Advances in Complex Systems

Emergence of Scale-Free Networks in Markets

Jie-Jun Tseng and Sai-Ping Li
Institute of Physics, Academia Sinica
Taipei, Taiwan

Shu-Heng Chen
AI-Econ Research Center and Department of Economics, National Chengchi University
Taipei, Taiwan

Sun-Chong Wang
Institute of Systems Biology and Bioinformatics, National Central University
Chungli, Taiwan

Received (received date)
Revised (revised date)

Financial markets are complex systems, all the information scattered around the market is fairly and dynamically reflected in the current prices. However, it is difficult to understand the dynamics of markets merely by traditional analyzing methods. We here propose a new concept inspired by complex networks to study the trading behavior and the dynamics of markets. A web-based platform for prediction market which trades the political futures contracts is built to monitor the trading behavior among the human players. Two experiments are conducted on this platform in parallel for 30 days. From the accumulated transaction data, we reconstruct the so-called cash-flow networks. By examining the degree distributions of these networks, we observe that the network structure is scale-free with a power-law exponent of 1.15±0.07, which means that there must be a non-trivial mechanism governing the network growth in our markets. Through carrying out a post-simulation modelling by a continuous double auction market with "zero-intelligence" traders, we demonstrate that such a simple model is capable of generating scale-free networks. We thus suggest that the scale-free nature of the cash-flow networks should rely on the institutional design and the structure of markets rather than on the traders' strategies.

Keywords: prediction market; complex networks; political futures.

1. Introduction

Complex networks, exhibit several non-trivial topological features (including a fat-tail in the degree distribution, a high clustering coefficient, community structure at many scales, and a hierarchical structure), emerge in many complex systems such as biological [1], social [2] and technological [3] systems. Although these systems have been modeled as random graphs in the past, more and more evidence suggests that the topology and evolution of these networks are governed by robust organizing principles [4]. We can say that the network topology evolves to fulfill system requirements. Studying network systems thus help us to have better insight into the complex systems.

In economics, financial markets are complex systems as well. The prices and individual wealth in the market are driven up and down by the so-called "invisible hand" coined by *Adam Smith*. Although we know that these fluctuations are resulting from the interactions among traders within the market, it is difficult for us to make any accurate prediction about the markets. In a naive thinking, since the network can be applied to explain the relations or interactions among each element, we shall be able to map the interactions among traders into networks and study them. Therefore, we conduct experiments for gathering the real information about the trading behavior in the market and introduce a new method to map the trading behavior into the networks [5].

In this contribution, we first introduce our system for markets with human traders and show the results of two experiments performed on this system. The trading behavior in these two experiments is mapped into a so-called cash-flow network and we then present the observation of these networks. Finally, a post-simulation, which is designed to reproduce the resulting network topology, will be mentioned. The simulation is modeled as a continuous double-auction (CDA) market with zero-intelligence traders.

2. Market with Human Traders

Markets are open systems where intelligent traders interact with each other with some simple trading rules. For an orderly market, the price reflects the underlying value of the market instruments. But when bubbles develop, the orderly behavior breaks down and markets become complex systems. In the bottom-up scheme, if we want to study a complex system, we need to learn how individual elements interact with each other in our system. Following this concept, we build a virtual market for human traders which allows us to monitor their trading behaviors in real-time. We believe that the transactions among traders represent the strength of the interactions (or relations) between them. Therefore, we might have deeper insight into the market with these transaction data.

2.1. *A Web-based Futures Exchange System*

Prediction market [6, 7] is a market designed to run for the primary purpose of mining and aggregating information scattered among traders. The aggregation of the information will therefore be reflected in the form of market prices so as to make predictions about specific future events. From the concept of prediction market, we design a system which allows the registers to trade the political futures contracts on web and enable us to monitor the transactions among the traders [8, 9, 10]. Although we use the virtual money for the trading, the principles and operation of our system follow those of major financial exchanges in the real world. Our system works as a web-based server which runs for 24 hours a day until we shut it down on the day of liquidation. Any web browser can participate in the trading by on line registration. An account with the user-provided login name is created for the participant after registration. An initial amount of (virtual) money is deposited by the server to the newly created account. The initial wealth for each participant is the same. The demographics about all the registers to date is also updated. The process takes place on the server automatically and a trader can start trading almost immediately after successful registration. The demography, price fluctuation and accumulated volumes plots are open to any Web surfer irrespective of her registration or not. However, only registered users can trade upon login.

