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Cooperative Communication in Wireless Systems

Elza Erkip, Andrew Sendonaris, Andrej Stefanov, and Behnaam Aazhang

ABSTRACT. The broadcast nature of wireless communications suggests that a source signal transmitted towards the destination can be "overheard" at neighboring nodes. Cooperative communication refers to processing of this overheard information at the surrounding nodes and retransmission towards the destination to create spatial diversity, thereby to obtain higher throughput and reliability. In this paper we describe information theoretic models suitable for cooperation, study achievable rate regions and outage probabilities, and describe channel coding techniques that allow us to exploit the diversity advantages of cooperation.

1. Introduction

Ad-hoc wireless networks are based on multi-hop communications, where the information from the source to the destination is relayed via other mobiles. An ad-hoc network does not have a fixed infrastructure, so this relaying operation is essential in order to overcome the path loss incurred over large distances. Multi-hop ideas are also utilized in cellular and wireless LAN systems to provide higher quality of service, power savings and extended coverage.

Information theory of multi-hop communication dates back to the relay channel model, which contains a source, a destination and a relay whose goal is to facilitate information transfer from the source to the destination. The relay channel was introduced by Van der Meulen [**23**] and investigated extensively by Cover and El Gamal [**3**]. Cover and El Gamal provided a number of relaying strategies, found achievable regions and provided upper bounds to the capacity of a general relay channel. They also provided an expression for the capacity of the degraded relay channel, in which the communication channel between the source and the relay is physically better than the source-destination link. The capacity of the general relay channel is still unknown. Motivated by the recent interest in multi-hop, a number of recent papers investigate the use of multiple relays. Some relevant references include [**7, 8, 9, 15, 18, 19, 26**].

Even though the information theoretic model allows for the destination to listen to both the source and the relay, in most multi-hop systems the destination only processes the signal coming from the relay. This is justified in a wireless channel

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where path loss has the dominant effect. Since the source is generally further away from the destination than the relay, the received signal at the destination due to the source would be much weaker than the relay signal. However, when fading is also taken into account, this scheme would incur considerable loss, especially in diversity, compared to one in which the destination processes both signals. Hence one can use multi-hop not just to overcome path loss, but also to provide diversity.

Motivated by the above observation, cooperative communication involves two main ideas: (i) Use relays (or multi-hop) to provide spatial diversity in a fading environment, (ii) Envision a collaborative scheme where the relay also has its own information to send so both terminals help one another to communicate by acting as relays for each other (called "partners").

One can think of a cooperative system as a virtual antenna array, where each antenna in the array corresponds to one of the partners. The partners can overhear each other's transmissions though the wireless medium, process this information and re-transmit to collaborate. This provides extra observations of the source signals at the destinations, the observations which are dispersed in space and usually discarded by current implementations of cellular, wireless LAN or ad-hoc systems. However, since the elements of this array are not co-located and are connected via noisy, fading links, it is not clear a priori how much the benefits of this cooperation would be. Our goal in this paper is to argue that the benefits, in terms of achievable data rates, diversity and error performance, are significant. In our discussion, we briefly describe some of our prior and ongoing work along with relevant literature, and provide directions for future research. We first provide an information theoretic model to describe the cooperative system, a set of achievable rates and an outage probability analysis. Then we talk about how one can design and analyze channel codes that can exploit the predicted benefits of a cooperative system.

2. Information Theoretic Model and Analysis

For ease of exposure, we consider a cooperative system consisting of two source terminals denoted by T_1 and T_2. These terminals could communicate with a common destination (such as a cooperative system in cellular) or more generally could have separate destinations. Even though the cooperative concept is equally applicable to both cases, we will constrain ourselves to the common destination case for the analysis. Our model is illustrated in Figure 1. Mathematically we have the discrete-time model

$$Y_0 = K_{10}X_1 + K_{20}X_2 + Z_0 \tag{2.1}$$

$$Y_1 = K_{21}X_2 + Z_1 \tag{2.2}$$

$$Y_2 = K_{12}X_1 + Z_2 \tag{2.3}$$

where Y_0 denotes the signal received at the common destination, Y_1 and Y_2 denote the overheard signals at each of the partners and X_1 and X_2 represent the signals transmitted by terminal 1 and 2 respectively at a particular time instant t. The noise terms Z_0, Z_1 and Z_2 are independent from each other and have iid complex Gaussian samples in time with variances σ_i^2, $i = 0, 1, 2$ respectively. We assume a flat fading Rayleigh channel with complex Gaussian channel gains K_{ij}. The gains K_{10}, K_{20} and K_{12} are independent as they correspond to different links. In general, the relationship between K_{12} and K_{21} depends on how the partners access the channel. We will assume that the channel between two partners is reciprocal

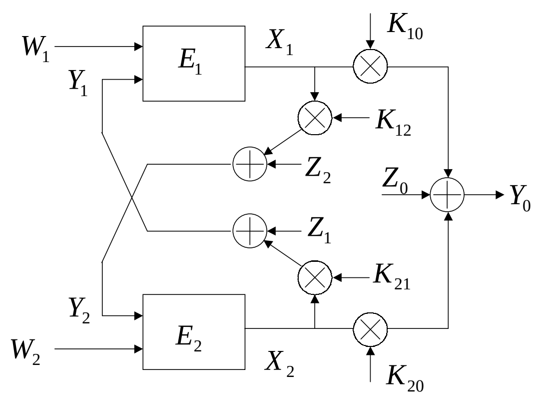


FIGURE 1. Block diagram of a fading Gaussian cooperative channel

leading to $K_{12} = K_{21}$. However, this assumption is not necessary for our results and can be relaxed. For the achievable rate region, we use the notion of ergodic capacity, so we do not make any assumptions about the time variations of the channel except that they form an ergodic random process. When we consider outage probabilities and frame error rates corresponding to particular channel codes, we will assume that the channel varies slowly and is constant during the communication session.

We allow for both partners to transmit and receive at the same time and we assume that each terminal can perform perfect echo cancelation. As illustrated in equations (2.2) and (2.3) this leads to Y_1 (Y_2) having no contribution from X_1 (X_2). We also assume that both terminals are synchronized. Even though these assumptions may not hold in a practical system, they are quite common in the information theory literature and they allow us to investigate the fundamental benefits of cooperation. We will argue in Section 3 that even when we relax some of these assumptions, cooperative communication still provides considerable improvement over non-cooperative systems.

All of the receivers have perfect channel state information, that is they can exactly measure the channel gain of the incoming signals. For the achievable rate region and outage probability analysis in this section, we also assume that there is some partial channel state information at transmitters in the form of the phase of the complex channel gains K_{10} and K_{20}. This allows the partners to form signals that coherently combine at the destination. We relax this assumption in Section 3.

2.1. Achievable Rate Region. This section summarizes some of our results from [**20, 21**]. We provide an achievable rate region for the two-user cooperative model above. We make the observation that this model is contained in the multiple access channel with generalized feedback which was analyzed for discrete memoryless and Gaussian channels in [**2, 24**]. The achievability results will make use of the signal structure of [**24**]. Our main contribution will be to illustrate the potential diversity and throughput benefits of cooperation in a wireless network with varying link qualities.

