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Research Article

Distance Based Entropy Measure of Interval-Valued Intuitionistic Fuzzy Sets and Its Application in Multicriteria Decision Making

Tabasam Rashid, Shahzad Faizi, and Sohail Zafar

Department of Mathematics, University of Management and Technology, Lahore 54770, Pakistan

Correspondence should be addressed to Tabasam Rashid; tabasam.rashid@gmail.com

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Fuzzy entropy means the measurement of fuzziness in a fuzzy set and therefore plays a vital role in solving the fuzzy multicriteria decision making (MCDM) and multicriteria group decision making (MCGDM) problems. In this study, the notion of the measure of distance based entropy for uncertain information in the context of interval-valued intuitionistic fuzzy set (IVIFS) is introduced. The arithmetic and geometric average operators are firstly used to aggregate the interval-valued intuitionistic fuzzy information provided by the decision makers (DMs) or experts corresponding to each alternative, and then the fuzzy entropy of each alternative is calculated based on proposed distance measure. Several numerical examples are solved to demonstrate the application to MCDM and MCGDM problems to show the effectiveness of the proposed approach.

1. Introduction

In fuzzy set theory, the measurement of the degree of fuzziness in fuzzy sets and other extended higher order fuzzy sets is an important concept in dealing with real world problems. Zadeh [1] proposed the fuzzy entropy theory in 1965 which has been realized as a great achievement in various fields of MCDM. To measure of the amount of information in decision processes, De Luca and Termini [2] introduced their axiomatic construction of entropy of fuzzy sets by using the idea of Shannon's probability entropy. Kaufmann [3] pointed out that the entropy of a fuzzy set can be found by calculating the distance between the fuzzy set and its nearest nonfuzzy set. However, Yager [4] constructed the entropy of a fuzzy set by using the distance of a fuzzy set to its complement. Based on the concept of entropy measure of a fuzzy set proposed by De Luca and Termini [2] and Zadeh [5], Loo [6] further extended the definition of entropy and developed new entropy measure of a fuzzy set. Liu [7] proposed the axiomatic definition of similarity, distance, and entropy measure of fuzzy set whereas Mi et al. [8] introduced a generalized axiomatic definition of entropy based on a distance in comparison with the axiomatic definition of Liu [7]. A great number of scholars have introduced various techniques of entropy measures for fuzzy set [9–11]. Szmidt [12] and Vlachos [13] introduced some entropy measures for intuitionistic fuzzy sets (IFSs) and discussed their applications in pattern recognition. Furthermore, Burillo and Bustince [14] proposed some entropy measures for interval-valued fuzzy sets (IVFSs) and IFSs while Zeng [15] and Zhang [16] proposed different entropy measures in the context of IVFSs. Liu [7], Zeng [17, 18], and Li [19] investigated some valuable relationships between similarity, distance, and entropy measures for fuzzy sets, IVFSs and IFSs. Farhadinia [20] came up with some entropy measures in the context of hesitant fuzzy sets and interval-valued fuzzy soft sets.

The IVIFS introduced by Atanassov [21] is a very useful generalization of IFS [22]. The IVIFS greatly helps in working under vagueness and uncertainty with a membership and nonmembership interval values of an element belonging to an IVIFS instead of real values in $[0,1]$. In the uncertain environments, IVIFS has played a powerful role and received great consideration from the researchers. Most of the researchers got useful results in solving the MCDM [23–26] and MCGDM [27, 28] problems in the context of IVIFSs. In the past few years, the distance, similarity, inclusion, and information entropy measures for IVIFSs were very important topics. Therefore, there are a large number of researchers who have investigated their studies considering all the measures as mentioned above. Huang et al. [29] proposed an idea of information entropy for IVIFS and used it in uncertain system control and decision making. Furthermore, more work on entropy measures for IVIFSs has been proposed by many researchers in different viewpoints; for example, Liu [30], Zhang [31–33], Wei [34], and Ye [35] have investigated and contributed a lot of work on this topic and discussed their applications to solve different MCDM problems in real life. However, many entropy measures for IVIFS are very complicated and unacceptable in the intuitive sense. In order to overcome this weakness in the existing entropy measures, Zhang et al. [36] proposed the distance based information entropy measure for IFS and IVIFSs and then established some relationships between the entropy, similarity, and inclusion measures of IVIFSs.

Recently, the studies on the similarity and entropy measures of different extensions of fuzzy sets are growing; for example, Pramanik et al. [37] discussed NS-cross entropy and designed a MAGDM strategy which is symmetric, free from undefined phenomena, and very useful for MCGDM problems. Dalapati et al. [38] further introduced IN-cross entropy and weighted IN-cross entropy and used them to design a MCGDM strategy. Pramanik et al. [39] discussed vector similarity measures of single-valued and interval neutrosophic sets by hybridizing the ideas of dice and cosine similarity measures and presented its applications in MCDM problems. Uluay et al. [40] proposed dice similarity, weighted dice similarity, hybrid vector similarity, and weighted hybrid vector similarity measures for bipolar neutrosophic sets and presented a MCDM strategy. Pramanik et al. [41] discussed cross entropy measures of bipolar neutrosophic sets and interval bipolar neutrosophic sets and designed MCDM strategies based on the cross entropy measures.

The distance between fuzzy sets and the entropy measure are very important to calculate the degree of fuzziness of a fuzzy set, and a lot of work as mentioned above has been contributed to the literature by various researchers. Therefore, motivated by the advantages of the entropy measures of different extensions of fuzzy sets, in this paper we proposes an entropy measure based on new distance measure for IVIFSs. In this approach, we first put forward a useful distance measure for IVIFSs which is completely different from the existing ones and then propose an entropy measure formula based on this distance measure. Furthermore, a useful comparison of the proposed entropy measure with the existing entropy measures formulae is performed to avoid any inconsistency in the proposed approach. In order to ensure the practicality and effectiveness of the proposed approach, we finally apply this approach to solve various MCDM and MCGDM problems using IVIFSs.

The remaining part of the paper can be summarized briefly as follows. Some basic concepts related to the work are presented in Section 2. A comparison of the performance of proposed entropy measure to those of all known entropy measures is established in Section 3. Different approaches are developed in Section 4 to handle MCDM and MCGDM problems using IVIFSs. Several numerical examples are provided to make out the practicality and effectiveness of the proposed approach in Section 5. We wind up the paper with some useful remarks in Section 6.

2. Preliminary

In this section, basic definitions of IVIFS, the weighted arithmetic and geometric average operators, and some distance measures for IVIFSs are presented. The entropy measure for IVIFS based on different distance measures along with some basic properties is also discussed in this section. Furthermore, throughout the paper, the set of all closed subintervals of $[0,1]$ is denoted by $[I]$, the domain or universe of discourse is denoted by a set $X = \{x_1, x_2, \ldots, x_n\}$, the collection of all the crisp sets is denoted by $P(X)$, and the set of all IVIFSs in X are denoted by IVIF(X).