Once the user login onto the server, he can buy bundles of contracts from the server for a guaranteed price per bundle or buy the futures contracts from the market directly. In our system, a given political futures contract is associated with the liquidation price which equals the percentage of votes that a candidate gets on the day of election. A bundle, by design, consists of futures contracts for each candidate in the race as well as for all the invalid casts. After the election, all the futures contracts in the account should be liquidated. The bundle price of 100 is fair since neither the user nor our server loses. Transactions are free in our system and no further service fees will be charged. Users can place market or limit orders to buy or sell futures contracts. Our system then stores and sorts the submitted bid (ask) orders in a bid (ask) queue and matches counterpart orders which are compatible with each other's price limits. If no matches are met, limit orders stay in the queue and wait for further matches with new orders. These limit orders would either expire or be canceled by users before the matches. Market orders do not stay if no matches are found. Order matching is via the process of continuous double auction (CDA) which realizes price formation by the law of supply and demand while maximizing transaction volumes. In brief, it materializes the concept that excessive buying/selling drives up/down prices. It is the price discovery system widely used by exchange markets in the world, including New York Stock Exchange, Tokyo Stock Exchange, SBF - Bourse de Paris, and the Stock Exchange of Hong Kong.

The server records user's trading activities and results. After a transaction, the account assets, including cashes and futures contracts, is balanced immediately. The price of the contract is decided by the current market price. Therefore, even if a user dose not transact anymore, as long as he owns the contracts, his assets would vary depending on current market prices of the contracts, The account in our server earns no interest. When a limit order is placed but the execution is not complete, the system will block further order submissions by this user. This rule is meant to prevent the server from reckless submissions since there are no transaction fees and the money is virtual.

2.2. *Experimental Design*

In this analysis, we take the data from the experiment on Taipei mayoral election in Taiwan on December 9, 2006. The system issued six futures contracts which consisted of five candidates ran for Taipei mayor and one for any invalid ballots cast on the election day. The sum of the prices of these six contracts are set to 100 at the beginning. Afterward, the sum should remain 100 if the traders are rational or if the market is efficient. The virtual money of amount 30,000 is deposited by the system for each account to begin with. Two servers ran in parallel for the experiment at that time. One is AI-ECON futures exchange (AI-ECON FX [a]) and the other is Taiwan Political Exchange (TAIPEX [b]). AI-ECON FX and TAIPEX are almost identical in design except for the traders of the former one can chose a preliminary software agent for the trading. Both servers started to run 30 days before the day of liquidation. At the end of the experiment, any contracts in the accounts were liquidated using the official result of votes from the government. Money prizes were then awarded to the top ten winners determined by the ultimate wealth in the players' accounts.

By analyzing the change of trading volumes in minutes, we observe that the market was active about 11% of the time in AI-ECON FX and 12% in TAIPEX. The number of registered players increased monotonically with time in both servers. Before the end of the experiments, AI-ECON FX and TAIPEX have accumulated 532 and 628 registrants respectively. The number of successful transactions totaled 7,440 in AI-ECON FX and 8,573 in TAIPEX. We further analyzed the transaction data to distinguish the active players from those who never traded with others throughout the whole experiment. After filtering, we found that there are 366 (427) active players left in AI-ECON FX (TAIPEX), which implies that only about 68% of registrants were active in both servers.

2.3. *The Cash-flow Network*

In a previous analysis [10], we showed that such a market, which accumulated typically 400 participants, exhibited power-law distributions of price fluctuation, net wealth and inter-transaction times that are characteristic of real world markets. Furthermore, predictions of the market have so far been consistent with election outcomes. In this contribution, being inspired by the recent development of complex networks, we introduce a new concept to study the trading behavior in a financial market. Our concept is detailed as follows.