In general, causality of the system suggests that $X_{1,t}$ the signal transmitted by T_1 at time t can only depend on the message of T_1, W_1 and what T_1 has "overheard" so far, $(Y_{1,1}, \ldots, Y_{1,t-1})$. However, we will use a more restricted signaling strategy based on superposition block Markov encoding [**4**], a technique which has been successfully used in multi-user systems involving feedback or some form of an inter-user link. We envision transmission for B blocks, each of length n. Both B and n are assumed to be large. In fact, for the achievable rate region, n has to be large enough to observe different fading levels so that ergodic capacity is the right notion to use. In each block, both terminals transmit a new message. The transmitted signal from T_1 in block j depends on $W_1(j)$, the message in that block and $(\mathbf{Y}_1(1), \ldots, \mathbf{Y}_1(j-1))$, all the signals received up to and including block $j-1$. Note that $\mathbf{Y}_1(i)$ denotes n observations of the output signal in block i. The destination uses backward decoding [**25, 27**], that is starts decoding from the last block and moves one by one to the first.

We next state the achievable region for the cooperative channel, and provide a sketch of the proof which illustrates the signal structure. We then provide comparisons with the non-cooperative capacity region.

THEOREM 2.1. *An achievable rate region for the cooperative system given in (2.1)-(2.3) is the closure of the convex hull of all rate pairs* (R_1, R_2) *such that* $R_1 = R_{10} + R_{12}$ *and* $R_2 = R_{20} + R_{21}$ *with*

$$R_{12} < E\left\{C\left(\frac{|K_{12}|^2 P_{12}}{|K_{12}|^2 P_{10} + \sigma_1^2}\right)\right\} \tag{2.4}$$

$$R_{21} < E\left\{C\left(\frac{|K_{21}|^2 P_{21}}{|K_{21}|^2 P_{20} + \sigma_2^2}\right)\right\} \tag{2.5}$$

$$R_{10} < E\left\{C\left(\frac{|K_{10}|^2 P_{10}}{\sigma_0^2}\right)\right\} \tag{2.6}$$

$$R_{20} < E\left\{C\left(\frac{|K_{20}|^2 P_{20}}{\sigma_0^2}\right)\right\} \tag{2.7}$$

$$R_{10} + R_{20} < E\left\{C\left(\frac{|K_{10}|^2 P_{10} + |K_{20}|^2 P_{20}}{\sigma_0^2}\right)\right\} \tag{2.8}$$

$$R_{10} + R_{20} + R_{12} + R_{21} < E\left\{C\left(\frac{|K_{10}|^2 P_1 + |K_{20}|^2 P_2 + 2|K_{10}||K_{20}|\sqrt{P_{U1} P_{U2}}}{\sigma_0^2}\right)\right\}, \tag{2.9}$$

for some power assignment satisfying $P_1 = P_{10} + P_{12} + P_{U1}$, $P_2 = P_{20} + P_{21} + P_{U2}$. *The function* $C(x) = \frac{1}{2}\log(1 + x)$ *is the capacity of an additive white Gaussian noise channel with signal to noise ratio* x *and* E *denotes expectation with respect to the fading parameters* K_{ij}.

PROOF. The proof follows [**24**]. Here we sketch it briefly to illustrate the signal structure and decoding scheme.

In the cooperative system, unlike the degraded relay channel, one cannot ensure the partner will always be able to receive more information than the destination. Hence for every block j, T_1 divides its information $W_1(j)$ into two parts, one to be relayed through the partner ($W_{12}(j)$ at rate R_{12}), the other to be transmitted directly to the destination ($W_{10}(j)$ at rate R_{10}). The transmitted signal X_1 consists of three parts X_{10}, used to send W_{10}, X_{12} used to send W_{12}, and U_1 which ensures cooperation. The respective powers allocated to these signal components are P_{10}, P_{12} and P_{U1}. We have

$$\mathbf{X}_{10}(j) = \sqrt{P_{10}}\tilde{\mathbf{X}}_{10}(W_{10}(j), W_{12}(j-1), W_{21}(j-1)) \tag{2.10}$$

$$\mathbf{X}_{12}(j) = \sqrt{P_{12}}\tilde{\mathbf{X}}_{12}(W_{12}(j), W_{12}(j-1), W_{21}(j-1)) \tag{2.11}$$

$$\mathbf{U}_{1}(j) = \sqrt{P_{U1}}e^{-j\theta_{10}}\tilde{\mathbf{U}}(W_{12}(j-1), W_{21}(j-1)), \tag{2.12}$$

with $\mathbf{X}_1(j) = \mathbf{X}_{10}(j) + \mathbf{X}_{12}(j) + \mathbf{U}_1(j)$ and $P_1 = P_{10} + P_{12} + P_{U1}$. We assume $K_{10} = |K_{10}|e^{j\theta_{10}}$, hence θ_{10} is the phase of the complex fading between T_1 and the destination. Here $\mathbf{X}_1(j)$ denotes n transmitted symbols for block j. The entries of the random codebooks are generated by choosing $\tilde{\mathbf{X}}_{10}$, $\tilde{\mathbf{X}}_{12}$ and $\tilde{\mathbf{U}}$ independently, all consisting of n iid samples of $\mathcal{N}(0,1)$ distribution. The signal transmitted by T_2, $\mathbf{X}_2(j)$, is generated in a similar fashion. Note that $\tilde{\mathbf{U}}$ is used by both terminals in their codebooks.

We observe that the signal $\mathbf{X}_1(j)$ not only depends on $W_1(j) = (W_{10}(j), W_{12}(j))$, but also on $W_{12}(j-1)$ and $W_{21}(j-1)$. Obviously, T_1 knows $W_{12}(j-1)$. We will also ensure that R_{21}, the rate of transmission for W_{21}, is chosen so that W_{21} can be perfectly recovered at T_1. Therefore at block j $(W_{12}(j-1), W_{21}(j-1))$ will be known at both terminals and can be used to form the basis for cooperation.

In order to find out the conditions on R_{12} (R_{21}) for perfect recovery of W_{12} (W_{21}) at the partner, we first look at block 1. We assume $(W_{12}(0), W_{21}(0)) = (0, 0)$. Terminal 2 receives $\mathbf{Y}_2(1) = (Y_{2,1}(1), Y_{2,2}(1), \ldots, Y_{2,n}(1))$ where $Y_{2,t}(1)$ is given by equation (2.3) for all $t = 1, \ldots, n$. The signal $\mathbf{X}_1(1)$ depends on $W_{12}(0)$ and $W_{21}(0)$ both of which are known, $W_{12}(1)$ which T_2 is interested in, and $W_{10}(1)$ which T_2 will not attempt to find out. Using the representation in equations (2.10)-(2.12), we observe that $\mathbf{U}_1(1)$ is known and hence can be canceled, $\mathbf{X}_{10}(j)$ will be treated as noise and $\mathbf{X}_{12}(1)$ will be decoded at terminal 2. Assuming the fading is ergodic over the block length n, provided that

$$R_{12} < \mathrm{E}\left\{C\left(\frac{|K_{12}|^2 P_{12}}{|K_{12}|^2 P_{10} + \sigma_1^2}\right)\right\},$$

terminal 2 can estimate $W_{12}(1)$ with arbitrarily low probability of error. Note that this condition is given by equation (2.4) of Theorem 2.1. Based on equation (2.5) we can make similar arguments for perfect recovery of $W_{21}(1)$ at T_1. Moving sequentially from block $j-1$ to block j, $j = 2, \ldots, n$ we can ensure that at the beginning of block j, both partners know $(W_{12}(j-1), W_{21}(j-1))$.