Definition 1 (see [14]). Let $[a,b]$, $[c,d] \in [I]$. For the comparison of two elements of $[I]$, one defines

$$\begin{aligned} [a,b] \le [c,d] &\Longleftrightarrow a \le c, b \le d; \\ [a,b] \preceq [c,d] &\Longleftrightarrow a \le c, b \ge d; \\ [a,b] = [c,d] &\Longleftrightarrow a = c, b = d. \end{aligned} \tag{1}$$

Definition 2 (see [21]). An IVIFS A in X is defined by a set $A = \{\langle x_i, a(x_i), a'(x_i)\rangle \mid x_i \in X\}$, where $a(x_i) = [\underline{a}(x_i), \overline{a}(x_i)], a'(x_i) = [\underline{a'}(x_i), \overline{a'}(x_i)] \in [I]$ are called the interval membership and nonmembership degrees of an element x_i to A, respectively, provided that $0 \le \overline{a}(x_i) + \overline{a'}(x_i) \le 1$ for any $x_i \in X$.

Definition 3. For $A, B \in$ IVIF(X), where $A = \{\langle x_i, a(x_i), a'(x_i)\rangle \mid x_i \in X\}$ and $B = \{\langle x_i, b(x_i), b'(x_i)\rangle \mid x_i \in X\}$, some basic operations and relations are defined as follows:

(1) $A \cup B = \{\langle x_i, [\underline{a}(x_i) \vee \underline{b}(x_i), \overline{a}(x_i) \vee \overline{b}(x_i)], [\underline{a'}(x_i) \wedge \underline{b'}(x_i), \overline{a'}(x_i) \wedge \overline{b'}(x_i)]\rangle \mid x_i \in X\}$;

(2) $A \cap B = \{\langle x_i, [\underline{a}(x_i) \wedge \underline{b}(x_i), \overline{a}(x_i) \wedge \overline{b}(x_i)], [\underline{a'}(x_i) \vee \underline{b'}(x_i), \overline{a'}(x_i) \vee \overline{b'}(x_i)]\rangle \mid x_i \in X\}$;

(3) $A^c = \{\langle x_i, [\underline{a'}(x_i), \overline{a'}(x_i)], [\underline{a}(x_i), \overline{a}(x_i)]\rangle \mid x_i \in X\}$;

(4) $A \subseteq B \Longleftrightarrow [\underline{a}(x_i), \overline{a}(x_i)] \le [\underline{b}(x_i), \overline{b}(x_i)]$ and $[\underline{a'}(x_i), \overline{a'}(x_i)] \ge [\underline{b'}(x_i), \overline{b'}(x_i)] \forall x_i \in X$;

(5) $A = B, \Longleftrightarrow [\underline{a}(x_i), \overline{a}(x_i)] = [\underline{b}(x_i), \overline{b}(x_i)]$ and $[\underline{a'}(x_i), \overline{a'}(x_i)] = [\underline{b'}(x_i), \overline{b'}(x_i)] \forall x_i \in X$.

Definition 4 (see [27, 42]). Let $A, B \in$ IVIF(X) and $\lambda \ge 0$. Then

(1) $A \oplus B = \{\langle x_i, [\underline{a}(x_i) + \underline{b}(x_i) - \underline{a}(x_i)\underline{b}(x_i), \overline{a}(x_i) + \overline{b}(x_i) - \overline{a}(x_i)\overline{b}(x_i)], [\underline{a'}(x_i)\underline{b'}(x_i), \overline{a'}(x_i)\overline{b'}(x_i)]\rangle \mid x_i \in X\}$;

(2) $A \otimes B = \{\langle x_i, [\underline{a}(x_i)\underline{b}(x_i), \overline{a}(x_i)\overline{b}(x_i)], [\underline{a'}(x_i) + \underline{b'}(x_i) - \underline{a'}(x_i)\underline{b'}(x_i), \overline{a'}(x_i) + \overline{b'}(x_i) - \overline{a'}(x_i)\overline{b'}(x_i)]\rangle \mid x_i \in X\}$;

(3) $\lambda A = \{\langle x_i, [1 - (1 - \underline{a}(x_i))^\lambda, 1 - (1 - \overline{a}(x_i))^\lambda], [(\underline{a'}(x_i))^\lambda, (\overline{a'}(x_i))^\lambda]\rangle \mid x_i \in X\}$;

(4) $A^{\lambda} = \{\langle x_i, [(\underline{a}(x_i))^{\lambda}, (\overline{a}(x_i))^{\lambda}], [1-(1-\underline{a'}(x_i))^{\lambda}, 1-(1-\overline{a'}(x_i))^{\lambda}]\rangle \mid x_i \in X\}$.

In the following, Xu [43] introduced two weighted aggregation operators related to IVIFSs as follows.

Definition 5 (see [43]). The weighted arithmetic average operator for $A_j = \{\langle x_i, a_j(x_i), a'_j(x_i)\rangle \mid x_i \in X\} \in \text{IVIF(X)}$, $j = 1, 2, \ldots, n$ is defined by

$$F_w(A_1, A_2, \ldots, A_n) = \left(\left[1 - \prod_{j=1}^{n}\left(1 - \underline{a_j}(x)\right)^{w_j}, 1 - \prod_{j=1}^{n}\left(1 - \overline{a_j}(x)\right)^{w_j}\right], \left[\prod_{j=1}^{n}\left(\underline{a'_j}(x)\right)^{w_j}, \prod_{j=1}^{n}\left(\overline{a'_j}(x)\right)^{w_j}\right]\right), \quad (2)$$

where $w_j \in [0,1]$ and $\sum_{j=1}^{n} w_j = 1$. The element w_j is denoted as the weight of $A_j\,(j = 1, 2, \ldots, n) \in \text{IVIF(X)}$. In particular, F_w is called the arithmetic average operator for IVIFSs if $w_j = 1/n\,(j = 1, 2, \ldots, n)$.

Definition 6 (see [43]). The weighted geometric average operator for $A_j \in \text{IVIF(X)}$, $j = 1, 2, \ldots, n$, is defined by

$$G_w(A_1, A_2, \ldots, A_n) = \left(\left[\prod_{j=1}^{n}\left(\underline{a_j}(x)\right)^{w_j}, \prod_{j=1}^{n}\left(\overline{a_j}(x)\right)^{w_j}\right], \left[1 - \prod_{j=1}^{n}\left(1 - \underline{a'}(x)\right)^{w_j}, 1 - \prod_{j=1}^{n}\left(1 - \overline{a'}(x)\right)^{w_j}\right]\right), \quad (3)$$

where $w_j \in [0,1]$ and $\sum_{j=1}^{n} w_j = 1$. The element w_j is denoted as the weight of $A_j\,(j = 1, 2, \ldots, n) \in \text{IVIF(X)}$. In particular, G_w is called the geometric average operator for IVIFSs if $w_j = 1/n\,(j = 1, 2, \ldots, n)$.

