Comm.*, vol. 45, no. 3, pp322-333, Mar. 1997

[3] P.A.Bello, “Characterization of Randomly Time-Variant Linear Channels,” *IEEE Trans. Commun. Sys.*, vol. 11, pp360-393, Dec. 1962

[4] J.H.Lodge and M.J.Moher, “Maximum Likelihood Sequence Estimation of CPM Signals Transmitted over Rayleigh Flat-Fading Channels,” *IEEE Trans. Commun.*, vol. 38, no. 6, pp787-794, June 1990

[5] M.K.Tsatsanis, G.B.Giannakis, G.Zhou, “Estimation and Equalisation of Fading Channels with Random Coefficients,” *Signal Processing*, vol. 53, pp211-229, Sept. 1996

[6] L.M. Davis, I.B. Collings and R.J. Evans, "Identification of Time-Varying Linear Channels", to appear in *Int. Conf. on Acoustics, Speech and Signal Processing, ICASSP* 97, Munich, Germany, 1997

[7] B.D.Hart and D.P.Taylor, “Maximum Likelihood Synchronisation, Equalisation and Sequence Estimation for Unknown, Time-Varying, Frequency-Selective, Rician Channels,” submitted to *IEEE Trans. Commun.*

[8] R.H.Clarke, “A Statistical Theory of Mobile-Radio Reception,” *Bell Syst. Tech. J.*, vol 47, pp957-1000, July-Aug. 1968

[10] B.D.Hart, “MLSE Diversity Receiver Structures,” PhD thesis, University of Canterbury, Christchurch, New Zealand, 1996

[11] D.Verdin and T.C.Tozer, “Generating a Fading Process for the Simulation of Land-Mobile Radio Communications,” *Electronics Letters*, vol. 29, no. 23, pp2011-2012, 11 Nov. 1993

[12] Y.Li, Z.Ding, “A Simplified Approach to Optimum Diversity Combining and Equalization in Digital Data Transmission,” *IEEE Trans. Commun.*, vol. 43, no. 8, pp2285-2288, Aug. 1995

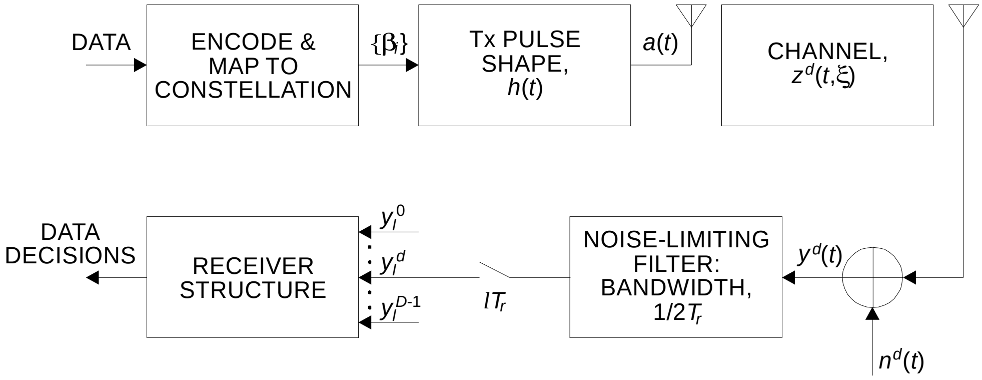

Figure 1: Structural representation of the communications system.