We now illustrate how equations (2.6)-(2.9) lead to reliable transmission of W_{10}, W_{12}, W_{20} and W_{21} to the destination. The decoding starts from the last block [**25, 27**] in which no new information is transmitted. Therefore we can set

$$(W_{10}(B), W_{12}(B), W_{20}(B), W_{21}(B)) = (0, 0, 0, 0).$$

Although this reduces the overall information rate by the factor $(B-1)/B$, the rate loss is negligible for large B. In this block the destination wishes to decode $W_{12}(B-1)$ and $W_{21}(B-1)$. Since $W_{10}(B), W_{12}(B), W_{20}(B), W_{21}(B)$ are known, the condition

$$R_{12} + R_{21} < \mathrm{E}\left\{C\left(\frac{|K_{10}|^2 P_1 + |K_{20}|^2 P_2 + 2|K_{10}||K_{20}|\sqrt{P_{U1}P_{U2}}}{\sigma_0^2}\right)\right\} \tag{2.13}$$

is sufficient for reliable reproduction. The factor $\sqrt{P_{U1}P_{U2}}$ results from the coherent addition of the cooperative signals U_1 and U_2.

Moving into block $B-1$, the destination now has to decode $(W_{10}(B-1), W_{20}(B-1), W_{12}(B-2), W_{21}(B-2))$. Following multiuser achievability results in [**5**] and using [**6, 1**] to replace all the bounds on the rates by their expected values where expectation is over the fading amplitudes, sufficient conditions on R_{10}, R_{20}, R_{12} and R_{21} for asymptotically error free transmission are given by equations (2.6)-(2.9) and (2.13).

Dependency of all the signal components (namely X_{10}, X_{12}, U_1, X_{20}, X_{21} and U_2) on the cooperation information $(W_{12}(B-2), W_{21}(B-2))$ leads to equations (2.9) and (2.13). Since (2.13) is dominated by (2.9) it will be omitted in the final formulation. The destination then proceeds backwards in the same manner until all the blocks are decoded.

By considering different power assignments P_{10}, P_{12}, P_{U1} (and respective powers for T_2) satisfying the total power constraint and time-sharing among different strategies, we obtain the achievable region stated in Theorem 2.1. □

In order to illustrate potential benefits of a cooperative system, we draw the achievable rate region for the proposed cooperative strategy of Theorem 2.1, together with the capacity region of the non-cooperative system in Figure 2. The non-cooperative case corresponds to the usual Gaussian fading multiple access channel. We also include the "ideal" cooperative scenario in which the inter-user channel is perfect. This represents the capacity of a two-antenna system with total power $P_1 + P_2$ and channel phase information at the transmitter. We assume both terminals have the same power constraint and the quality of the channel between each terminal to the destination (represented as the mean of the fading levels K_{10} and K_{20}) is the same. We illustrate the achievable rate region of Theorem 2.1 for various qualities of the inter-user channel, with higher mean of K_{12} corresponding to a better channel. We observe that even when the inter-user channel has similar or worse quality than the user-destination channel, cooperation results in a larger rate region. As the quality of the inter-user channel improves, the achievable rate region greatly increases and gets closer to the ideal multi-antenna system.

Performance of the suggested cooperative system for an asymmetric scenario is shown in Figure 3. The fading for T_1 has higher mean than T_2. This, for example could occur if T_1 is closer to the destination. We observe that cooperation continues to increase the set of achievable rates. Even though the user with severe fading (lower fading mean) benefits more from cooperation, the equal rate $(R_1 = R_2)$ and the maximum rate sum point $(R_1 + R_2)$ is increased considerably with cooperation. This suggests that depending on the system requirements (such as maximizing fairness or the total throughput) even the good user can benefit from cooperation. Section 3 will provide further examples on the symmetric and asymmetric cooperative systems.

We note that relay channel corresponds to $R_1 = 0$ or $R_2 = 0$. Hence in Figures 2 and 3, the points at which any rate curve intersects the x or the y-axis give achievable rates for the corresponding relay channel. Since the cooperative curve is above the line joining $R_1 = 0$ and $R_2 = 0$ points, we argue that cooperation performs better than time sharing between the two relays.

2.2. Probability of Outage. While the ergodic capacity region, or the set of achievable rates, tell us about the long term average throughputs, outage probability [**17**] shows us the robustness of the system to the variations in the channel. In

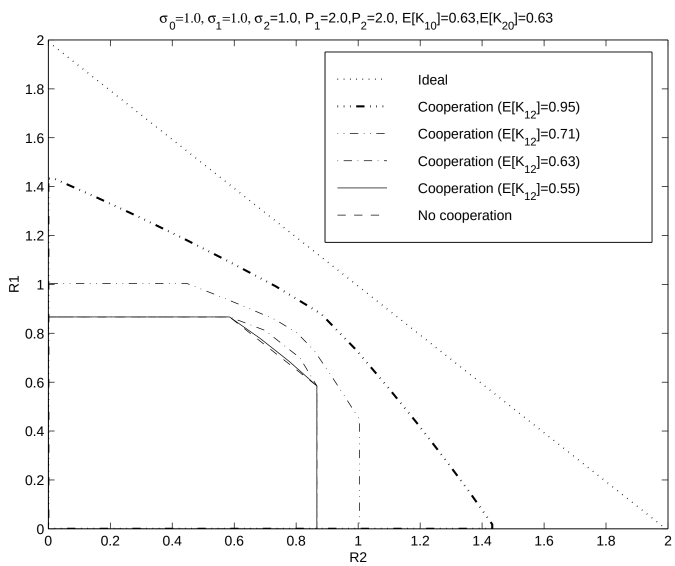


FIGURE 2. Achievable rate region for two-user cooperation. Both users have similar average quality links towards the destination.