Definition 7. The distance measure of IVIFSs on X is a real valued function $d : \text{IVIF(X)} \times \text{IVIF(X)} \longrightarrow [0,1]$ which satisfies the following properties:

(1) If A is a crisp set, then $d(A, A^c) = 1$;

(2) $d(A, B) = 0 \Longleftrightarrow$ if $A = B$;

(3) $d(A, B) = d(B, A), \forall A, B \in \text{IVIF(X)}$;

(4) If $A \subseteq B \subseteq C$, then $d(A, C) \geq d(A, B)$ and $d(A, C) \geq d(B, C)$.

Definition 8. For any two IVIFSs in X, the following six distance measures were discussed in [36].

Consider $A, B \in \text{IVIF(X)}$, where $X = \{x_1, x_2, \ldots, x_n\}$.

(1) Normalized Euclidean distance

$$d_1(A, B) = \left\{\frac{1}{4n}\sum_{i=1}^{n}\left[\left(\underline{a}(x_i) - \underline{b}(x_i)\right)^2 + \left(\underline{a'}(x_i) - \underline{b'}(x_i)\right)^2 + \left(\overline{a}(x_i) - \overline{b}(x_i)\right)^2 + \left(\overline{a'}(x_i) - \overline{b'}(x_i)\right)^2\right]\right\}^{1/2}. \quad (4)$$

(2) Normalized hamming distance

$$d_2(A, B) = \frac{1}{4n}\sum_{i=1}^{n}\left[\left|\underline{a}(x_i) - \underline{b}(x_i)\right| + \left|\underline{a'}(x_i) - \underline{b'}(x_i)\right| + \left|\overline{a}(x_i) - \overline{b}(x_i)\right| + \left|\overline{a'}(x_i) - \overline{b'}(x_i)\right|\right]. \quad (5)$$

(3) Normalized hamming distance measure induced by Hausdorff metric

$$d_3(A, B) = \frac{1}{2n} \cdot \sum_{i=1}^{n}\left[\max\left\{\left|\underline{a}(x_i) - \underline{b}(x_i)\right|, \left|\overline{a}(x_i) - \overline{b}(x_i)\right|\right\} + \max\left\{\left|\underline{a'}(x_i) - \underline{b'}(x_i)\right|, \left|\overline{a'}(x_i) - \overline{b'}(x_i)\right|\right\}\right]. \quad (6)$$

(4) Normalized distance measure induced by Hausdorff metric

$$d_4(A, B) = \frac{1}{n} \cdot \sum_{i=1}^{n}\max\left\{\frac{1}{2}\left(\left|\underline{a}(x_i) - \underline{b}(x_i)\right| + \left|\overline{a}(x_i) - \overline{b}(x_i)\right|\right), \frac{1}{2}\left(\left|\underline{a'}(x_i) - \underline{b'}(x_i)\right| + \left|\overline{a'}(x_i) - \overline{b'}(x_i)\right|\right)\right\}. \quad (7)$$

(5) Fifth distance measure

$$d_5(A, B) = \frac{1}{2n}\sum_{i=1}^{n}\left\{\frac{1}{4}\left(\left|\underline{a}(x_i) - \underline{b}(x_i)\right| + \left|\overline{a}(x_i) - \overline{b}(x_i)\right| + \left|\underline{a'}(x_i) - \underline{b'}(x_i)\right| + \left|\overline{a'}(x_i) - \overline{b'}(x_i)\right|\right) + \frac{1}{2}\max\left(\left|\underline{a}(x_i) - \underline{b}(x_i)\right| + \left|\overline{a}(x_i) - \overline{b}(x_i)\right|, \left|\underline{a'}(x_i) - \underline{b'}(x_i)\right| + \left|\overline{a'}(x_i) - \overline{b'}(x_i)\right|\right)\right\}. \quad (8)$$

(6) Generalized distance measure, for any $p \geq 2$,

$$d_6(A,B) = \left\{\frac{1}{2n}\sum_{i=1}^{n}\left[\left(\max\left\{\left|\underline{a}(x_i)-\underline{b}(x_i)\right|,\ \left|\overline{a}(x_i)-\overline{b}(x_i)\right)\right)^p\right\} + \left(\max\left\{\left|\underline{a'}(x_i)-\underline{b'}(x_i)\right|, \left|\overline{a'}(x_i)-\overline{b'}(x_i)\right\}\right)^p\right\}^{1/p}. \quad (9)$$

Definition 9 (see [31]). The entropy measure of IVIFSs is a function E : IVIF(X) $\longrightarrow [0,1]$ which satisfies the following four properties.

(1) $E(A) = 0$ if A is a crisp set;

(2) $E(A) = 1 \Longleftrightarrow a(x_i) = a'(x_i), \forall x \in X$;

(3) $E(A) = E(B)$, if A is less fuzzy than B; i.e.,

$a(x_i) \leq b(x_i), a'(x_i) \geq b'(x_i)$, for $b(x_i) \leq b'(x_i)$;

$a(x_i) \geq b(x_i), a'(x_i) \leq b'(x_i)$, for $b(x_i) \geq b'(x_i)$;

$a(x_i) \leq b(x_i), a'(x_i) \geq b'(x_i)$, for $b(x_i) \leq b'(x_i)$;

$a(x_i) \geq b(x_i), a'(x_i) \leq b'(x_i)$, for $b(x_i) \geq b'(x_i)$;

(4) $E(A) = E(A^c)$.

Definition 10 (see [36]). The entropy measure of IVIFS on X can be defined as a function E : IVIF(X) $\longrightarrow [0,1]$ which fulfills the properties as follows:

(1) $E(A) = 0, \forall A \in P(X)$, i.e., if A is a crisp set;

(2) $E(A) = 1, \Longleftrightarrow a(x_i) = a'(x_i) = [1/2, 1/2], \forall x_i \in X$, i.e., $A = ([1/2,1/2],[1/2,1/2])_X$;

(3) If $d(A, \langle[1/2,1/2],[1/2,1/2]\rangle_X) \geq d(B, \langle[1/2,1/2],[1/2,1/2]\rangle_X)$, then $E(A) \leq E(B), \forall A, B \in$ IVIF(X);

(4) $E(A) = E(A^c)$.

Remark 11. The properties as discussed above reveal the following important points:

(1) a crisp set is always nonfuzzy; i.e., an IVIFS has no vagueness when it degenerates to a crisp set;

(2) the fuzziest IVIFS is $F_X = \langle[1/2,1/2],[1/2,1/2]\rangle_X$;

(3) the IVIFS is more fuzzier when it is closer to F_X;

(4) the IVIFS and its complement have the same fuzziness.

Zhang [36] introduced the following useful method to obtain an entropy measure of IVIFSs, which possesses all the above-mentioned four properties.

Definition 12 (see [36]). Suppose the six distance measures between IVIFSs are denoted by $d_i, i = 1,2,\ldots,6$ as mentioned before. Then, for any $A \in$ IVIF(X), $E_i(A) = 1 - 2d_i(A, F_X), i = 1,2,\ldots,6$, are known as the entropy measures of IVIFSs where $d_i (1 \leq i \leq 6)$ are the distance measures as discussed in Definition 8.