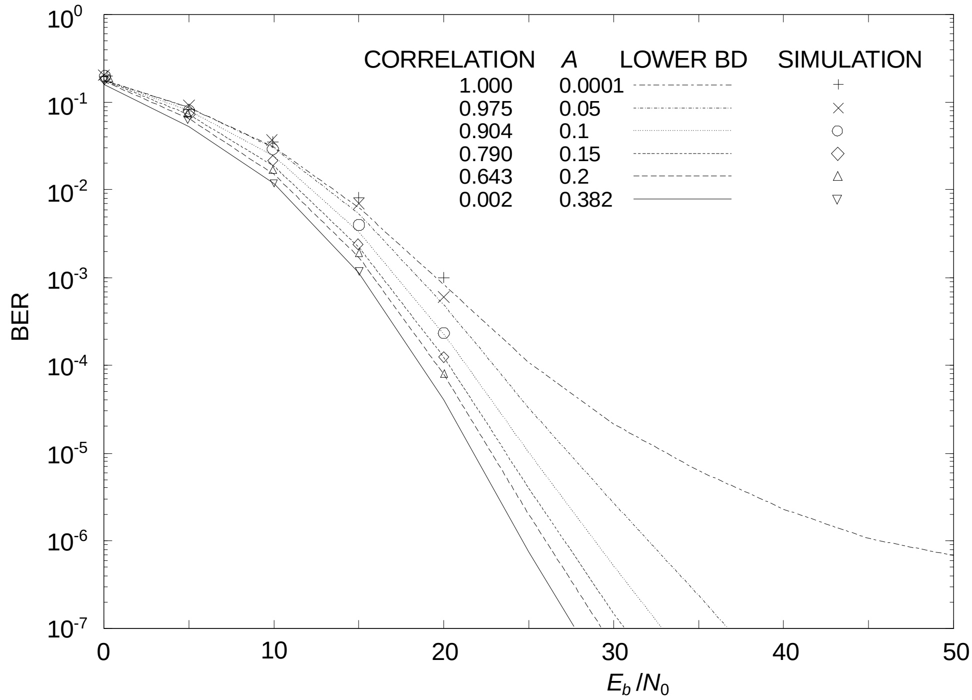

Figure 2: The BER as a function of E_b/N_0 of a receiver employing joint prediction, with antenna separation or inter-antenna correlation as a parameter. BPSK, $r = 3$, $B = 6$, $D = 2$, $f_D T = 0.1$, $\tau/T = 0.5$.

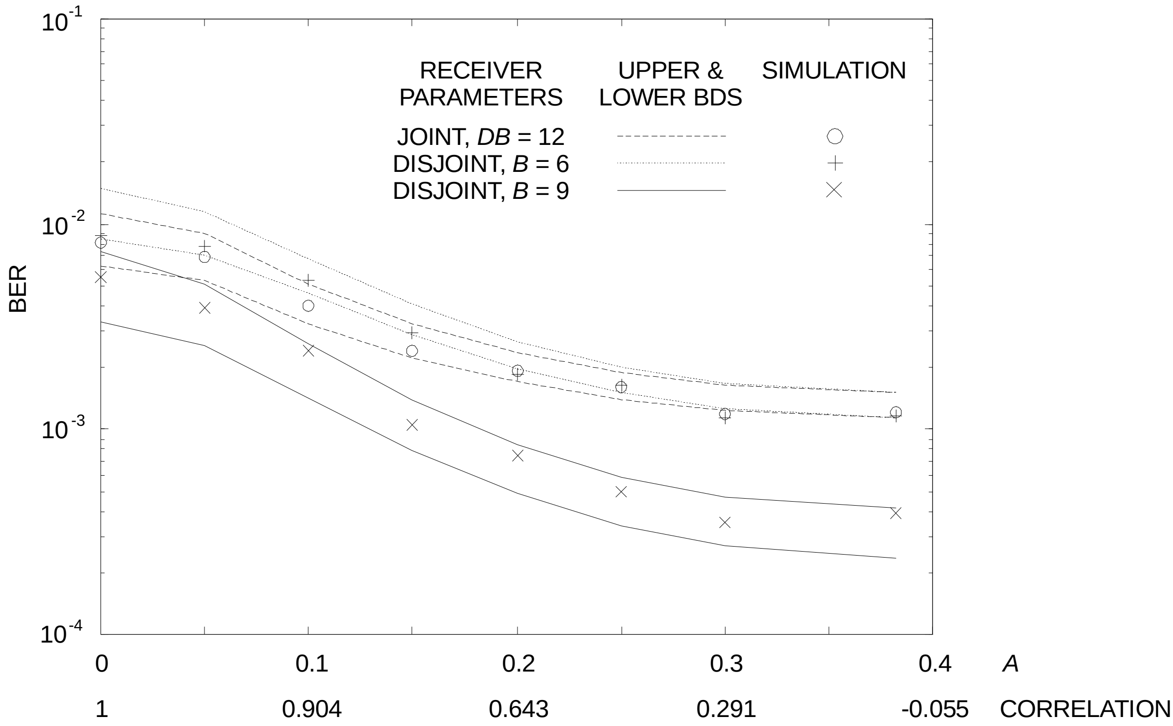

Figure 3: The BER as a function of antenna spacing or inter-antenna correlation for receivers employing joint and disjoint prediction. BPSK, E_b/N_0 = 15dB, r = 3, B = 6, 9, D = 2, $f_D T$ = 0.1, τ/T = 0.5. Every symbol period, computing the predictions requires $2^5/2\times2\times3\times12$ operations for joint prediction and B = 6 (DB = 12); $2^5/2\times2\times3\times6$ operations for disjoint prediction and B = 6; and $2^6/2\times2\times3\times9$ operations for disjoint prediction and B = 9.