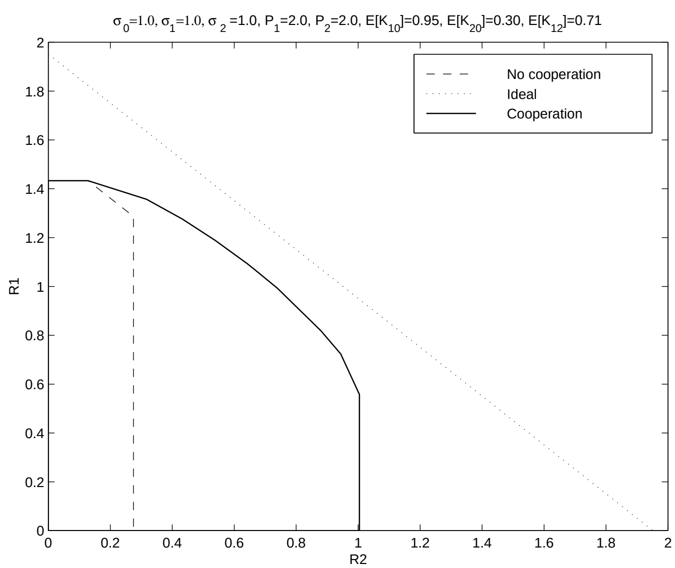


FIGURE 3. Achievable rate region for two-user cooperation. Users have different average quality links towards the destination.

order to calculate the outage probability of the cooperative system, we assume that the fading is slow and remains constant for the duration of B blocks. In that case, the achievable rate region depends on the current values of the fading levels K_{ij} and is random. Assuming the transmit signal structure and the decoding scheme in the proof of Theorem 2.1, equations (2.4)-(2.9) without the expectations lead to an achievable rate region. To illustrate the improvement in outage probability with cooperation, we consider a symmetric system as in Figure 2 and focus on the largest equal rate operating point $R_1 = R_2 = R_{max}$ that can be sustained by co-operation. Note that R_{max} is random and depends on the fading levels of all the links involved. The outage probability $P_{out} = P(R_{max} < r)$ tells us the probability of a cooperative system being able to sustain an equal operating rate of r for both users.

In Figure 4 we show the outage probability of the cooperative scheme as a function of the operating rate r. For comparison, we also plot the equal rate outage probabilities of a non-cooperative (multi-access) system and an ideal system where the inter-user channel is error free. We observe that the outage probability for the cooperation scheme is smaller than that of no cooperation. This is true despite the fact that the increase in achievable rate due to cooperation is moderate for the scenario depicted in Figure 4, as can be seen from Figure 2 ($\mathrm{E}[K_{12}] = 0.63$). Hence even in cases when it does not significantly increase achievable rates, cooperative communication is still able to increase robustness against channel variations. This is due to increased spatial diversity of the system; with cooperation, partners are able to utilize each other's links towards the destination in case their own links fail. To illustrate this further, Figure 5 shows the equal rate outage probability as a function of the user signal to noise ratio (SNR) for a fixed rate $r = 0.18$. We observe that the cooperation curve falls steeper than no cooperation and is parallel to the ideal case. Since the slope of this curve for high SNR illustrates the level of diversity, we can conclude that cooperation provides diversity equivalent to that of a two-antenna array. Comparison with a worse inter-user channel ($\mathrm{E}[K_{12}] = 0.45$) shows that the diversity gains are still present. However now the coding gain is less, that is the outage curve exhibits a shift to the right. We of course expect the performance to depend on the inter-user channel quality. This will be illustrated further in Section 3.

3. Coding for Cooperative Systems

In this section we illustrate how cooperation continues to provide substantial performance gains over a non-cooperating wireless system when we incorporate some practical constraints. We will review some recent literature and describe some of our ongoing work on channel coding for cooperation.

In order to model a more practical system, we have to change some of the assumptions of Section 2. In Section 2 we had allowed both nodes to listen to each other's transmissions and transmit their own information at the same time. Even though this is standard in information theory literature, in practice a wireless device cannot perform perfect echo-cancelation, hence it cannot simultaneously transmit and receive. To incorporate this constraint, we assume that the terminals transmit and receive at different times. Also, in order to have a simple receiver which does not employ multiuser decoding, we assume the terminals have an orthogonal multiple accessing scheme. As an example we consider time division among nodes, that is when there is no cooperation, each terminal has a separate time slot consisting of N uses of the channel as in Figure 6(a). When they cooperate, as shown in Figure 6(b), each terminal divides its time slot into two equal parts. In the first $N/2$ channel uses, the terminal that owns the time slot transmits a signal. This signal is received

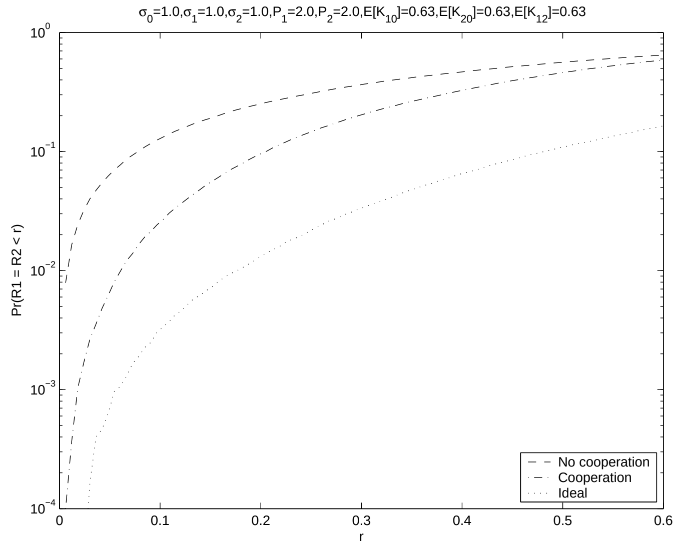


FIGURE 4. Probability of outage versus common rate r for two-user cooperative system. Both users have similar average quality links towards the destination.

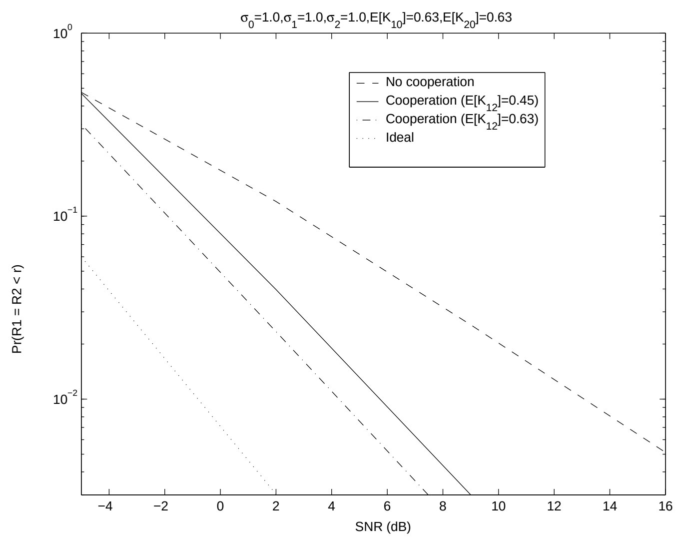


FIGURE 5. Probability of outage versus signal to noise ratio for two-user cooperative system. Both users have similar average quality links towards the destination. The common operating rate is $r = 0.18$.

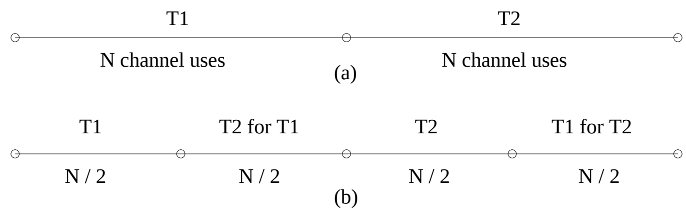


FIGURE 6. Cooperation through time division. (a) Non-cooperative orthogonal transmissions, (b) Cooperative scheme.

by both the partner and the destination. The second $N/2$ channel uses are reserved for cooperation. We will describe a number of different cooperative strategies the partner can utilize below.