3. Comparison of Entropy Measures for IVIFSs

Liu [7] developed few distance measures of fuzzy sets and discussed some of their desired properties. Zeng [17] and Grzegorzewski [44] developed a number of normalized distance measures for IVFSs. Park [45] investigated various distances over IVIFSs in X. Motivated by the above work, we can set our investigation over another useful distance measure of IVIFSs as follows.

Definition 13. For $A, B \in$ IVIF(X), where $A = \{\langle x_i, a(x_i), a'(x_i)\rangle \mid x_i \in X\}$ and $B = \{\langle x_i, b(x_i), b'(x_i)\rangle \mid x_i \in X\}$, one defines

$$\begin{aligned}\underline{d} &= \frac{1}{n}\sum_{i=1}^{n}\min\left\{\left|\underline{a}(x_i)-\underline{b}(x_i)\right|, \left|\underline{a'}(x_i)-\underline{b'}(x_i)\right|,\right.\\ &\qquad \left.\left|\overline{a}(x_i)-\overline{b}(x_i)\right|, \left|\overline{a'}(x_i)-\overline{b'}(x_i)\right|\right\}\\ \overline{d} &= \frac{1}{n}\sum_{i=1}^{n}\max\left\{\left|\underline{a}(x_i)-\underline{b}(x_i)\right|, \left|\underline{a'}(x_i)-\underline{b'}(x_i)\right|,\right.\\ &\qquad \left.\left|\overline{a}(x_i)-\overline{b}(x_i)\right|, \left|\overline{a'}(x_i)-\overline{b'}(x_i)\right|\right\}.\end{aligned} \quad (10)$$

The proposed distance measure of IVIFSs A and B is defined by $d_{FZ}(A,B) = (1/2)(\underline{d} + \overline{d})$.

Remark 14. It is easy to show that the new proposed distance d_{FZ} satisfies all the four properties discussed in Definition 7.

Some important entropy measures of IVIFSs have been introduced in the literature by various researchers to handle MCDM problems of real-life applications [33–35]. Zhang [31] extended the intuitionistic fuzzy entropy axioms presented in [12] and introduced a new definition of entropy for IVIFSs. Zhang [36] further extended the entropy concept for fuzzy set [2, 7] to introduce another useful distance based entropy measure for IVIFS. Now, we introduce another entropy measure of IVIFSs based on a proposed distance measure d_{FZ} as follows.

Definition 15. Let d_{FZ} be the proposed distance measure (Definition 13) of IVIFSs. Then, $E_{FZ}(A) = 1 - 2d(A, F_X)$ is defined as an entropy measure of IVIFSs for any $A \in$ IVIF(X) based on the distance measure d_{FZ}.

Theorem 16. *Let d_{FZ} be the proposed distance measure (Definition 13) between two IVIFSs. Then, $E_{FZ}(A) = 1 - 2d_{FZ}(A, F_X)$ is the entropy measure of IVIFS A for any $A \in$ IVIF(X).*

Table 1: Comparison of the proposed entropy measure with existing entropy measures of IVIFSs.

IVIFS	E_{FZ}	E_1	E_2	E_3	E_4	E_5	E_6	E_L	E_{ZJ}	$E_{W,\lambda=0.5}$	E_{ZM}	E_Y
A	0.3657	0.2825	0.3601	0.2936	0.2980	0.3290	0.2422	0.2904	0.2939	0.3239	0.1139	0.5009
$A^{1/2}$	0.4000	0.3203	0.4100	0.3200	0.3000	0.3550	0.2624	0.3294	0.2819	0.3365	0.1547	0.5651
A^2	0.3320	0.2726	0.3390	0.2360	0.2260	0.2825	0.1999	0.2886	0.2103	0.2717	0.1771	0.5095
A^3	0.3000	0.2250	0.2941	0.1876	0.1982	0.2461	0.1579	0.27	0.2102	0.2662	0.0592	0.4514
A^4	0.2224	0.1911	0.2519	0.1792	0.1572	0.2045	0.1372	0.2416	0.1517	0.2217	0.194	0.4099

Proof. We will try to prove that E_{FZ} fulfills all the properties discussed in Definition 10.

(1) For $A(x) = \langle [1,1],[0,0]\rangle$ or $A(x) = \langle [0,0],[1,1]\rangle$ $\forall x \in X$, i.e., if A is a crisp set, then, from Definition 13 we have $d_{FZ}(A, F_X) = 1/2$.

Hence, $E_{FZ}(A) = 1 - 2d_{FZ}(A, F_X) = 0$.

(2) $E_{FZ}(A) = 1 \Longleftrightarrow 1 - 2d_{FZ}(A, F_X) = 1 \Longleftrightarrow d_{FZ}(A, F_X) = 0 \Longleftrightarrow A = F_X$ (by Definition 13).

(3) $d_{FZ}(A, F_X) \geq d_{FZ}(B, F_X)$; then it immediately follows that $E_{FZ}(A) \leq E_{FZ}(B)$.

(4) $A^c = \{\langle x_i, [\underline{a}'(x_i), \overline{a}'(x_i)], [\underline{a}(x_i), \overline{a}(x_i)]\rangle \mid x_i \in X\}$. Therefore, it can be easily observed that $d_{FZ}(A^c, F_X) = d_{FZ}(A, F_X)$. Hence $E_{FZ}(A^c) = E_{FZ}(A)$. □

Zhang [36] compared the performances of the proposed entropies E_1, E_2, E_3, E_4, E_5, and E_6 to those of E_L [30], E_{ZJ} [31], $E_{W,\lambda}$ [34], E_{ZM} [33], and E_Y [35]. Now we shall also compare the performance of proposed entropy measure to those of all entropy measures as mentioned above with the help of the following example.

Example 17. We will consider here an example, as adapted by Hung and Yang in [46]. Suppose that $A = \{\langle x_i, [\underline{a}(x_i), \overline{a}(x_i)], [\underline{a}'(x_i), \overline{a}'(x_i)]\rangle \mid x_i \in X\} \in$ IVIF(X). For any $n \in R^+$, define the IVIFS A^n as follows:

$$A^n = \Big\{\Big\langle x_i, \left[\underline{a}(x_i)^n, \overline{a}(x_i)^n\right], \left[1 - \left(1 - \underline{a}'(x_i)\right)^n, 1 - \left(1 - \overline{a}'(x_i)\right)^n\right]\Big\rangle \mid x_i \in X\Big\}. \tag{11}$$

