Analytic Solutions of Shear Lag on Steel-Concrete Composite T-girder under Simple Bending

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Abstract: The composite T-girders include concrete flange plates and steel beams, which are connected by shear connectors. The longitudinal stress about concrete plate is a non-uniform distribution on cross section because of the shear lag effect. A differential equation of longitudinal forces at transverse section flange and cantilever flange is separately established according to the strain compatibility and the force equilibrium conditions about a composite T-girder. The method of separation of variables is used to solve the differential equation about the simply supported composite T-girder. The shear lag coefficient is decided by the ratio between stress calculated by this method and stress decided by elementary beam theory. An example of such calculation is given to approve its applicability.

Keywords: Composite T-girders, longitudinal stress, differential equations, shear lag coefficient.

1. INTRODUCTION

The composite T-girders are commonly used in construction, due to taking advantage of the mechanical characteristic of steel and concrete, namely the use of concrete compressive and steel tensile capacity. It is well known that the uneven deformation of the wider T-girder flanges can produce an uneven distribution of the longitudinal stresses [1] under symmetrical bending. The shear lag effect can result in the obvious increase of longitudinal stress near the edge of the flange and cause stress concentration. Due to shear lag effects, it can cause stress concentration in structure, structural damage [2].

Guo jinqiong, Wei Lina, etc. had put forward some practical theories of computation and computational methods, such as variational methods [3,4], finite strip method [5,6] and finite element method [6-8] etc. Lawrence F K, Adam S, Khaled M. Sennah etc. had researched on the shear resistance of the surface of the steel-concrete composite beams with cantilever flange [9,10].

Those methods have different characteristics, but difficulty of more calculation and analysis exists with these methods to calculate composite section. The same method is used to calculate shear lag effect of composite box girders in reference [6]. It is also applied to calculate shear lag effect of the composite T-girder. Parameters of a composite T-girder are shown in Fig. (1).

2. DIFFERENTIAL EQUATIONS ABOUT FLANGE EQUILIBRIUM

As shown in Fig. (1), It is assumed that relative slip between the concrete plate and steel girder is not happened

Fig. (1). A composite T-girder.

Fig. (2). A flange element of the T-girder.

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under symmetrical bending. Both of them bear the applied loads. Any element of the flange, as shown in Fig. (2), bears a shear flow q_e and a normal force $n_{x,i}$ ($i=1$ represents one cantilever flange, $i=2$ represents the other). It is assumed that the shear flow q_e is resisted by the composite flange plate itself, without considering the help from the stiffeners. But both the composite flange plate and stiffeners bear the normal longitudinal forces. Without special provisions, the subscript i has the same meaning as the subscript i of $n_{x,i}$ in the followings.

2.1. Stress and Strain

In the elastic range, an equivalent thickness t_s of steel flanges can be obtained according to the Match-plate method. Then, an equivalent thickness $\bar{t}$ for the composite flange can be obtained according to the equivalent conversion principle, where concrete slab thickness of flange is converted to steel equivalent thickness.

$$\bar{t} = t_1 + \frac{A_s}{a} + \frac{A_{sx}}{a_1} + t_c \frac{E_c}{E_s} = t_s + t_c \frac{E_c}{E_s} \quad (1)$$

As in Fig. (1), t_1 represents the steel plate thickness of flange, a and a_1 are the spacing of longitudinal and transverse stiffener respectively, and A_s is the transverse section area of each longitudinal stiffener, A_{sx} is the transverse section area of each transverse stiffener, t_c is the thickness of the concrete layer; E and E_c represent the Young's moduli of steel and concrete respectively.

As shown in Fig. (2), the direct strains in the longitudinal direction are then given by

$$\varepsilon_{x,i} = \frac{\sigma_{x,i}}{E_s} = \frac{n_{x,i}}{\bar{t}E_s} \quad i=1,2 \quad (2)$$

Also, it may be assumed that the transverse direct strain in the steel plate and concrete slab are the same so that

$$\varepsilon_{y,i} = -\frac{\nu_s n_{xs,i}}{t_s E_s} = -\frac{\nu_c n_{xc,i}}{t_c E_c} \quad i=1,2 \quad (3)$$