Another assumption we relax from Section 2 is the channel phase knowledge at the transmitters. The accurate knowledge of the phase may be hard to obtain in practice, so we assume that the transmitters do not have any knowledge about the fading amplitudes except for their statistics. As before, we assume that receivers have perfect channel state information. We will observe that even with the above set of assumptions cooperation continues to provide substantial performance gains.

The above cooperation scheme through time division was first suggested in [**13, 14**] which also provided a number strategies for cooperation. All the links were assumed to be slowly fading, hence the proper information theoretic measure is the outage probability. The emphasis was on the diversity, the large signal–to–noise ratio (SNR) exponent of 1/SNR in the outage probability, of the suggested cooperative strategies.

Amplify-and-forward and adaptive decode-and-forward are two cooperative strategies investigated in [**13, 14**] that result in full (in this case two level) diversity. In the amplify-and-forward scheme, the partner simply scales its received signal to satisfy its own power constraint and re-transmits to cooperate. The destination combines the first $N/2$ symbols coming from the original terminal with the $N/2$ forwarded symbols from the partner. For the adaptive decode-and-forward strategy, the partner attempts to decode the original information based on the signal it receives. As long as the inter-user channel has a received SNR high enough to support the desired rate, there is no outage and Shannon-type perfect decoding is possible. In that case, the partner re-encodes the information and transmits in the remaining $N/2$ symbols where N is large. Otherwise, the original terminal simply repeats the same codeword in the second half of its time slot. Note that both of the protocols described involve some kind of repetition at the partner. However, we know from channel coding techniques that there are more effective ways of designing codes. Coding gain as well as diversity is important in the performance of a channel code.

For cooperative channel coding, we will work with finite block lengths N and focus on designing and analyzing codes suitable for cooperation. We follow the time division scheme in Figure 6. We assume slow or quasi-static fading, that is each link has a constant fading level for N symbols. Our protocol is similar to the adaptive-decode-and forward of [**13, 14**] and that of [**11**]. We make use of the cyclic redundancy check (CRC) commonly used for error detection in wireless communication systems. Excluding the CRC, in a non-cooperative system each

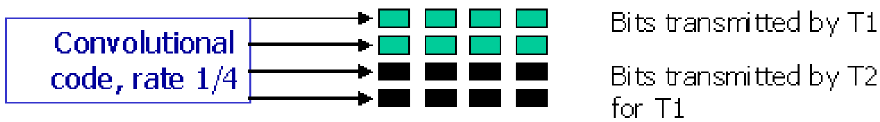


FIGURE 7. An example of the bit mapping for cooperation for a rate 1/4 convolutional code

terminal sends N coded bits per frame. In order to cooperate, T_1 multiplexes these N bits properly and only sends half of its coded bits. If the original channel code had rate R, this corresponds to an effective coding rate of $2R$. These bits are received by both the destination and the partner. The partner, T_2, decodes these $N/2$ bits and detects whether there are any errors using the CRC. If the partner has the correct information, it re-encodes and sends the *additional* $N/2$ coded bits T_1 did not transmit. This is illustrated in Figure 7 for a convolutional code of rate 1/4. Otherwise, T_1 is informed and it continues its transmission of the remaining $N/2$ coded bits. The destination waits until the end of the frame and combines both $N/2$ observations to decode the information bit stream. Assuming the destination estimates the current fading level every $N/2$ bits, there is no need to notify it as to whether the partner received the information correctly or not. Then T_1 and T_2 change roles, so T_2 acts as the source while T_1 relays the information.

Note that in the suggested cooperative coding scheme, when the partner cannot decode correctly, T_1 suffers no loss from the non-cooperative case. This feature is not present in the adaptive decode-and-forward protocol, as it forces the source to repeat information rather than designing the "best" codebook for the whole N uses of the channel.

From the perspective of the destination, when partner decodes correctly, the first $N/2$ coded bits observe a fading amplitude of K_{10}, the second $N/2$ bits an independent fading amplitude K_{20}. Hence the overall effect at the destination is that of block fading, with two fading blocks. Based on this observation, we argue that codes designed for block fading channels (such as those in [**12**]) are suitable for cooperation. However, the cooperative systems impose additional constraints. In order to cooperate often, that is for the partner to correctly decode, the first half of the coded bits (or the punctured code) should form a good code in the quasi-static inter-partner channel. Also, when the cooperation does not take place, all the coded bits face the same fading level, so the code must be good for quasi-static channel as well as the cooperative block fading channel. We will provide an example of such a code in Section 3.2, more can be found in [**22**].

We note here that Hunter and Nosratinia [**11**] suggested the use of rate compatible punctured convolutional (RCPC) codes [**10**] for cooperation. However, RCPC codes were not designed with diversity schemes or block fading channels in mind. We think that the block fading framework allows us to work with a richer family of codes and also enables us to provide a performance analysis illustrating the diversity and coding gains of cooperative coding. Also, by starting from a good code in the inter-user channel and adding additional parity bits to get a cooperative block fading code, we can in fact improve the performance over a good code (say a code designed for a block fading channel) punctured to be used in the inter-user channel. Hence we think *overlaying* a code with additional parity bits is more suitable than puncturing a code to be used in the inter-user channel. More details on this can be found in [**22**]. The overlay ideas can also be used to take into account the cases in which source and partner divide up the N bits unequally as suggested in [**11**].

In the next subsection, we analyze the diversity achieved by user cooperation by studying upper bounds on the frame error rates of cooperative codes. We then provide some simulations illustrating the suggested diversity and coding gains.

3.1. Performance Analysis of Cooperative Codes. In order to find bounds on the performance of cooperative codes, we will focus on the frame error probability of the cooperative coding scheme described above. We concentrate on T_1 and let P_f^C denote the overall frame error rate of a channel code when used for cooperation. We also let P_f^{in} denote the frame error probability of the portion of the code used in the inter-user channel (corresponding to block length $N/2$), P_f^{QS} denote the frame error probability over the quasi-static channel of T_1 to destination (corresponding to a block length of N) and P_f^{BF} denote the frame error probability over the cooperative block fading channel (first $N/2$ coded bits face T_1-destination link, second $N/2$ bits face T_2-destination link). We can then write

$$P_f^C = (1 - P_f^{in})P_f^{BF} + P_f^{in}P_f^{QS}.$$

We can upper bound P_f^C as

$$P_f^C \leq P_f^{BF} + P_f^{in}P_f^{QS}. \tag{3.1}$$

We now investigate each of the terms in this upper bound. Let SNR_1 denote the average (averaged over the Rayleigh fading) received signal–to–noise ratio at the destination corresponding to the transmission from T_1. The value for SNR_1 depends on P_1, σ_0^2 and $E[K_{10}]$. Similarly, let SNR_2 denote the average received signal–to–noise ratio at the destination corresponding to the transmission from T_2 and SNR_{in} denote the average received signal–to–noise ratio at partner (T_2) corresponding to the transmission from T_1.