The IVIFS A was defined on the universal set $X = \{6, 7, 8, 9, 10\}$ in [46] as follows:

$$A = \{\langle 6, [0.1, 0.2], [0.6, 0.7]\rangle, \langle 7, [0.3, 0.5], [0.4, 0.5]\rangle, \langle 8, [0.6, 0.7], [0.1, 0.2]\rangle, \langle 9, [0.8, 0.9], [0, 0.1]\rangle, \langle 10, [1.0, 1.0], [0, 0]\rangle\}, \tag{12}$$

and, similarly, we can determine the following IVIFSs:

$$\begin{aligned}
A^{1/2} = \{&\langle 6, [0.3162, 0.4472], [0.3675, 0.4523]\rangle, \\
&\langle 7, [0.5477, 0.7071], [0.2254, 0.2929]\rangle, \\
&\langle 8, [0.7746, 0.8367], [0.0513, 0.1056]\rangle, \\
&\langle 9, [0.8944, 0.9487], [0, 0.0513]\rangle, \\
&\langle 10, [1.0, 1.0], [0, 0]\rangle\}, \\
A^2 = \{&\langle 6, [0.01, 0.04], [0.84, 0.91]\rangle, \\
&\langle 7, [0.09, 0.25], [0.64, 0.75]\rangle, \\
&\langle 8, [0.36, 0.49], [0.19, 0.36]\rangle, \\
&\langle 9, [0.64, 0.81], [0, 0.19]\rangle, \langle 10, [1.0, 1.0], [0, 0]\rangle\}, \\
A^3 = \{&\langle 6, [0.0010, 0.0080], [0.9360, 0.9730]\rangle, \\
&\langle 7, [0.0270, 0.1250], [0.7840, 0.8750]\rangle, \\
&\langle 8, [0.2160, 0.3430], [0.2710, 0.4880]\rangle, \\
&\langle 9, [0.5120, 0.7290], [0, 0.2710]\rangle, \\
&\langle 10, [1.0, 1.0], [0, 0]\rangle\}, \\
A^4 = \{&\langle 6, [0.0001, 0.0016], [0.9744, 0.9919]\rangle, \\
&\langle 7, [0.0081, 0.0625], [0.8704, 0.9375]\rangle, \\
&\langle 8, [0.1296, 0.2401], [0.3439, 0.5904]\rangle, \\
&\langle 9, [0.4096, 0.6561], [0, 0.3439]\rangle, \\
&\langle 10, [1.0, 1.0], [0, 0]\rangle\}.
\end{aligned} \tag{13}$$

In order to express the characterization of these sets in linguistic variables, De [47] considered A as "LARGE" in X. By using the above-mentioned operations,

$A^{1/2}$ can be considered as "more or less LARGE";
A^2 can be considered as "very LARGE";
A^3 can be considered as "quite very LARGE";
A^4 can be considered as "very very LARGE".

Based on the mathematical operations, the entropy measures of IVIFSs as discussed above should fulfill the condition as mentioned in [46]; i.e., $E(A^{1/2}) > E(A) > E(A^2) > E(A^3) > E(A^4)$. Now we try to compare the performance of proposed entropy measures E_{FZ} by using the IVIFSs as mentioned above to those of $E_1, E_2, E_3, E_4, E_5, E_6, E_L, E_{ZJ}, E_{W,\lambda}, E_{ZM}$, and E_Y. The entropy values by using all the corresponding distance measures can be seen in Table 1.

It can be easily seen that the purposed entropy measure also looks quite reasonable as compared with existing entropy measures of IVIFSs.

4. Optimal Decision Making Approaches Using IVIFSs

In this section, we develop some procedures to solve various MCDM and MCGDM problems based on a proposed entropy measure formula in the context of interval-valued intuitionistic fuzzy environment.

4.1. For Single DM. Let $A = \{A_1, A_2, \ldots, A_m\}$ be a set of m alternatives and $C = \{C_1, C_2, \ldots, C_n\}$ be a set of n criteria. Suppose each alternative $A_i\ (i = 1, 2, \ldots, m)$ is assessed by the DM D with respect to each criterion $C_j\ (j = 1, 2, \ldots, n)$ using the IVIFS

$$A_i = \left\{ \left\langle C_j, \left[\underline{a}(x_i), \overline{a}(x_i)\right], \left[\underline{a'}(x_i), \overline{a'}(x_i)\right] \right\rangle \mid x_i \in X \right\}, \tag{14}$$

where $0 \leq \overline{a}(x_i) + \overline{a'}(x_i) \leq 1$. Suppose that $R = (r_{ij})_{m\times n} = ([a_{ij}, b_{ij}], [c_{ij}, d_{ij}])_{m\times n}$ is an interval-valued intuitionistic fuzzy decision matrix provided by the DM D during the assessment process. The intervals $[a_{ij}, b_{ij}]$ and $[c_{ij}, d_{ij}]$ represent the degrees that the alternative $A_i\ (i = 1, 2, \ldots, m)$ satisfies and does not satisfy, respectively, the attribute $C_j (j = 1, 2, \ldots, n)$ during the assessment process.

By using the proposed entropy formula of IVIFSs (see Definition 15), the entropy value of each alternative corresponding to each criterion can be easily computed. Suppose $E_1, E_2, \ldots, E_m$ are the corresponding entropy measures of each alternative $A_i\ (i = 1, 2, \ldots, m)$, respectively. As it is generally known, the DM can get more useful information from an alternative if its entropy value corresponding to each criterion is smaller. Therefore, the less the entropy value of an alternative, the more it will be considered as a better option among all the given alternatives. Thus, with respect to the descending order of the entropy values as calculated above, we can find the ranking order of all the alternatives.

4.2. For Single DM with Given Weights of Criteria. Let A, C, and $R = (r_{ij})_{m\times n}$ be as previously mentioned in Section 4.1. Suppose the DM D entered the weight value $w_j, w_j \in [0, 1]$, $\sum_{j=1}^{n} w_j = 1$ for each $C_j\ (j = 1, 2, \ldots, n)$. Now, we try to develop a procedure to solve this MCDM problem, according to the defined measures in the following two steps.

Step 1. According to each row in the decision matrix $R = (r_{ij})_{m\times n}$, calculate the aggregated interval-valued intuitionistic fuzzy number $\alpha_i\ (i = 1, 2, \ldots, m)$ where $\alpha_i = F_{iw}(r_{i1}, r_{i2}, \ldots, r_{in})$ or $\alpha_i = G_{iw}(r_{i1}, r_{i2}, \ldots, r_{in})$ by using Definitions 5 and 6.

Step 2. Calculate $d_i(\alpha_i, F_X)$ by using the proposed distance measure (see Definition 13) for each $i = 1, 2, \ldots, m$, and then find their corresponding entropy measures $E_1, E_2, \ldots, E_m$ of each alternative by using Definition 15. The ranking order of all the alternatives can be obtained by using again the concept of entropy theory.

4.3. Procedure to Solve MCGDM Problems with IVIFSs. Let A and C be again considered as previously mentioned in Section 4.1. Suppose $D = \{d_1, d_2, \ldots, d_t\}$ is a set of DMs. Let $[a_{ij}^k, b_{ij}^k]$ be an interval which represents the degree to which the alternative $A_i\ (i = 1, 2, \ldots, m)$ meets the criteria $C_j\ (j = 1, 2, \ldots, n)$ and $[c_{ij}^k, d_{ij}^k]$ represents the degree to which the alternative $A_i (i = 1, 2, \ldots, m)$ does not meet the criteria $C_j\ (j = 1, 2, \ldots, n)$ assessed by the DM $d_k\ (k = 1, 2, \ldots, t)$. Let $R^k = (r_{ij}^k)_{m\times n} = ([a_{ij}^k, b_{ij}^k], [c_{ij}^k, d_{ij}^k])_{m\times n}$ be an intuitionistic fuzzy decision matrix provided by a DM $d_k\ (k = 1, 2, \ldots, t)$ for an alternative $A_i\ (i = 1, 2, \ldots, m)$ under the criteria $C_j\ (j = 1, 2, \ldots, n)$. The concrete steps of MCGDM procedure in the IVIF setting can be organized as follows.