Where $\sigma_{x,i}$ represents the x -direction stress of different element, ν_s and ν_c express the Poisson's ratio of the steel and concrete respectively; $n_{xs,i}$ and $n_{xc,i}$ are the sections of the normal force ($n_{x,i}$) carried by the steel and concrete components respectively, such that

$$n_{x,i} = n_{xs,i} + n_{xc,i} \quad i=1,2 \quad (4)$$

According to equations (3),(4), equation (5) gives

$$n_{xs,i} = \frac{\frac{\nu_c n_{x,i}}{t_c E_c}}{\frac{\nu_s}{t_s E_s} + \frac{\nu_c}{t_c E_c}} \quad i=1,2 \quad (5)$$

By substitution from equations (3),(5), the $\varepsilon_{y,i}$ can be written as

$$\varepsilon_{y,i} = -\nu_s \nu_c \frac{n_{x,i}}{\nu_c t_s E_s + \nu_s t_c E_c} \quad i=1,2$$

It can be simplified by using an 'equivalent' Poisson's ratio (r), equation(6) gives.

$$\varepsilon_{y,i} = -r \frac{n_{x,i}}{\bar{t} E_s} = -r \varepsilon_{x,i} \quad i=1,2 \quad (6)$$

$$\text{where, } r = \frac{\nu_s \nu_c (t_s E_s + t_c E_c)}{\nu_c t_s E_s + \nu_s t_c E_c} \quad i=1,2.$$

2.2. Shear Stress and Strain

It is assumed that no relative slippage between the, the shear strains (γ_i) of elements about steel and concrete slab are the same, then

$$\gamma_i = \frac{q_{s,i}}{t_1 G_s} = \frac{q_{c,i}}{t_c G_c} \quad (i=1,2) \quad (7)$$

Where, G_s and G_c express the shear moduli of the steel and concrete respectively. $q_{s,i}$ and $q_{c,i}$ are the sections of the shear flow(q_i) carried by the steel and concrete components respectively. The sum of $q_{s,i}$ and $q_{c,i}$ equals the shear flow(q_i). By substitution, equation (8) gives

$$\gamma_i = \frac{q_i}{t_1 G_s + t_c G_c} \quad i=1,2 \quad (8)$$

That may be written as

$$\gamma_i = \frac{q_i}{t_1^* G_s} \quad i=1,2 \quad (9)$$

Where the equivalent thickness of the shear action is given by

$$t_1^* = t_1 + t_c \frac{G_c}{G_s}$$

2.3. Compatibility and Equilibrium Equations

As shown in Fig. (2), the equilibrium of the flange element in the longitudinal direction is given by

$$\frac{\partial n_{x,i}}{\partial x} + \frac{\partial q_i}{\partial y} = 0 \quad i=1,2 \quad (10)$$

As shown in Fig. (1), the composite flange can be regarded as a plane stress problem, by the Hooke's Law [3], the equation controlling the condition of compatibility may be written as [11]

$$\frac{\partial^2 \varepsilon_{x,i}}{\partial y^2} + \frac{\partial^2 \varepsilon_{y,i}}{\partial x^2} = \frac{\partial^2 \gamma_i}{\partial x \partial y} \quad i=1,2 \quad (11)$$

Substituting equations (2), (6) and (9) for the strains, equation (11) gives

$$\frac{\partial^2 n_{x,i}}{\partial y^2} - r \frac{\partial^2 n_{x,i}}{\partial x^2} = \frac{\bar{t} E_s}{t_1^* G_s} \frac{\partial^2 q_i}{\partial x \partial y} \quad i=1,2$$

and the shear flow q_i may be substituted by the normal force $n_{x,i}$ from equation (10), thus,

$$\frac{\partial^2 n_{x,i}}{\partial y^2} + \left(\frac{\bar{t} E_s}{t_1^* G_s} - r \right) \frac{\partial^2 n_{x,i}}{\partial x^2} = 0 \quad i=1,2 \quad (12)$$

3. SOLUTION OF DIFFERENTIAL EQUATION

For the partial differential equation, the normal force $n_{x,i}(x,y)$ can be written as the Fourier series by adopting the method of separation of variables [8] under the simply supported border, such that

$$n_{x,i}(x,y) = \sum_{k=1}^{\infty} N_{k,i}(y) \sin \frac{k\pi x}{L} \quad i=1,2 \quad (13)$$

Where, L represents the length of the simply supported span and $N_{k,i}(y)$ expresses an amplitude function which represents the variation in the cross-section direction of the normal force. Where, x represents the distance from the origin to the calculated section. k is the series term.

