Analysis of Class D-E Resonant dc/dc Converter Using Thinned-out Method

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Abstract

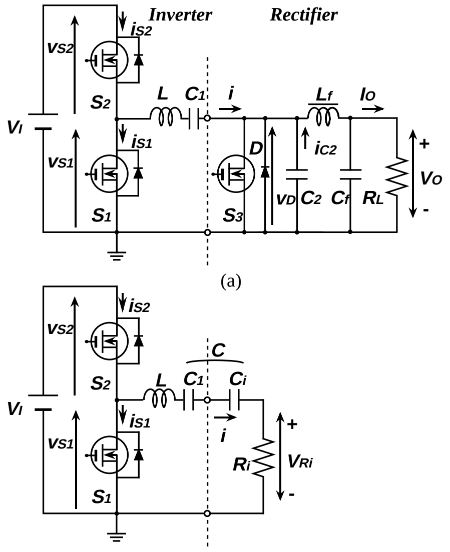
Previous analytical descriptions of the Class D-E resonant dc/dc converter using thinned-out method have been based on the assumptions simplifying the effect of thinned-out control on the equivalent impedance. Therefore the difference between the calculated values and the measured values is significant in large thinned-out ratio. This paper presents a new analysis for this circuit. The equivalent impedance's variation by thinned-out control is taken into consideration. The measured performance shows good agreement with the analytical performance in wide range.

1 Introduction

Class D inverter [1] is one of valuable high-density power sources because of the high dc/ac power conversion efficiency which is achieved by their low switching losses at several hundred kHz, furthermore Class D inverter has also low switch device voltage stresses, equal to the dc input voltage.

Class E rectifiers have been introduced in [2]. These circuits also offer high-frequency high-efficiency low-noise rectification because of the Class E switching conditions, i.e. the switch voltage and its slope are zero at turn on or off transition.

Class D-E converter [3] is composed of a Class D inverter and a Class E rectifier. In the early works, it was regulated against load and line variations by varying the switching frequency. However, this generates a wide and unpredictable noise spectrum and poor utilization of magnetic components. As a remedy of these problems, thinned-out control was applied to the converter [4]. Its output power is regulated in the rectifier side by eliminating the diode voltage pulse using only one active device added in parallel with the diode. Therefore the resonant converter using this circuit can be driven at a fixed operating frequency with high power conversion efficiency because of keeping Class E switching conditions. However, its analytical description is based on a simplified assumptions.



(b)

Figure 1: Class D-E resonant dc/dc converter using thinned-out method. (a) Basic circuit, (b) equivalent circuit.

In this paper we presents a new analysis of Class D-E resonant dc/dc converter using thinned-out method. The analysis is carried out using a parameter r of the remained-pulse ratio. The rectifier is treated as a C_i-R_i series circuit in the Class D inverter. The measured performance shows good agreement with the theoretical predictions in wide control range.

2 Circuit description

A basic circuit of the Class D-E resonant dc/dc converter with thinned-out method is shown in Fig. 1. The converter of Fig. 1(a) is composed of a Class D inverter and a Class E rectifier using thinned-out method. The inverter is loaded by the input impedance Z_i of a Class E rectifier as shown in Fig. 1(b). Each switch is comprised of a transistor and an anti-parallel diode. Switches S_1 and S_2 become on and off alternately with a duty cycle of 0.5 applying a nearly square-wave voltage to the series-resonant circuit. The series resonant circuit forces a sinusoidal current, so only the power at the fundamental frequency is transferred from the inverter to the rectifier.

A basic circuit of the Class E rectifier using

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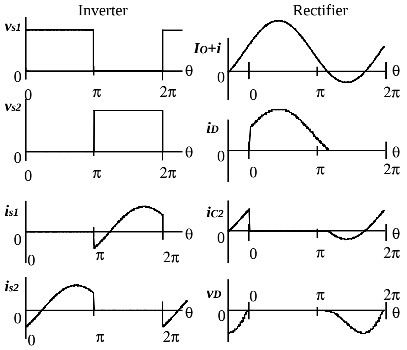

Figure 2: Waveforms in Class D-E resonant dc/dc converter using thinned-out method

thinned-out method is shown on the right side of Fig. 1. The rectifier is driven by a nearly sinusoidal current i. Assuming a large inductance of the output filter L_f-C_f, the current through the inductance is approximately constant and equal to the dc output current I_O. Thus, the current flowing through the parallel combination of the diode D, the thinning-out switch S_3, and the capacitor C_2 is equal to $I_O - i$. During non-thinned out periods the input current $I_O - i$ flows into the shunt capacitor C_2 and the diode switch alternatively as shown in Fig. 2. The diode turns on automatically when its voltage increases to the threshold voltage and turns off when it's current decreases to zero. While the diode is on, the current i_D flows through the diode. Therefore the voltage across the diode is kept to be zero. While the diode is off, the current i_{C2} flows into the capacitor C_2, producing the voltage v_D across the parallel combination. During the thinned-out period, the current that is supposed to charge the shunt capacitor C_2 flows through the thinning-out switch S_3. So the voltage across the diode is kept to be zero. The output power is regulated by the thinning-out control.