Step 1. Determine the aggregated decision matrix S of the decision matrices $R_1, R_2, \ldots, R_t$ where $S = R_1 \oplus R_2 \oplus \cdots \oplus R_t$ by using Definition 4.

Step 2. By using the proposed entropy formula of IVIFSs (see Definition 15), it is easy to compute the entropy value of each possible alternative in the aggregated decision matrix S and then deduce the ranking order of all the alternatives $A_i\ (i = 1, 2, \ldots, m)$ using again the concept of entropy theory.

5. Numerical Examples

In this section, various practical MCDM and MCGDM problems are offered to show the concrete applications of the proposed method. First, we study the same MCDM problem as presented by Nayagam in [22].

Example 1 (see [22]). Assume there exists a panel with four possible alternatives for investment: (1) A_1: a car company; (2) A_2: a food company; (3) A_3: a computer company; (4) A_4: an arms company. The investment entity must make a decision according to the following three criteria: C_1 (risk), C_2 (growth), and C_3 (environmental impact). The interval-valued intuitionistic fuzzy assessment values of four alternatives provided by the DMs under the above-mentioned three criteria can be seen as in the following matrix:

$$R = \begin{array}{c} \\ A_1 \\ A_2 \\ A_3 \\ A_4 \end{array} \begin{array}{c} \begin{array}{ccc} C_1 & C_2 & C_3 \end{array} \\ \begin{pmatrix} \langle [0.4, 0.5], [0.3, 0.4] \rangle & \langle [0.4, 0.6], [0.2, 0.4] \rangle & \langle [0.1, 0.3], [0.5, 0.6] \rangle \\ \langle [0.6, 0.7], [0.2, 0.3] \rangle & \langle [0.6, 0.7], [0.2, 0.3] \rangle & \langle [0.4, 0.8], [0.1, 0.2] \rangle \\ \langle [0.3, 0.6], [0.3, 0.4] \rangle & \langle [0.5, 0.6], [0.3, 0.4] \rangle & \langle [0.4, 0.5], [0.1, 0.3] \rangle \\ \langle [0.7, 0.8], [0.1, 0.2] \rangle & \langle [0.6, 0.7], [0.1, 0.3] \rangle & \langle [0.3, 0.4], [0.1, 0.2] \rangle \end{pmatrix} \end{array}. \tag{15}$$

Corresponding to an alternative $A_i\,(i = 1, 2, 3, 4)$ with respect to each criterion $C_j\,(j = 1, 2, 3)$, each element of the matrix R is an IVIFS. As we know, the less the uncertainty information about an alternative with respect to given criteria, the better the alternative is. Therefore, the optimal ranking of the alternatives can be obtained according to the measure of the entropy values of corresponding IVIFSs.

By employing the proposed entropy formula (see Definition 15) of IVIFSs, the corresponding entropy measures of the four alternatives $A_i\,(i = 1, 2, 3, 4)$ are, respectively, $E_1 = 0.6667, E_2 = 0.5667, E_3 = 0.7000$, and $E_4 = 0.4667$. Thus, with respect to descending order of the entropy values, the ranking order of all the four alternatives is $A_4 > A_2 > A_1 > A_3$. Hence, the best alternative is A_4, which agrees with the ranking results as obtained in [22].

Next, we study the same problem as discussed by Xu in [42].

Example 2 (see [42]). Suppose we are given five alternatives $A_i\,(i = 1, 2, 3, 4, 5)$ and six appropriate criteria $C_j\,(j = 1, 2, 3, 4, 5, 6)$. Assume that the criterion weight vector provided by the DM is $W = (0.20, 0.10, 0.25, 0.10, 0.15, 0.20)$. The decision matrix R containing the assessment values of all the alternatives with respect to six criteria provided by the DM using IVIFSs is then constructed and given as follows:

$$R = \begin{array}{c} \\ A_1 \\ A_2 \\ A_3 \\ A_4 \\ A_5 \end{array}\begin{array}{c} \begin{array}{cccccc} C_1 & C_2 & C_3 & C_4 & C_5 & C_6 \end{array} \\ \begin{pmatrix} \langle[0.2,0.3], [0.4,0.5]\rangle & \langle[0.6,0.7], [0.2,0.3]\rangle & \langle[0.4,0.5], [0.2,0.4]\rangle & \langle[0.7,0.8], [0.1,0.2]\rangle & \langle[0.1,0.3], [0.5,0.6]\rangle & \langle[0.5,0.7], [0.2,0.3]\rangle \\ \langle[0.6,0.7], [0.2,0.3]\rangle & \langle[0.5,0.6], [0.1,0.3]\rangle & \langle[0.6,0.7], [0.2,0.3]\rangle & \langle[0.6,0.7], [0.1,0.2]\rangle & \langle[0.3,0.4], [0.5,0.6]\rangle & \langle[0.4,0.7], [0.1,0.2]\rangle \\ \langle[0.4,0.5], [0.3,0.4]\rangle & \langle[0.7,0.8], [0.1,0.2]\rangle & \langle[0.5,0.6], [0.3,0.4]\rangle & \langle[0.6,0.7], [0.1,0.3]\rangle & \langle[0.4,0.5], [0.3,0.4]\rangle & \langle[0.3,0.5], [0.1,0.3]\rangle \\ \langle[0.6,0.7], [0.2,0.3]\rangle & \langle[0.5,0.7], [0.1,0.3]\rangle & \langle[0.7,0.8], [0.1,0.2]\rangle & \langle[0.3,0.4], [0.1,0.2]\rangle & \langle[0.5,0.6], [0.1,0.3]\rangle & \langle[0.7,0.8], [0.1,0.2]\rangle \\ \langle[0.5,0.6], [0.3,0.5]\rangle & \langle[0.3,0.4], [0.3,0.5]\rangle & \langle[0.6,0.7], [0.1,0.3]\rangle & \langle[0.6,0.8], [0.1,0.2]\rangle & \langle[0.6,0.7], [0.2,0.3]\rangle & \langle[0.5,0.6], [0.2,0.4]\rangle \end{pmatrix} \end{array}. \quad (16)$$

The proposed method is now applied to solve the given problem, according to the computational practice as follows.