By substituting the k^{th} term of the series into equation (13), the ordinary differential equation is written as

$$\frac{d^2 N_{k,i}}{dy^2} - (\xi_k)^2 N_{k,i} = 0 \quad i=1,2$$

where, $\xi_k = \frac{k\pi}{L} \sqrt{\frac{\bar{t} E_s}{t_1^* G_s} - r}$, the common solution of the equation is written as

$$N_{k,i}(y) = C_{i,k1} \cosh \xi_k y + C_{i,k2} \sinh \xi_k y \quad i=1,2 \quad (14)$$

where the coefficient $C_{i,k1}$ and $C_{i,k2}$ are constants that decided by the supporting and loading conditions at the longitudinal edges of the flange. As shown in Fig. (1), normal force $n_{x,i}(x,y)$ with shear lag has a symmetric distribution in the cross section under symmetrical bending. As shown in Fig. (2), the coefficient $C_{i,k2}$ is zero due to symmetry. The constant $C_{i,k1}$ can be determined according to the conditions of shear flow equivalent and normal force continuity. By substituting equation (14) into equation(13), the normal longitudinal force can be expressed as

$$n_{x,i}(x,y) = \sum_{k=1}^{\infty} C_{i,k1} \cosh \xi_k y \sin \frac{k\pi x}{L} \quad i=1,2 \quad (15)$$

The normal longitudinal force of the cut-off point about two cantilever flange must meet continuous conditions. Then, the normal longitudinal force is written as

$$\sum_{k=1}^{\infty} C_{1,k1} \cosh \xi_k c \sin \frac{k\pi x}{L} = \sum_{k=1}^{\infty} C_{2,k1} \cosh \xi_k c \sin \frac{k\pi x}{L} \quad (16)$$

By substituting equation(15) into equation(10), the equation of the first-order derivative of y on q_i is given as

$$\frac{\partial q_i}{\partial y} = -\frac{\partial n_{x,i}}{\partial x} = -\sum_{k=1}^{\infty} \frac{k\pi}{L} C_{i,k1} \cosh \xi_k y \cos \frac{k\pi x}{L} \quad i=1,2 \quad \text{By}$$

integrating the y and noting the value of ξ_k , the shear flow at any point may be written as

$$q_i(x,y) = -\frac{k\pi}{\xi_k L} \sum_{k=1}^{\infty} C_{i,k1} \sinh \xi_k y \cos \frac{k\pi x}{L} \quad i=1,2 \quad (17)$$

At the central of T-girder flange (where $y=c$) as shown in Fig. (2), the sum of the shear flow on two cantilever flanges equals the shear flow (q_e) transferred from the web to the flange, which can be expressed as

$$q_e(x) = q_{e,1}(x) + q_{e,2}(x) \quad (18)$$

Where, the shear flow $q_{e,1}(x)$ and $q_{e,2}(x)$ expresses the shear flow transferred from the web to the flange, which can be expressed as

$$q_e(x) = q_{e,1}(x) + q_{e,2}(x) = -\frac{k\pi}{\xi_k L} \sum_{k=1}^{\infty} C_{1,k1} \sinh \xi_k c \cos \frac{k\pi x}{L} - \frac{k\pi}{\xi_k L} \sum_{k=1}^{\infty} C_{2,k1} \sinh \xi_k c \cos \frac{k\pi x}{L} \quad (19)$$

The shear flow (q_e) can be approximately decided from elementary beam theory, it can be written as

$$q_e(x) = V(x) \frac{\bar{t} b e}{2I}$$

Where, $V(x)$ is the total shear force imposed on the T-girder cross section at the position x , and I is the inertial moment of the cross section, e is the distance from the neutral axial to the centroidal axis of the flange. For the simply supported span, the shear flow $q_e(x)$ can be expressed by the Fourier series, such that

$$q_e(x) = \sum_{k=1}^{\infty} Q_{ek} \cos \frac{k\pi x}{L} \quad (20)$$

$$\text{where, } Q_{ek} = \frac{2}{L} \int_0^L q_e(x) \cos \frac{k\pi x}{L} dx = \frac{\bar{t} b e}{IL} \int_0^L V(x) \cos \frac{k\pi x}{L} dx.$$