3 Circuit analysis

In previous papers, the performance of the converter was described under the simple assumptions, that is, $C_i = C_{iopt}/r$, $R_i = rR_{iopt}$ and $|M| \propto 1/\sqrt{r}$ [4]. In this paper an analysis of the converter is carried out under the following assumptions. 1. The active devices perform as ideal bidirectional switches. 2. The rectifier diode is ideal, i.e. the device acts as one directional switch with zero threshold voltage. 3. The rectifier is driven by an ideal sinusoidal current source. 4. The ac ripple on the dc current I_O through the output filter is negligible. 5. All the circuit elements are ideal.

3. 1 Class D inverter

Referring to Fig. 2, the input voltage of the series-resonant circuit v_{S1} is a square wave of amplitude V_I. The fundamental component of this voltage waveform is $v_1 = V_o \sin\omega t$. From Fourier analysis, the amplitude V_o is given as $V_o = 2/\pi \cdot V_I \simeq 0.6366V_I$. Hence, the rms value of v_1 is $V_{orms} \equiv V_o/\sqrt{2} = \sqrt{2}/\pi \cdot V_I \simeq 0.4502V_I$. Therefore, the voltage transfer function from the input of the inverter to the input of the series-resonant circuit is

$$M_1 \equiv \frac{V_{orms}}{V_I} = \frac{\sqrt{2}}{\pi} \simeq 0.4502. \tag{1}$$

The voltage transfer function of the series-resonant circuit is defined as the ratio of the rms value V_{Ri} of the sinusoidal voltage v_{Ri} across R_i to V_{orms}. That is expressed as

$$M_2 \equiv \frac{V_{Ri}}{V_{orms}} = \frac{R_i}{Z_i} = \frac{R_i}{R_i + j(\omega L - 1/\omega C)}. \tag{2}$$

As shown in Fig. 1(b), the capacitive component is composed of a fixed value of capacitance C_1 and an equivalent capacitance of the rectifier. The relationship between these elements is $1/j\omega C = 1/j\omega C_1 + 1/j\omega C_i$. In the designed condition, resonant circuit L-C composes a BPF at the operating frequency, $2\pi f = 1/\sqrt{LC_{opt}}$. In this case, M_2=1 is given by (2) and the equation, $1/j\omega C_{opt} = 1/j\omega C_1 + 1/j\omega C_{iopt}$ is satisfied. Thus, the reactive component is expressed as $jX = 1/j\omega C - 1/j\omega C_{opt} = 1/j\omega C_i - 1/j\omega C_{iopt} = j(C_i - C_{iopt})/\omega C_i C_{iopt}$. Using this, the load impedance Z_i is shown as follows.

$$Z_i = R_i + j\frac{C_i - C_{iopt}}{\omega C_i C_{iopt}} \tag{3}$$

From (2) and (3),

$$|M_2| = \frac{R_i}{\sqrt{R_i^2 + \dfrac{(C_i - C_{iopt})^2}{\omega^2 C_i^2 C_{iopt}^2}}} = |\cos\psi|,$$

$$\psi = \arctan\left(\frac{\dfrac{C_i - C_{iopt}}{\omega C_i C_{iopt}}}{R_i}\right). \tag{4}$$

So the voltage transfer function of the Class D inverter is then expressible as

$$\begin{aligned} |M_I| &= \frac{V_{Ri}}{V_I} = \frac{V_{Ri}V_{orms}}{V_{orms}V_I} = M_1|M_2| \\ &= \frac{\sqrt{2}}{\pi}\frac{R_i}{\sqrt{R_i^2 + \dfrac{(C_i - C_{iopt})^2}{\omega^2 C_i^2 C_{iopt}^2}}}. \end{aligned} \tag{5}$$

The current through the series-resonant circuit is given by $i = I_m \sin(\omega t - \psi)$, where $I_m = 2V_I/\pi|Z_i|$.

3. 2 Class E rectifier using thinned-out method

Analysis of Class E rectifier using thinned-out method was carried out in [5]. In this paper we show some characteristics by applying the calculation technique introduced in [5].