Note that in the cooperative block fading channel, the first block observes an average signal–to–noise ratio of SNR_1 while the second block has SNR_2. Even though this is different than the usual block fading model in which all blocks have the same average SNR, the pairwise error probability can be derived in a form similar to [**12**]. Hence, utilizing the pairwise error probability expression for the block Rayleigh fading channel and the union upper bound on the frame error probability, we have

$$P_f^{BF} \leq \sum_{\boldsymbol{c}} \sum_{\boldsymbol{e} \neq \boldsymbol{c}} \frac{1}{\Xi^2(\boldsymbol{c}, \boldsymbol{e})\, (\text{SNR}_1/4)^{\mathcal{I}_{d_1^2}}\, (\text{SNR}_2/4)^{\mathcal{I}_{d_2^2}}} \tag{3.2}$$

In order to define the terms in the above expression, we first define the code Euclidian distances. Let us consider two codewords $\boldsymbol{c} = (c_1, \ldots, c_N)$ and $\boldsymbol{e} = (e_1, \ldots, e_N)$ each consisting of N bits. The distance $d_i^2(\boldsymbol{c}, \boldsymbol{e}) = \sum_{n=1}^{N/2} |c_{n+(i-1)N/2} - e_{n+(i-1)N/2}|^2$, $i = 1, 2$, denotes the squared Euclidean distance among first and second $N/2$ bits of the two codewords respectively. Note that for the cooperative block fading channel, these two parts of the codewords face independent fading. The term $\Xi^2(\boldsymbol{c}, \boldsymbol{e})$ denotes the product of the non–zero squared Euclidean distances $d_1^2(\boldsymbol{c}, \boldsymbol{e})$ and $d_2^2(\boldsymbol{c}, \boldsymbol{e})$ and $\mathcal{I}$ denotes the indicator function.

For the quasi-static frame error probability P_f^{QS} of the T_1-destination channel, the relevant distance term is $\Psi^2(\boldsymbol{c},\boldsymbol{e}) = d_1^2(\boldsymbol{c},\boldsymbol{e})+d_2^2(\boldsymbol{c},\boldsymbol{e})$ and it denotes the squared Euclidean distance between the two entire codewords $\boldsymbol{c}$ and $\boldsymbol{e}$. This gives us the upper bound

$$P_f^{QS} \leq \sum_{\boldsymbol{c}} \sum_{\boldsymbol{e}\neq\boldsymbol{c}} \frac{1}{\Psi^2(\boldsymbol{c},\boldsymbol{e})\mathrm{SNR}_1/4} \tag{3.3}$$

The inter-user channel is also quasi-static, but it only utilizes the first $N/2$ coded bits. Hence,

$$P_f^{in} \leq \sum_{\boldsymbol{c}} \sum_{\boldsymbol{e}\neq\boldsymbol{c}} \frac{1}{\Phi^2(\boldsymbol{c},\boldsymbol{e})\mathrm{SNR}_{in}/4} \tag{3.4}$$

where $\Phi^2(\boldsymbol{c},\boldsymbol{e}) = d_1^2(\boldsymbol{c},\boldsymbol{e})$.

Using equations (3.2)-(3.4) in the upper bound of (3.1), we can bound the cooperative coding frame error probability as

$$\begin{aligned} P_f^C \quad \leq \quad & \left(\sum_{\boldsymbol{c}} \sum_{\boldsymbol{e}\neq\boldsymbol{c}} \frac{1}{\Xi^2(\boldsymbol{c},\boldsymbol{e})\left(\mathrm{SNR}_1/4\right)^{\mathcal{I}_{d_1^2}}\left(\mathrm{SNR}_2/4\right)^{\mathcal{I}_{d_2^2}}} \right) \\ + \quad & \left(\sum_{\boldsymbol{c}} \sum_{\boldsymbol{e}\neq\boldsymbol{c}} \frac{1}{\Phi^2(\boldsymbol{c},\boldsymbol{e})\mathrm{SNR}_{in}/4} \right) \left(\sum_{\boldsymbol{c}} \sum_{\boldsymbol{e}\neq\boldsymbol{c}} \frac{1}{\Psi^2(\boldsymbol{c},\boldsymbol{e})\mathrm{SNR}_1/4} \right). \end{aligned} \tag{3.5}$$

We envision cooperative systems to act as virtual antenna arrays and provide us with additional spatial diversity. However, this depends crucially on the quality of the inter-user channel. We now investigate two extreme scenarios for the inter-user channel quality to study the potential gains of cooperation.

Case I: Good Inter–User Channel. First, we consider the performance when the inter–user channel is very good. If the inter–user channel has very high average signal–to–noise ratio, $\mathrm{SNR_{in}}$, then P_f^{in} will be very small and the equation (3.1) is dominated by P_f^{BF}. Therefore we have $P_f^C \approx P_f^{BF}$ and full second order diversity is obtained. This is of course, expected. When the inter-user channel is almost perfect, cooperative communication (using time division) corresponds to a regular block fading channel.

Case II: Poor Inter–User Channel. In this case, we assume that the inter–user channel has poor quality, that is P_f^{in} is high. We note that for high signal–to–noise ratios, block fading error probability P_f^{BF} will be lower than that of P_f^{QS} as it provides two level diversity. Hence the upper bound on P_f^C is dominated by the term $P_f^{in}P_f^{QS}$. Therefore,

$$P_f^C \leq \left(\sum_{\boldsymbol{c}} \sum_{\boldsymbol{e}\neq\boldsymbol{c}} \frac{1}{\mathrm{SNR}_{in}/4} \frac{1}{\Phi^2(\boldsymbol{c},\boldsymbol{e})} \right) \left(\sum_{\boldsymbol{c}} \sum_{\boldsymbol{e}\neq\boldsymbol{c}} \frac{1}{\mathrm{SNR}_1/4} \frac{1}{\Psi^2(\boldsymbol{c},\boldsymbol{e})} \right). \tag{3.6}$$

As the inter–user channel quality is low, we can assume that for the range of transmit powers of interest, the received inter–user channel average signal–to–noise ratio SNR_{in} is at most equal to some value C_{in}. Then the tightest upper bound for P_f^C is obtained for $\mathrm{SNR}_{in} \approx C_{in}$. For high signal–to–noise ratios in T_1-destination channel, this results in the approximation P_f^C,

$$P_f^C \approx \frac{1}{C_{in}} \left(\frac{1}{\text{SNR}_1/4} \right) \frac{1}{\min_{\boldsymbol{c},\boldsymbol{e}} \{\Phi^2(\boldsymbol{c},\boldsymbol{e})\Psi^2(\boldsymbol{c},\boldsymbol{e})\}} \tag{3.7}$$

Hence the minimum product distance $\min_{\boldsymbol{c},\boldsymbol{e}} \left\{\Phi^2(\boldsymbol{c},\boldsymbol{e})\Psi^2(\boldsymbol{c},\boldsymbol{e})\right\}$ dominates the performance at large values of the signal–to–noise ratio SNR_1..

We observe that when the inter-user channel quality is very poor, the diversity of the cooperative system is one and is limited by the diversity of the quasi-static T_1-destination link. However, despite the limited diversity, there is still some coding gain with respect to the non–cooperative transmission, as indicated by the squared Euclidean distance product in equation (3.7). In the next section we illustrate via simulations that this coding gain enables the cooperative scheme to outperform non-cooperative communication even when the inter-user link is very noisy and severely faded. For more moderate inter-user channel qualities, the performance of cooperative coding substantially improves, and provides diversity as well as coding gains.