Step 1. If we focus on the groups' influence, the following weighted arithmetic average values α_i of all the alternatives $A_i\,(i = 1, 2, 3, 4, 5)$ are obtained by using a definition of weighted arithmetic average operator (see Definition 5) as presented in Section 2:

$$\begin{aligned} \alpha_1 &= ([0.4165, 0.5597], [0.2459, 0.3804]), \\ \alpha_2 &= ([0.5176, 0.6574], [0.1739, 0.2947]), \\ \alpha_3 &= ([0.4703, 0.5900], [0.1933, 0.3424]), \\ \alpha_4 &= ([0.6070, 0.7203], [0.1149, 0.2400]), \\ \alpha_5 &= ([0.5375, 0.6536], [0.1772, 0.3557]). \end{aligned} \quad (17)$$

Step 2. Let E_i's denote the entropy of α_i's $(i = 1, 2, 3, 4, 5)$, respectively. Then by using our proposed distance measures of IVIFSs (see Definition 13), the corresponding entropy measures of α_i's are obtained, respectively, as $E_1 = 0.6863, E_2 = 0.6563, E_3 = 0.6636, E_4 = 0.5078$, and $E_5 = 0.6397$. Based on these entropy measures, the optimal ranking order of the five alternatives is $A_4 > A_5 > A_2 > A_3 > A_1$ and the most suitable alternative is A_4. The ranking results coincide with the ones obtained by Xu in [42].

If we focus on the individual influence, the weighted geometric average values α_i of all the alternatives are calculated by applying Definition 6. The IVIF values α_i for $A_i\,(i = 1, 2, 3, 4, 5)$ are computed as follows:

$$\begin{aligned} \alpha_1 &= ([0.3257, 0.4848], [0.2878, 0.4132]), \\ \alpha_2 &= ([0.4896, 0.6338], [0.2185, 0.3301]), \\ \alpha_3 &= ([0.4398, 0.5673], [0.2260, 0.3533]), \\ \alpha_4 &= ([0.5733, 0.6868], [0.1210, 0.2467]), \\ \alpha_5 &= ([0.5204, 0.6307], [0.1991, 0.3782]). \end{aligned} \quad (18)$$

The corresponding entropy measures of α_i's can be calculated again as, respectively, $E_1 = 0.7726, E_2 = 0.7081, E_3 = 0.6657, E_4 = 0.5477$, and $E_5 = 0.6786$, and therefore the ranking order of the given alternatives is $A_4 > A_3 > A_5 > A_2 > A_1$ and, obviously, A_4 is again the best alternative as we have seen earlier.

Now, we discuss a MCGDM problem adapted from Park et al. [27] concerning a manufacturing company, searching the finest international dealer for one of its main key parts used in assembling procedure.

Example 3 (see [27]). Suppose there are four potential international dealers (attributes) $A = (A_1, A_2, A_3, A_4)$ and these are evaluated with respect to five criteria: (1) C_1: overall cost of the product; (2) C_2: quality of the product; (3) C_3: service performance of dealer; (4) C_4: dealer's profile; (5) C_5: risk factor. Let us assume that a group of four experts d_k $(k = 1, 2, 3, 4)$ is formed from each strategic decision area and they provide the following four decision matrices based on their judgements:

$$R_1 = \begin{array}{cc} & \begin{array}{ccccc} C_1 & C_2 & C_3 & C_4 & C_5 \end{array} \\ \begin{array}{c} A_1 \\ A_2 \\ A_3 \\ A_4 \end{array} & \begin{pmatrix} \langle [0.5, 0.6], [0.2, 0.3] \rangle & \langle [0.3, 0.5], [0.4, 0.5] \rangle & \langle [0.6, 0.7], [0.2, 0.3] \rangle & \langle [0.5, 0.7], [0.1, 0.2] \rangle & \langle [0.1, 0.4], [0.3, 0.5] \rangle \\ \langle [0.3, 0.4], [0.4, 0.6] \rangle & \langle [0.1, 0.3], [0.2, 0.4] \rangle & \langle [0.3, 0.4], [0.4, 0.5] \rangle & \langle [0.2, 0.4], [0.5, 0.6] \rangle & \langle [0.7, 0.8], [0.1, 0.2] \rangle \\ \langle [0.4, 0.5], [0.3, 0.5] \rangle & \langle [0.7, 0.8], [0.1, 0.2] \rangle & \langle [0.5, 0.8], [0.1, 0.2] \rangle & \langle [0.4, 0.6], [0.2, 0.3] \rangle & \langle [0.5, 0.6], [0.2, 0.3] \rangle \\ \langle [0.3, 0.5], [0.4, 0.5] \rangle & \langle [0.1, 0.2], [0.7, 0.8] \rangle & \langle [0.1, 0.2], [0.5, 0.8] \rangle & \langle [0.2, 0.3], [0.4, 0.6] \rangle & \langle [0.2, 0.3], [0.5, 0.6] \rangle \end{pmatrix} \end{array}$$

$$R_2 = \begin{array}{cc} & \begin{array}{ccccc} C_1 & C_2 & C_3 & C_4 & C_5 \end{array} \\ \begin{array}{c} A_1 \\ A_2 \\ A_3 \\ A_4 \end{array} & \begin{pmatrix} \langle [0.4, 0.5], [0.2, 0.4] \rangle & \langle [0.3, 0.4], [0.4, 0.6] \rangle & \langle [0.6, 0.7], [0.1, 0.2] \rangle & \langle [0.5, 0.6], [0.1, 0.3] \rangle & \langle [0.1, 0.3], [0.3, 0.5] \rangle \\ \langle [0.3, 0.5], [0.4, 0.5] \rangle & \langle [0.1, 0.3], [0.3, 0.7] \rangle & \langle [0.3, 0.4], [0.4, 0.5] \rangle & \langle [0.2, 0.3], [0.6, 0.7] \rangle & \langle [0.6, 0.8], [0.1, 0.2] \rangle \\ \langle [0.4, 0.6], [0.3, 0.4] \rangle & \langle [0.6, 0.8], [0.1, 0.2] \rangle & \langle [0.7, 0.8], [0.1, 0.2] \rangle & \langle [0.4, 0.6], [0.3, 0.4] \rangle & \langle [0.5, 0.6], [0.2, 0.4] \rangle \\ \langle [0.3, 0.4], [0.4, 0.6] \rangle & \langle [0.1, 0.2], [0.6, 0.8] \rangle & \langle [0.1, 0.2], [0.7, 0.8] \rangle & \langle [0.3, 0.4], [0.4, 0.6] \rangle & \langle [0.2, 0.4], [0.5, 0.6] \rangle \end{pmatrix} \end{array} \quad (19)$$