The input current i is sinusoidal $i = I_m \sin(\omega t + \phi)$, where I_m is the amplitude and ϕ is the phase angle. The current difference of i and output dc current I_O flows into the shunt capacitor C_2 and the diode D, which is expressed as $i_d = I_O - I_m \sin(\omega t + \phi)$. As indicated in Fig. 2, $i_d(0) = 0$, thus,

$$I_O = I_m \sin\phi. \tag{6}$$

The voltage across the capacitor C_2 for $(0 < \omega t < 2\pi(1 - D_D))$ is

$$\begin{aligned} v_D &= \frac{1}{\omega C_2} \int_0^{2\pi(1-D_D)} i_d d(\omega t) \\ &= \frac{V_O}{\omega C_2 R_L} \left\{ \omega t + \frac{\cos(\omega t + \phi) - \cos\phi}{\sin\phi} \right\}. \end{aligned} \tag{7}$$

Because diode turns on at zero voltage, that is $v_d(2\pi(1 - D_D)) = 0$, the relationship between the diode-on-duty-ratio D_D and the phase angle ϕ is given by

$$\tan\phi = \frac{1 - \cos(2\pi D_D)}{2\pi(1 - D_D) + \sin 2\pi D_D}. \tag{8}$$

During the thinned-out period, the voltage across the capacitor C_2 and the diode is kept to be zero. On the other hand, during normal periods, the diode voltage v_D is the same as (7). Consequently, V_O is equal to the average value of the voltage v_d. Therefore, V_O is expressed as

$$\begin{aligned} -V_O &= \frac{r}{2\pi} \int_0^{2\pi(1-D_D)} v_d d(\omega t) \\ &= \frac{rV_O}{2\pi\omega C_2 R_L} \left[2\pi^2(1 - D_D)^2 - 1 + \cos 2\pi D_D \right. \\ &\qquad \left. - \frac{2\pi(1 - D_D) + \sin 2\pi D_D}{\tan\phi} \right], \end{aligned} \tag{9}$$

where r is the ratio of remained pulse. It is defined as

$$r \equiv \frac{N_{remainned}}{N_{thinned} + N_{remainned}}. \tag{10}$$

Assuming 100 percent power conversion efficiency, we have $I_O^2 R_L = I_m^2 R_i/2$. Using this relation and (6),

$$\frac{R_i}{R_L} = 2\sin^2\phi \tag{11}$$

is given. Applying the Fourier formula to (7) at the operating frequency gives

$$\begin{aligned} V_{Xm1} &= \frac{r}{\pi} \int_0^{2\pi} v_d \cos(\omega t + \phi) d(\omega t) = \frac{rV_O}{\pi\omega C_2 R_L} \\ &\cdot \left\{ \frac{\cos\phi[\pi(1 - D_D) + \sin 2\pi D_D - \frac{1}{4}\sin 4\pi D_D)]}{\tan\phi} \right. \\ &+ \sin\phi \left[\pi(1 - D_D) + \frac{1}{4}\sin 4\pi D_D + \sin 2\pi D_D \right] \\ &- \cos\phi \sin^2 2\pi D_D \\ &\left. -2\pi(1 - D_D)\sin(2\pi D_D - \phi) \right\}. \end{aligned} \tag{12}$$

The equivalent input capacitance of the rectifier at the operating frequency is given by

$$\begin{aligned} C_i &= \frac{\pi C_2}{r \sin\phi} \\ &\cdot \left\{ \frac{\cos\phi[\pi(1 - D_D) + \sin 2\pi D_D - \frac{1}{4}\sin 4\pi D_D)]}{\tan\phi} \right. \\ &+ \sin\phi \left[\pi(1 - D_D) + \frac{1}{4}\sin 4\pi D_D + \sin 2\pi D_D \right] \\ &- \cos\phi \sin^2 2\pi D_D \\ &\left. -2\pi(1 - D_D)\sin(2\pi D_D - \phi) \right\}^{-1} \end{aligned} \tag{13}$$

The duty ratio D_D is given by applying the Newton-Raphson method to (9). Relationships between the important parameters are calculated, which are shown in Fig. 3 as functions of normalized thinned-out ratio $r/\omega C_2 R_L$.

The ac/dc voltage transfer function of the rectifier is

$$M_R = \frac{V_O}{V_{rms}} = \sqrt{\frac{R_L}{R_i}} = \frac{1}{\sqrt{2}\sin\phi}, \tag{14}$$

which is plotted in Fig. 3 (d) as a function of $r/\omega C_2 R_L$.

The voltage transfer function of the converter is given by

$$M \equiv \frac{V_O}{V_I} = \sqrt{\eta_{tot}} |M_I| M_R, \tag{15}$$

where η_{tot} is the total converter efficiency.