3.2. Simulation Results. In this section we present the performance of the proposed cooperative coding scheme via simulations to illustrate the potential benefits. As discussed above, we assume a Rayleigh slow fading channel. Hence, we use the quasi–static model, where the fading coefficients remain the same for the duration of the entire frame for each user. Note, however that the users observe independently faded channels. As an illustrative example, we use a constraint length $K = 4$ convolutional code with generator polynomials (13,15,15,17) in octal notation and BPSK modulation [**12**]. This is an appealing solution due to the widespread use of convolutional codes and the simple maximum likelihood decoding algorithm [**16**]. The extensions to higher order modulations are also possible [**12**].

The way in which the transmission of bits is organized in order to form a cooperative code is illustrated in Figure 7 for the rate 1/4 convolutional code. As discussed, we would like to have the best convolutional code operating in the quasi–static inter–user channel and user–destination channel. The rate 1/2, (15,17) code provides the best performance in the quasi-static inter–user channel [**16**]. When partner fails to decode and the original terminal continues with transmission, the rate 1/4, (13,15,15,17) code provides the best performance in the quasi–static user–destination channel [**16**]. When cooperation indeed takes place, the same rate 1/4 code, also provides full diversity and excellent coding gain over the cooperative block fading channel [**12**]. Hence, with proper multiplexing of the coded bits, T_1 first transmits the bits corresponding to the (15,17) generator polynomials. Terminal 2 receives this transmission, decodes the information bits, and then re–encodes them using the (13,15) generator polynomials. Note that in the case when T_2 cannot successfully decode the transmission from T_1, T_1 transmits the rest of the coded bits, corresponding to the (13,15) generator polynomials, by itself.

Similar to the situations analyzed in Figures 2 and 3, we consider both a symmetric scenario, in which terminals have similar quality links towards the destination, and an asymmetric scenario. We will also investigate the effect of inter-user channel quality on the performance of the suggested cooperative scheme thereby confirming the intuitions gained in Section 3.1.

In the symmetric scenario, both terminals have channels of similar quality, as represented by equal average signal–to–noise ratios SNR_1 and SNR_2, to the destination. We demonstrate the performance of the (13,15,15,17) code in terms of the frame error rate (FER) versus this common signal-to-noise ratio. Similar results could also be obtained in terms of the bit error rate. We compare the frame error rates of single user scheme (non-cooperative), cooperative with perfect inter-user channel ("ideal"), and cooperative with various inter-user quality channels (represented by inter-user FER of 0.01, 0.1 and 0.5). Our results are summarized in Figure 8(a).

We observe that the suggested cooperative coding scheme with perfect inter–user channel provides a performance improvement of about 4 dB at FER of 10^{-1} and about 10 dB at FER of 10^{-2} with respect to the non–cooperative, single user transmission. As the inter-user channel quality degrades, or the inter-user FER increases, the performance gracefully degrades from the perfect inter-user channel case, and approaches to that of single–user transmission. For low inter-user FER (0.01), the performance is very close to the perfect inter-user channel. Also, slope of the FER curves for high SNR suggest that cooperation for low inter-user frame error rates can indeed achieve full second order diversity as suggested by Section 3.1. When the inter-user link has poor quality (FER=0.5) the diversity is limited to one. Nevertheless, as predicted in Section 3.1, there is still some coding gain with respect to the non–cooperative transmission, which results in performance improvements. We would like to note that the FER trends of cooperative coding, as a function of the inter-partner link quality, are similar to those of the achievable rate region and outage probability studied in Section 2.

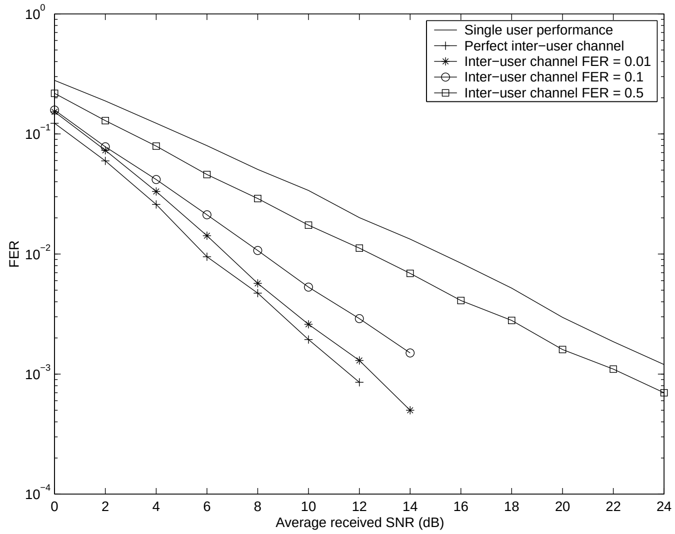
We next focus on the asymmetric scenario, which happens when, say, SNR_1 is much larger than SNR_2. We consider this case by fixing one of the terminal's channel to the destination at a relatively high average signal to noise ratio. We then vary the quality of the other terminal's channel to the destination and observe the performance of both cooperating users.

For the rate 1/4, (13,15,15,17) convolutional code, we fix SNR_1 at 15 dB, which results in a frame error rate of about 10^{-2} in the non–cooperative case. The inter–user channel frame error rate is 0.5 which corresponds to a poor inter-user channel. We vary the average signal–to–noise ratio of T_2 and plot the cooperative and non-cooperative FER's of both terminals as a function of SNR_2 in Figure 8(b). We observe that with cooperation, T_1 achieves the frame error rate of 10^{-2} when the signal–to–noise ratio of T_2 is only about -4 dB. At higher signal–to–noise ratios cooperative performance for T_1 is better than in the non–cooperative case. Terminal 2 also improves its performance by about 3 dB with respect to the non–cooperative case for all the signal–to–noise ratios investigated. Hence cooperation benefits *both* users even in an asymmetric case and poor inter-user channel.

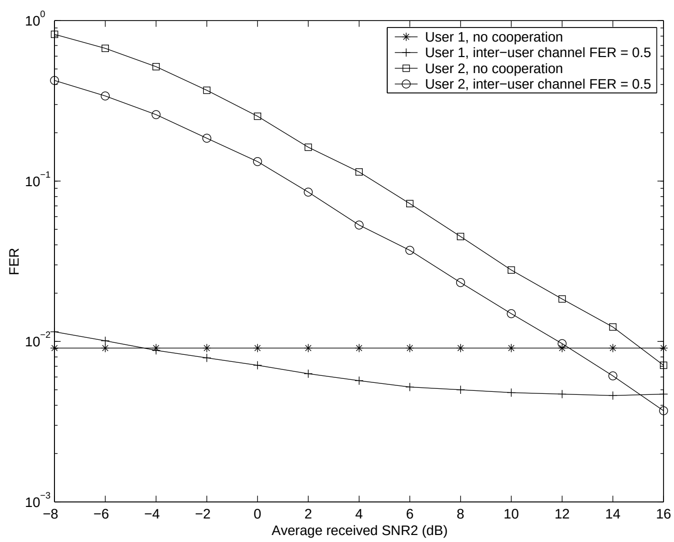
We would like to compare this asymmetric situation with that of Figure 3. In Figure 3 we had observed that even though the maximum rate of the good user does not improve by much, the set of achievable rates increase considerably for an asymmetric cooperative scenario. For the coding scheme discussed above, the gains are even more dramatic, even when the inter-user has a high FER of 0.5 the good also user benefits from cooperation.