If we treat each trader in the market as a node, and subsequently the transactions among them could be referred to as the edges. Therefore, we can reconstruct a network with traders and transactions. In our experiments, players trade futures contracts. When a transaction was made between traders i and j with volume v and

[a] http : //futures.nccu.edu.tw/exchange/exchange_eng.html
[b] http : //socioecono.phys.sinica.edu.tw/exchange/exchange_eng.html

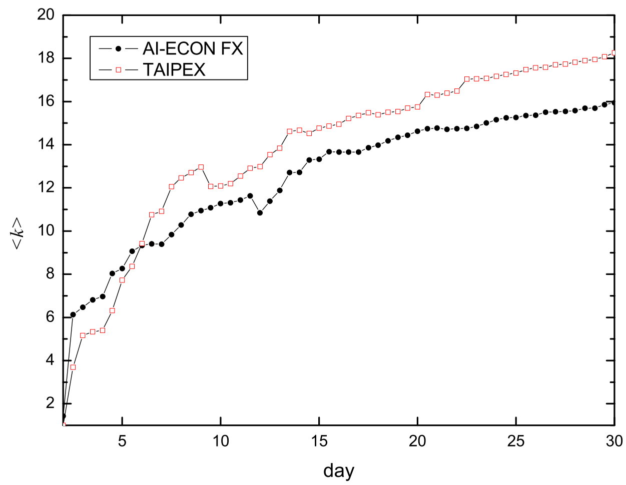

Fig. 1. The growth of $\langle k \rangle$ with time in AI-ECON FX (dot) and TAIPEX (square).

price p, an amount of cash $p \times v$ flowed from i to j. Because the flow is directional and accompanied with certain amount of cash, the resulting cash-flow networks should be directed, weighted and contains no self-loops. In order to scale down the complexity of our problem and to extract the essence from the trading behavior, we simplify our cash-flow network into an undirected and unweighted network throughout this analysis. The preliminary results for cash-flow networks with directed and weighted edges are discussed in Ref. 5. We also assume that non-active players, who never trade during the whole experiment, would scarcely affect our results. We therefore neglect all the isolated nodes in our following analysis. During the experiment, both servers output the accumulated cash flow among traders in every 12 hours, from which we reconstructed 60 networks for each server. To understand the growth rate of the edges in these networks, we plot the value of $\langle k \rangle$, the average number of edges per node, in time series. In Fig. 1, one can see that the value of $\langle k \rangle$ grew with time, topping at 15.94 (18.26) in AI-ECON FX (TAIPEX) on day 30. We observe that the growth rate in our experiment keeps almost a constant (about 0.2 per day) after the first 15 days of the running. Fig. 2 shows the network on day 3 in AI-ECON FX. One can easily identifies hubs in networks like the one here, which usually accompany with the small world properties. To figure out whether the cash-flow networks are scale-free or not, we calculate the degree distribution of our networks. The degree distribution, $p(k)$, describes the number of nodes to have k edges. In Fig. 3, we show the resulting $p(k)$ of the cash-flow networks on day 15 and day 30 in logarithmic scale with the linear fits. The distributions $p(k)$ plot

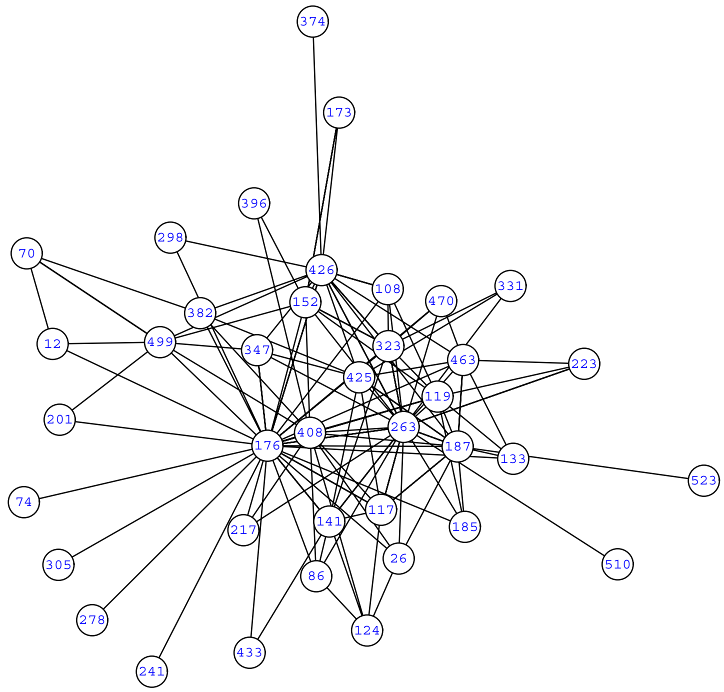

Fig. 2. The structure of the cash-flow network developed in AI-ECON FX on day 3. The number on the vertex corresponds to the ID for different traders assigned in the last day.