4 Deign procedure

To design a Class D-E resonant dc/dc converter using thinned-out method, we have to deign a conventional one. We give the following conditions, f=200 kHz, V_I=12.0 V, V_O=5.0 V, Q_L=5 and R_L=20 Ω. We assume the overall converter efficiency η_{tot}=80 %. The power relationships are, $P_O = V_O^2/R_L = 5.0^2/20 = 1.25$ W; $P_I = P_O/\eta = 1.56$ W; $I_O = V_O/R_L = 0.25$ A; $I_I = P_I/V_I = 0.13$ A. From the power relations, $R_{iopt} = (0.6366 V_I)^2/2V_o^2 \cdot R_L \cdot \eta = 18.67\Omega$. It gives $D_D = 0.6$ using (8) and (11). From the reference [2], $\omega C_2 R_L = 0.114$ and $C_{iopt}/C_2 = 11.831$ are given. Hence $C_2 = 4.43$ nF and $C_{iopt} = 11.83$ nF are calculated. For the inverter, $L = QR_{iopt}/\omega = 74.3\mu$H and $C = 1/\omega^2 L = 1/\omega Q R_{iopt} = 8.52$ nF. From the series combination of the capacitors, $C_1 = 1/(1/C - 1/C_{iopt})$=13.38nF.

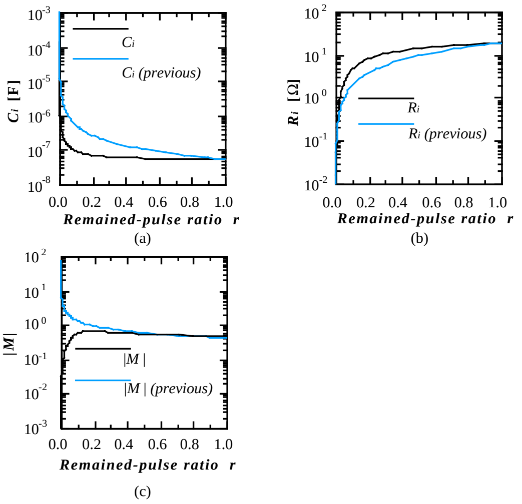

Figure 3: Calculated parameters for the designed circuit. (a)Input capacitance C_i, (b)input resistance R_i, (c)total voltage transfer function $|M|$ as functions of the remained-pulse ratio r.

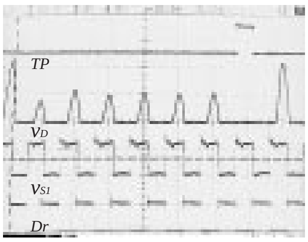

Figure 4: Observed waveforms at f=200kHz, V_I=12V, V_O=4.48V, R_L=20 Ω and r=7/8. Vertical: v_D 20V/div., v_{S1}= 10V/div., and thinned-out pulse (TP) and driving waveform of the inverter (D_r) 5V/div.; horizontal: 5μs/div.

Using the above parameters, the variations of C_i, R_i and $|M|$ are calculated as functions of r by the analytical equations. These are shown in Fig. 4. Calculated values from the previous equations are also plotted in Fig. 4. Especially about $|M|$ the difference is significant for small r.

5 Experimental results

Based on the above calculations, an experimental circuit was built and tested. The MOSFETs used as the switch devices were 2SK982 and the Schottky barrier diode was 11DQ04. Figure 5 shows the observed waveforms in case of the 7/8 remained pulse ratio. At this time the measured efficiency of dc to dc power conversion was 72 %. The causes of power losses might be occured in parasitic resistances of the circuit elements and during the MOSFET's non-zero transition time. Figure 6(a) shows the measured output voltage V_O and efficiency η. The output voltage was changed from 4.37 V to 4.75 V by control of the remained pulse ratio from 1 to 0.5 and in this period the power conversion efficiency was approximately constant, which was more than 70 %. Figure 6(b) shows the calculated output voltages based on this analysis and the previous work as functions of remained pulse ratio r. The measured voltage revised by the efficiency η is also plotted in it. The experimental result is good agreement with the theoretical value based on the new analysis. That is especially different from the previous works for small r.

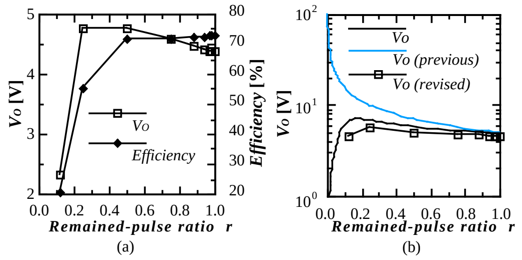

Figure 5: (a) Measured output voltage V_O and efficiency, (b) calculated output voltages based on the analysis, previous one and measured one revised by the efficiency.

6 Conclusions

A new analysis and experimental results of Class D-E resonant dc/dc converter using thinned-out method has been described. An equivalent circuit of the converter and its principle of operation have been explained. Design equations as a dc/dc converter have been given. The analytical results show good agreement with the experimental results. The converter's important parameters and characteristics are approximately explained by the analysis. Relations between the parameters have been illustrated. The calculated values based on the analysis are more correct than those based on the previous one.

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