4. Conclusions

In a cooperative communication system two or more active users in a network share their information and jointly transmit their messages, either at the different



(a)



(b)

FIGURE 8. Two user coded cooperation for different inter-user channel qualities: (a) symmetric, (b) asymmetric users. (13,15,15,17) convolutional codes, quasi-static fading.

times or simultaneously, to obtain greater reliability and efficiency than they could obtain individually. In this paper we consider a simple two-user cooperative system to illustrate the benefits. We provide both an information theoretic and a coding perspective to cooperation. Our achievable rate region and coding protocol illustrate few of the many possible schemes of collaboration among wireless terminals, yet the benefits of cooperation are clear. Through cooperation both terminals are able to simultaneously increase their throughputs and reliabilities even when they are connected via low quality links, or when one terminal has a much better link then the other. Cooperation enables the terminals to make use of each other's antennas, an extra spatial dimension which is typically not utilized.

Throughout this paper we assumed the partners are fixed and we focused on possible ways of cooperating. In order to successfully use the cooperative principles in a wireless network, one has to be able to choose a good partner. Our ongoing work investigates how partner choice should be made and what the geometry of cooperation is. We are also investigating the added diversity benefits as the number of partners increase, and how one can design and analyze cooperative space-time codes when terminals have multiple antennas.

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References

[1] E. Biglieri, J. Proakis, and S. S. (Shitz). Fading channels: Information theoretic and communications aspects. *IEEE Transactions on Information Theory*, vol. 44, pp. 2619–2692, October 1998.

[2] A. Carleial. Multiple-access channels with different generalized feedback signals. *IEEE Transactions on Information Theory*, vol. 28, pp. 841–850, November 1982.

[3] T.M. Cover and A. El Gamal. Capacity theorems for the relay channel. *IEEE Transactions on Information Theory*, vol. 25, pp. 572–584, September 1979.

[4] T. Cover and C. Leung. An achievable rate region for the multiple-access channel with feedback. *IEEE Transactions on Information Theory*, vol. 27, pp. 292–298, May 1981.

[5] T.M. Cover and J.A. Thomas, *Elements of Information Theory*, John Wiley & Sons, 1991.

[6] R. Gallager. An inequality on the capacity region of multiaccess fading channels. In *Communications and Cryptography–Two Sides of One Tapestry*, pp. 129–139, Kluwer, 1994.

[7] M. Gastpar, G. Kramer and P. Gupta, The multiple-relay channel: Coding and antenna-clustering capacity. In *Proceedings of IEEE International Symposium on Information Theory*, Lausanne, Switzerland, July 2002.

[8] M. Gastpar and M. Vetterli. On the capacity of wireless networks: The relay case. In *Proceedings of IEEE Infocom* , New York, June 2002.

[9] P. Gupta and P. R. Kumar. Towards an information theory of large networks: An achievable rate region. *IEEE Transactions on Information Theory* vol. 49, pp. 1877–1894, August 2003.

[10] J. Hagenauer. Rate-compatible punctured convolutional codes (RCPC codes) and their applications. *IEEE Transactions on Communications,* vol. 36, pp. 389–400, April 1988.

[11] T. Hunter and A. Nosratinia. Cooperation diversity through coding. In *Proceedings of IEEE International Symposium on Information Theory*, Lausanne, Switzerland, July 2002.

[12] R. Knopp and P. A. Humblet. On coding for block fading channels. *IEEE Transactions on Information Theory*, vol. 46, pp. 189–205, January 2000.

[13] J. N. Laneman, G. W. Wornell, and D. N. C. Tse. An efficient protocol for realizing cooperative diversity in wireless networks. In *Proceedings of IEEE International Symposium on Information Theory*, Washington D.C., June 2001.

[14] J. N. Laneman, D.N.C. Tse, and G. W. Wornell. Cooperative diversity in wireless networks: Efficient protocols and outage behaviour. Accepted *IEEE Transactions on Information Theory.*

[15] J. N. Laneman and G. W. Wornell, Distributed space-time coded protocols for exploiting cooperative diversity in wireless networks. *IEEE Transactions on Information Theory*, vol 49, pp. 2415–2425, October 2003.

[16] S. Lin and D.J. Costello, *Error Control Coding: Fundamentals and Applications.* Prentice-Hall, 1983.

[17] L. Ozarow, S. Shamai, and A. Wyner. Information theoretic considerations for cellular mobile radio. *IEEE Transactions on Vehicular Technology*, vol. 43, pp. 359–377, May 1994.

[18] A. Reznik, S. Kulkarni and S. Verdu. Capacity and optimal resource allocation in the degraded Gaussian relay channel with multiple relays. In *Proceedings of Fortieth Annual Allerton Conference on Communication, Control, and Computing,* Monticello, IL, Oct. 2002.

[19] B. Schein and R. Gallager. The Gaussian parallel relay network. In *Proceedings of IEEE International Symposium on Information Theory*, Sorrento, Italy, June 2000.

[20] A. Sendonaris, E. Erkip, and B. Aazhang. Increasing uplink capacity via user cooperation diversity. In *Proceedings of IEEE International Symposium on Information Theory*, Boston, August 1998.

[21] A. Sendonaris, E. Erkip, and B. Aazhang. User cooperation diversity–Part I: System description. *IEEE Transactions on Communications*, vol. 51, pp. 1927–1938, November 2003.

[22] A. Stefanov and E. Erkip. Cooperative coding for wireless networks. Under review.

[23] E.C. Van Der Meulen. Three-terminal communication channels. *Advances in Applied Probability*, 3:120–154, 1971.

[24] Frans M. J. Willems, Edward C. van der Meulen, and J. Pieter M. Schalkwijk. Achievable rate region for the multiple access channel with generalized feedback. In *Annual Allerton Conference on Communication, Control, and Computing*, 1983.

[25] F. Willems and E. V. D. Meulen. The discrete memoryless multiple-access channel with cribbing encoders. *IEEE Transactions on Information Theory*, vol. 31, pp. 313–327, May 1985.

[26] L. Xie and P. R. Kumar. A network information theory for wireless communication: Scaling laws and optimal operation. Submitted to *IEEE Transactions on Information Theory.*

[27] C. Zeng, F. Kuhlmann, and A. Buzo. Achievability proof of some multiuser channel coding theorems using backward decoding. *IEEE Transactions on Information Theory*, vol. IT–35, pp. 1160–1165, November 1989.

Department of Electrical and Computer Engineering, Polytechnic University, Five Metrotech Center, Brooklyn, NY 11201

E-mail address: elza@poly.edu

Qualcomm Inc., Campbell, CA

E-mail address: sendos@qualcomm.com

Department of Electrical and Computer Engineering, Polytechnic University, Five Metrotech Center, Brooklyn, NY 11201

E-mail address: stefanov@poly.edu

Department of Electrical and Computer Engineering, Rice University, Houston, TX 77005

E-mail address: aaz@rice.edu