here have been divided by the number of nodes for comparison. We found that the degree distributions of these networks can be well explained by a power-law decay, which has the following form:

$$p(k) \sim k^{-\gamma} \tag{1}$$

We have $\gamma = 1.13 \pm 0.08$ and $\gamma = 1.17 \pm 0.06$ for AI-ECON FX and TAIPEX on day 30 respectively consistent with each other. We also found that these exponents almost remain the same during the last 15 days. This result might have something to do with the fact that the growth of $\langle k \rangle$ also remains roughly the same rate from day 15 to the end of experiment. A power-law decay of $p(k)$ with k suggests excessive presence of hubs in our network. In other words, the networks reconstructed from the transactions among traders in our markets are scale-free. Since the traders in our markets are not supposed to communicate with each other, it is hard to imagine that why the transactions among them could develop into such a scale-free structure. One explanation is that the aggressive traders transact many times in order to make profits from others. But why are the distributions of these aggressive traders (i.e., hubs in our networks) almost the same in both servers is not that clear. To further

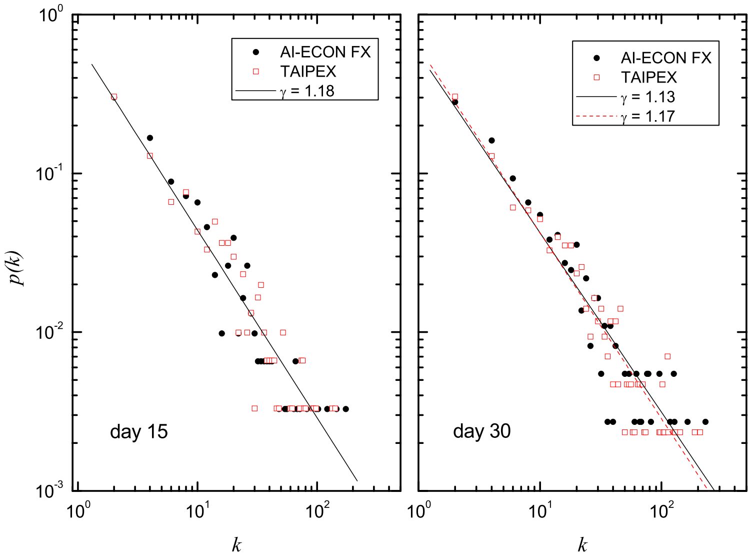

Fig. 3. The degree distribution of the cash-flow networks on day 15 (left) and day 30 (right) for AI-ECON FX (dot) and TAIPEX (square). On day 15, the $p(k)$ in both servers could be well fitted by a power law decay with $\gamma \sim 1.18$. While on day 30, the best fit is $\gamma = 1.13 \pm 0.08$ and $\gamma = 1.17 \pm 0.06$ for AI-ECON FX and TAIPEX respectively.

explain the observed phenomenon, we conduct a simulation to figure out whether the scale-free behavior in our cash-flow networks is due to the interactions among traders or due to the institutional design of our market.

3. Markets with Zero-Intelligence Traders

Inspired by the approach of agent-based modeling, in the first attempt, we model our simulation as a continuous double-auction (CDA) market with zero-intelligence traders (ZI traders). The ZI traders, by definition, are agent traders without any intelligence. In the markets, they will submit random bids and offers, therefore the resulting price never converges toward any specific level. We here adopt the definition by *D. K. Gode* and *S. Sunder* [11], which demonstrate that in a symmetrically structured market, by imposing a simple budget constraint (i.e., the ZI-C traders who must profit from the transaction), the allocative efficiency of these transactions could be raised close to 100%. Hence, the trader in our simulation is the ZI trader with a simple budget constraint.