According to the equations(16), (19) and (20), the constant $C_{i,k1}$ can be gotten, such that

$$C_{1,k1} = C_{2,k1} = -\frac{Q_{ek} L \xi_k}{2k\pi \sinh \xi_k c}$$

By substituting the constant $C_{i,k1}$ into equation (13) and equation (14), the normal force $n_{x,i}(x,y)$ may be gotten. Thus, the normal stresses of the steel flange and the concrete plate can respectively be given, such that

$$\sigma_{sx,i}(x,y) = \frac{n_{x,i}(x,y)}{\bar{t}} \quad i=1,2$$

$$\sigma_{cx,i}(x,y) = \sigma_{sx,i}(x,y) \frac{E_c}{E_s} \quad i=1,2 \quad (21)$$

The shear stress of the steel flange and the concrete plate can be given as

$$\tau_{sx,i}(x,y) = \frac{q_i(x,y)}{t_1^*} \quad i=1,2$$

$$\tau_{cx,i}(x,y) = \tau_{sx,i}(x,y) \cdot \frac{G_c}{G_s} \quad i=1,2 \quad (22)$$

4. CALCULATION AND CONCLUSIONS

According to the approximate calculation method for shear lag effect of composite T-girder, a simple manual or the program can get the shear lag coefficient of flange. In order to analyze the convergency, precision of the method and the parametric influence about the shear lag effect of composite T-girder, the transverse section of an example is written as in Fig. (3). The area of the longitudinal and horizontal stiffeners is 100Square millimeters. The depth of concrete flange is 60mm. The height of the composite T-girder is 1200mm. The thickness of the steel slab is 12mm, and other parameters are illustrated in Table 1. According to the ratio of wide-span, the flange width c is respectively 600mm, 800mm, 1200mm and 1800mm.

Fig. (3). Cross section of the example.

(a) Stress of cross point between web and flange

(b) Stress of lateral point about flange

Fig. (4). Relationship between width-span ratio ,number of term in series and stress-ratio about cross section of midspan (σ_1 -analytic solutions of this text, σ_2 -Solutions of primary beam theory).

Table 1. Parameters of the Example

Parameter	Uniform Load w (N/mm)	Span L (m)	Steel Elastic Modulus E (N/mm ²)	Concrete Elastic Modulus E_c (N/mm ²)	Steel Poisson's Ratio ν	Concrete Poisson's Ratio ν_c
Value	50	10	210000	32500	0.27	0.2

As shown in, Fig. (4), it shows the relationship between width-span ratio , number of term in Series and the stress-ratio of flange about mid-span section, by using the method to calculate the shear lag effect, where the vertical coordinate expresses the ratio of the longitudinal stress about concrete slab and about elementary beam theory.

As shown in Fig. (4), the convergency is quite fast for a uniformly distributed load. And the wide-span ratios of T-girder flange has little influence on the convergency, where the convergency of the longitudinal stress about the concrete plate of mid-span is shown for different wide-span ratios and the shear lag coefficient(λ) is defined. Shear lag coefficients about concrete slab increase or reduce with respect to its wide-span ratio.

As shown in Fig. (5), it expresses the normal longitudinal distribution of concrete plate stress, where the finite elements method (FEM) refers to ANSYS program. The calculated values with FEM is larger than that with the proposed method because of stress concentration at the intersection with the flange of the Web.

Fig. (5). Transverse distribution of concrete plate stress about mid-span.

5. CONCLUSIONS

The proposed method for solving the shear lag effect of the composite T-girders has fast convergency from simple calculations. The precision of result is enough for engineering. The has great effect on the shear lag effect of the composite T-girder. And the shear lag coefficient (λ) increases or decreases with the change of the wide-span ratios. Thus, it may be paid more attention to the wider T-girders. The method can also be applied for multi-span girders which may be divided into the simply supported and cantilevered girders.

CONFLICT OF INTEREST

The author confirm that this article content has no conflict of interest.

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