$$R_3 = \begin{array}{cc} & \begin{array}{ccccc} C_1 & C_2 & C_3 & C_4 & C_5 \end{array} \\ \begin{array}{c} A_1 \\ A_2 \\ A_3 \\ A_4 \end{array} & \begin{pmatrix} \langle [0.4, 0.7], [0.1, 0.2] \rangle & \langle [0.3, 0.5], [0.3, 0.4] \rangle & \langle [0.6, 0.7], [0.1, 0.2] \rangle & \langle [0.5, 0.6], [0.1, 0.3] \rangle & \langle [0.3, 0.5], [0.4, 0.5] \rangle \\ \langle [0.4, 0.5], [0.2, 0.4] \rangle & \langle [0.2, 0.4], [0.4, 0.5] \rangle & \langle [0.4, 0.5], [0.3, 0.4] \rangle & \langle [0.1, 0.2], [0.7, 0.8] \rangle & \langle [0.6, 0.7], [0.2, 0.3] \rangle \\ \langle [0.2, 0.4], [0.3, 0.4] \rangle & \langle [0.6, 0.8], [0.1, 0.2] \rangle & \langle [0.5, 0.7], [0.1, 0.3] \rangle & \langle [0.5, 0.7], [0.2, 0.3] \rangle & \langle [0.6, 0.8], [0.1, 0.2] \rangle \\ \langle [0.3, 0.4], [0.2, 0.4] \rangle & \langle [0.1, 0.2], [0.6, 0.8] \rangle & \langle [0.1, 0.3], [0.5, 0.7] \rangle & \langle [0.2, 0.3], [0.5, 0.7] \rangle & \langle [0.1, 0.2], [0.6, 0.8] \rangle \end{pmatrix} \end{array}$$

$$R_4 = \begin{array}{cc} & \begin{array}{ccccc} C_1 & C_2 & C_3 & C_4 & C_5 \end{array} \\ \begin{array}{c} A_1 \\ A_2 \\ A_3 \\ A_4 \end{array} & \begin{pmatrix} \langle [0.6, 0.7], [0.2, 0.3] \rangle & \langle [0.3, 0.4], [0.3, 0.4] \rangle & \langle [0.7, 0.8], [0.1, 0.2] \rangle & \langle [0.5, 0.6], [0.1, 0.3] \rangle & \langle [0.1, 0.2], [0.5, 0.7] \rangle \\ \langle [0.4, 0.5], [0.4, 0.5] \rangle & \langle [0.1, 0.2], [0.2, 0.3] \rangle & \langle [0.3, 0.4], [0.5, 0.6] \rangle & \langle [0.2, 0.3], [0.4, 0.6] \rangle & \langle [0.6, 0.7], [0.1, 0.2] \rangle \\ \langle [0.4, 0.5], [0.3, 0.4] \rangle & \langle [0.6, 0.7], [0.1, 0.3] \rangle & \langle [0.5, 0.8], [0.1, 0.2] \rangle & \langle [0.4, 0.5], [0.2, 0.3] \rangle & \langle [0.5, 0.6], [0.3, 0.4] \rangle \\ \langle [0.3, 0.4], [0.4, 0.5] \rangle & \langle [0.1, 0.3], [0.6, 0.7] \rangle & \langle [0.1, 0.2], [0.5, 0.8] \rangle & \langle [0.2, 0.3], [0.4, 0.5] \rangle & \langle [0.3, 0.4], [0.5, 0.6] \rangle \end{pmatrix} \end{array}.$$

We follow the same methodology as discussed in Section 4.3 for solving the MCGDM problems by following the two steps as discussed.

Step 1. By using Definition 4, compute the aggregated matrix S of the decision matrices R_1, R_2, R_3, R_4 where $S = R_1 \oplus R_2 \oplus R_3 \oplus R_4$. The aggregated matrix S is given as follows:

$$S = \begin{array}{cc} & \begin{array}{ccccc} C_1 & C_2 & C_3 & C_4 & C_5 \end{array} \\ \begin{array}{c} A_1 \\ \\ A_2 \\ \\ A_3 \\ \\ A_4 \\ \\ \end{array} & \begin{pmatrix} \langle [0.9280, 0.9820], & \langle [0.7599, 0.9100], & \langle [0.9808, 0.9946], & \langle [0.9375, 0.9808], & \langle [0.4897, 0.8320], \\ [0.0008, 0.0072] \rangle & [0.0144, 0.0480] \rangle & [0.0002, 0.0024] \rangle & [0.0001, 0.0054] \rangle & [0.0180, 0.0875] \rangle \\ \langle [0.8236, 0.9250], & \langle [0.4168, 0.7648], & \langle [0.7942, 0.8920], & \langle [0.5392, 0.7648], & \langle [0.9808, 0.9964], \\ [0.0128, 0.0600] \rangle & [0.0048, 0.0420] \rangle & [0.0240, 0.0600] \rangle & [0.0840, 0.2016] \rangle & [0.0002, 0.0024] \rangle \\ \langle [0.8272, 0.9400], & \langle [0.9808, 0.9976], & \langle [0.9625, 0.9976], & \langle [0.8920, 0.9760], & \langle [0.9500, 0.9872], \\ [0.0081, 0.0320] \rangle & [0.0001, 0.0024] \rangle & [0.0001, 0.0024] \rangle & [0.0024, 0.0108] \rangle & [0.0012, 0.0096] \rangle \\ \langle [0.7599, 0.8920], & \langle [0.3439, 0.6416], & \langle [0.3439, 0.6416], & \langle [0.6416, 0.7942], & \langle [0.5968, 0.7984], \\ [0.0128, 0.0600] \rangle & [0.1512, 0.3584] \rangle & [0.0875, 0.3584] \rangle & [0.0320, 0.1260] \rangle & [0.0750, 0.1728] \rangle \end{pmatrix} \end{array}. \quad (20)$$

Step 2. By using the proposed entropy formula of IVIFSs (see Definition 15), the corresponding entropy values of all alternatives A_1, A_2, A_3, and A_4 can be obtained, respectively, as $E_1 = 0.1834, E_2 = 0.2810, E_3 = 0.0799$, and $E_4 = 0.4154$.

Again with respect to the descending order of these entropy measures, the ranking order of the alternatives is $A_3 > A_1 > A_2 > A_4$. We can see that the most desirable global supplier is A_3 and the results are in agreement with Park et al. [27].

Remark 4. If we find the aggregated matrix T of the decision matrices R_1, R_2, R_3, R_4, i.e., $T = R_1 \otimes R_2 \otimes R_3 \otimes R_4$ by using Definition 4 and by using again the proposed entropy formula of IVIFSs (see Definition 15), the entropy values of each possible alternative A_1, A_2, A_3, and A_4 are calculated, respectively, as $E_1 = 0.3630, E_2 = 0.2396, E_3 = 0.4154$, and $E_4 = 0.0799$.

Again, by the descending order of the entropy values, the ranking of the four alternatives is $A_4 > A_2 > A_1 > A_3$. We observe that the ranking order of the alternatives is reversed.

6. Conclusion

In this paper, we propose a new formula to compute the distance measure between IVIFSs. Based on the proposed distance measure, we further introduce a definition of entropy measure for IVIFS. The methodology proposed in this paper helps the DMs in providing their assessment about an alternative by considering the interval-valued intuitionistic fuzzy information in real-life applications. The effectiveness and applicability of the proposed approach is demonstrated by conducting a comparison analysis between the proposed entropy measure and different other entropy measures of IVIFSs. Finally, various practical MCDM and MCGDM problems are given to further validate the usefulness of the proposed method which can efficiently help the DMs at large. Future studies will now extend the methodology of investigating different entropy measures for linguistic decision environments, such as hesitant 2-tuple linguistic information model and intuitionistic 2-tuple linguistic information model.

Data Availability

All data generated or analyzed during this study are included in this manuscript.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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