There are many variations of CDA markets, we made two choices to simplify our simulation. Firstly, each bid, offer and transaction is valid for a single item. Secondly, there is no transaction cost and the items are durable. Thirdly, in each duration, every trader could make only one successful transaction (i.e., the buyer could only have one item and the seller only has one item to sell in each duration). The implementation of our simulation is as follows: For the structure of markets, the supply and demand functions are generated from *Smith*'s value mechanism [12] at the beginning for each run and will not change through the end of simulation. The price for the item is ranging from 1 to 100 in units of virtual dollars. Because the ZI traders could only perform well in a symmetrically structured market [13], we choose the markets of this type in our simulation. In these markets, the intersection of supply and demand curves determines the equilibrium price. For the traders, initially, there is a fixed number of ZI traders in our simulation. Half of them are classified as buyers and the remaining half of the traders are sellers. At each step, one buyer and one seller are chosen for the matching. Due to the budget constraint, the buyer must bid with the price lower than its redemption value given by the demand function and the seller must offer the commodity at the price higher than the cost generated by the supply function. Once the bidding price exceeds the offering price, the transaction between this buyer and seller is made. No transaction will be made otherwise. Whether there exists a successful transaction or not, the system will move forward to the next step and choose another pair of traders. The simulation lasts for p periods of a specific duration d and terminates after $p \times d$ steps. One transaction represents one edge in the network, but since our network is unweighted, repeated transactions between the same pair of traders will only be counted once. Each simulation runs for 100 times and the resulting degree distribution is an average over these 100 runs.

One should keep in mind that, although the number of traders is fixed at first, not all of them will make a successful transaction with others. The final number of nodes connecting to the whole network (i.e., traders with successful transactions) and $\langle k \rangle$ will also depend on the input value of period and duration. In comparison with the experimental result we have in the market with human traders, we require the resulting cash-flow network to have 400 nodes on average with the value of $\langle k \rangle$ around 16. The value of input parameters must satisfied with the above condition. Fig. 4 shows the average degree distributions of the cash-flow networks from the simulations with two different sets of input parameters. We observe that the distribution follows a perfect power law decay with an exponent $\gamma \sim 0.59 \pm 0.04$. The sudden drop of the distribution curve at $k \sim 30$ might be due to the finite size effects.

To further justify this observation, we change the value of the input parameters for obtaining a network with larger network size. One can see that the decay behavior of $p(k)$ remains unchanged even for $n = 3500$ in Fig. 5. The decay exponent for this large network is $\gamma \sim 0.62 \pm 0.02$ which is roughly the same as the exponent in networks with $n = 400$. From the above result, it suggests that the nature of power-law decay of $p(k)$ depends on neither the network size nor the value of input parameters. Although the power-law exponent resulting from the simulation could not explain the observed exponent in the markets with human traders, we might

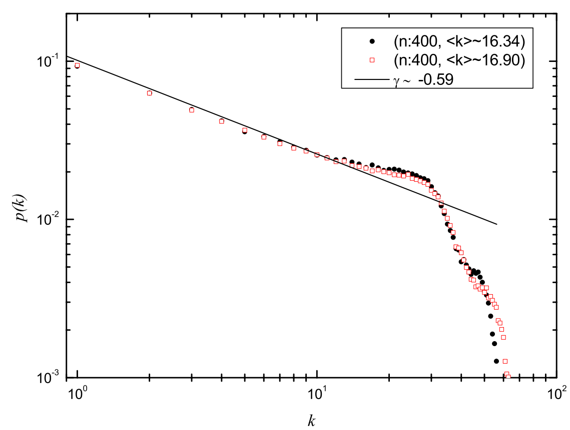

Fig. 4. The degree distributions of the cash-flow networks resulting from the simulations with the network size $n = 400$ and $\langle k \rangle \sim 16$. The solid line is the power law fitting with the exponent $\gamma \sim 0.59$.

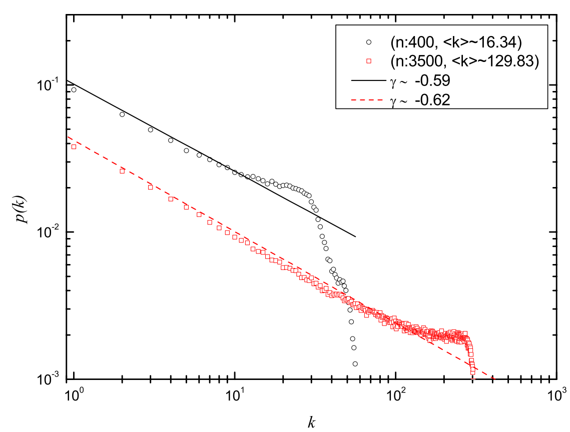

Fig. 5. The comparison of the degree distributions of the cash-flow networks with different network size. $n = 400$ for the circles and $n = 3500$ for the squares. The exponents for the power-law decay are $\gamma \sim 0.59$ (solid) and $\gamma \sim 0.62$ (dashed) respectively.

still come up with a conclusion that the scale-free nature of the cash-flow networks dose not rely on the intelligence of the traders. Therefore, we believe that the scale-free nature comes from the institutional design and the structure of markets (i.e., the supply and demand in the markets).

4. Conclusion

In this work, we introduce a new concept to analysis the trading behavior in a financial market. In order to realize this approach, we design a web-based futures exchange system in order to gather enough information about the transactions among traders in a market. Two experiments were conducted with our system on different servers (AI-ECON FX and TAIPEX) for 30 days. 7,440 (8,573) entries of transactions were accumulated and recorded in AI-ECON FX (TAIPEX). We reconstructed the cash-flow networks with these transaction data and found that these networks exhibited scale-free properties with a power-law exponent around 1.15. To further comprehend the underlying mechanism of the observed phenomena, we carry out a simple simulation involving a CDA market with zero-intelligence traders. We demonstrate that such a simple market is capable of forming a scale-free network structure. Although the power-law exponent resulting from this toy model, which is only around 0.6, could not explain the observed exponent in the markets with human traders. But the most important finding in this analysis is that the scale-free topology in the cash-flow networks might rely on the institutional design and the structure of markets rather than on the traders' strategies. In our simulation, all the agents are equiped with the same strategies. Therefore, it is the supply and demand function that determines which trader should play as the hub in the network. In addition, we also believe that the scale-free nature of the resulting networks might relate to the efficient market hypothesis (EMH)[14] and this relation should be revealed by future studies.

Acknowledgments

The research was supported in part by the National Science Council of Taiwan under grants NSC#95-2415-H-004-002-MY3 and NSC#96-2112-M-001-001.

References

[1] P. Uetz, et al., A comprehensive analysis of protein-protein interactions in Saccharomyces cerevisiae, *Nature* **403**, 623–627 (2000).
[2] Newman, M.E.J., The structure of scientific collaboration networks, *Proc. Natl. Acad. Sci. U.S.A.* **98**, 404–409 (2001).
[3] Guimera, R., Mossa, S., Turtschi, A. and Amaral, L.A.N., The worldwide air transportation network: Anomalous centrality, community structure, and cities' global roles, *Proc. Natl. Acad. Sci. U.S.A.* **102**, 7794–7799 (2005)
[4] Albert, R. and Barabasi, A.L., Statistical mechanics of complex networks, *Rev.Mod.Phys.* **74**, 47–97 (2002)

[5] Wang, S.C., Tseng, J.J., Tai, C.C., Lai, K.H., Wu, W.S., Chen, S.H. and Li, S.P., Network Topology of an Experimental Futures Exchange, arXiv:0705.2551 [physics.soc-ph] (2007), to appear in *Eur. Phys. J. B.*

[6] Berg, J.E., Nelson, F. and Rietz, T.A., Accuracy and Forecast Standard Error of Prediction Markets, working paper (2003).

[7] Berg, J.E. and Rietz, T.A., Prediction Markets as Decision Support Systems, *Information Systems Frontiers* **5**, 79–93 (2003)

[8] Wang, S.C., Yu, C.Y., Liu, K.P. and Li, S.P., A Web-based Political Exchange for Election Outcome Predictions, *Proc. IEEE/WIC/ACM International Conference on Web Intelligence (WI' 04)*, 173–178 (2004)

[9] Wang, S.C., Tseng, J.J., Li, S.P. and Chen, S.H., Prediction of Bird Flu A(H5N1) Outbreaks in Taiwan by Online Auction: Experimental Results, *New Mathematics and Natural Computation* **V2 N3**, 271–279 (2006)

[10] Wang, S.C., Li, S.P., Tai, C.C. and Chen, S.H., Statistical Properties of an Experimental Political Futures Market, to appear in *Quantitative Finance.*

[11] Gode, D.K., and Sunder, S., Allocative Efficiency of Markets with Zero-Intelligence Traders: Market as a Partial Substitute for Individual Rationality, *J. Polit. Econ.* **V101 N1**, 119–137 (1993)

[12] Cliff, D., and Bruten, J., Zero is not enough: on the lower limit of agent intelligence for continuous double auction markets, Technical report, **HPL-97-141** (1997)

[13] Smith, V.L., Experimental Economics: Induced Value Theory, *A.E.R. Papers and Proc.* **66**, 274–279 (1976)

[14] Fama, E.F., Efficient Capital Markets: A Review of Theory and Empirical Work, *Journal of Finance* **V25 N2**, 383–417 (1970)