

Kalman Filtering for Discrete-Time Networked Control Systems with Incomplete Observations: A Dropping Over-Delayed Packets Approach

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Abstract: In this paper, with regards to discrete-time networked control systems with incomplete observations, a novel optimal linear estimator design is presented. Through a threshold-length buffer coupled to the estimator and a dropping over-delayed packet strategy, the packet arrival process is artificially limited to time pre-threshold at current moment. According to statistics and probability theory, we model the packet arrival probability subject to the choice of threshold. Based on linear matrix inequality (LMI) theorem and modified Riccati equation with packet arrival probability, with the existence of a critical bound of packet arrival rate, we can completely establish the relation between the filter convergence and the threshold-based packet arrival process as well as characterize the filter performance. Examples demonstrate the feasibility and practicability of the proposed method.

Keywords: Networked Control Systems, Kalman Filter, Arrival Probability, Dropping Over-delayed, Riccati Equation, Linear Matrix Inequality (LMI)

1. INTRODUCTION

Networked Control Systems (NCS) (Wang *et al.*, 2008, and Hespanha *et al.*, 2007), which can offer access to information sharing and communication among all the devices of a system, can be seen in many areas such as vehicle control or industrial process. Known high performance depends on system monitoring and regulation, state estimation like Luenberger observer (Ellis, 2002) or Kalman filter (Grewal, and Andrews, 2008) is the necessity to main control strategies such as state feedback control or state-space based generalized predictive control. Hence, optimal state estimation (Simon, 2006) with respect to NCS is a problem of importance, giving rise to several challenges due to the loss of performance against network-induced uncertainties.

Along with its advantages, network also induced new problems of time delays, packet loss, and out-of-order. Zhu *et al.* (2005) began to focus on short time-delay. According to the previous literatures (Li *et al.*, 2006, and Luck *et al.*, 1990), Zhang *et al.* (2012) employed a compensation-based scheme subject to maximum delay to artificially complement long time-delays to maximum. Though the loss of performance is considered and the happening of packet re-ordering has been avoided, the study of buffer-free strategies (Seuret, 2008) was presented. He requested to know minimal and maximal bounds of delay and consider the process of one packet over N consecutive arriving at the destination. A first original contribution of the present study is that the pre-knowledge of delay is not required. Obviously, it still takes effect for a network with unbounded delays. The presented approach allows for minimizing a finite buffer and the issue of time delay is considered as that of packet loss. It constitutes a second contribution. Meanwhile, based on Kalman filtering theory, Schenato (2008) designed a finite

memory buffers needing multi-step iterations in one sampling period. Similar situations can be found in (Sahebsara *et al.*, 2007, and Sun, *et al.*, 2008). For the sake of avoiding the time-delay systems used by (Kruszewski, 2012, and Zhang, Kruszewski, 2012), it is expected to compensate for. Thus, we develop a network-faced estimator, which follows two points. Firstly, it is based on a delay-free controlled system. Secondly, the buffer size can be designed to match packet arrival process.

This paper is organized as follows: Section 2 gives an introduction to the system and modelling of packet arrival process. An optimal filter with a probabilistic approach is developed in section 3. Section 4 studies the relation among packet arrival probability, threshold-based buffer, and filter convergence. The proposed method is experimented in Section 5 and concluding remarks are given in Section 6.

2. PROBLEM STATEMENT

Let us consider a discrete-time model with sampling time T_s . The state-space model of a physical discrete-time system is assumed to satisfy:

$$\begin{cases} x_{k+1} = \Phi x_k + \Gamma u_k + \omega_k \\ y_k = C x_k + v_k \end{cases}, k \geq 1 \quad (1)$$

where at time kT_s , $x_k \in \mathfrak{R}^m$ is the state vector, $y_k \in \mathfrak{R}^n$ is the output vector, $u_k \in \mathfrak{R}^p$ is the control vector, and Φ , Γ , C are constant matrices of appropriate dimension. $\{\omega_k\}$, $\{v_k\}$ are Gaussian, white, and zero mean processes. $\{x_k\}$, $\{\omega_k\}$, and $\{v_k\}$ are uncorrelated, with respective positive-definite covariance P_k , Q , and R .

Hereby, we consider such a class of NCS with network-induced delays and rare packet loss guaranteed in the forward

channel (Fig. 1) like Can bus. Its priority principle of packet transmission avoids the happening of packet loss. Timestamp is contained in every data packet to indicate its ‘‘age’’ (Nilsson, 1998). Obviously, any out-of-order arrival packet can be reordered within one buffer span.

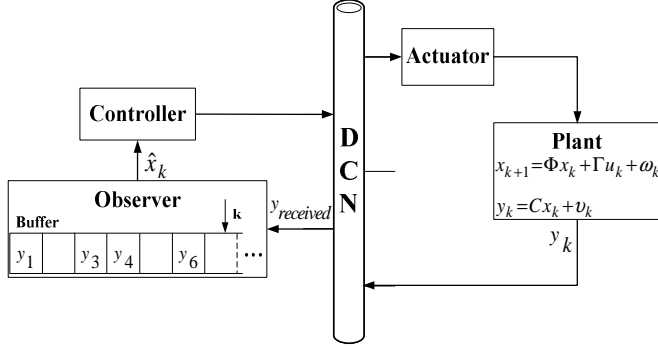


Fig. 1. Modelling of a feedback networked control systems with a buffer-based state observer.

In Figure (1), observations y are delivered via a digital communication network (DCN) existing stochastic delays and packet loss. Its arrival process at the observer site can be modelled by defining a binary random variable γ_n .

$$\gamma_n = \begin{cases} 1, & \text{if packet arrives} \\ 0, & \text{otherwise} \end{cases} \quad (2)$$

which simply states whether y_n is present in the receiver buffer at the current time kT_s .

3. OPTIMAL FILTERING WITH A PROBABILISTIC APPROACH

In this section, a novel strategy brings realization to shorten the buffer size of the observer. The dropping strategy is as follows: *Given a threshold T_{hold} , at the current time kT_s , if the timestamp of an arrival packet $y(t)$ is older than $(k - T_{hold})T_s$, this packet will be discarded immediately.* Hence, over-delayed packets are treated as packet loss and the maximal delay has been limited by T_{hold} .

This novel strategy artificially splits packet arrival process into two parts. One is a T_{hold} -length active sequence stored in the buffer, and the other is an infinite disabled process. Using Kalman filter, the states at time $(k - T_{hold})T_s$ are estimated in one step iteration. Then, a multi-step iteration is finished in one sampling period based on all the available information in the buffer. It aims at computing the optimal mean square estimate $\hat{x}(n)$ based on $\{\gamma_0 y_0, \gamma_1 y_1, \dots, \gamma_n y_n\}$, where $n = k - T_{hold}$, whose correction step is given by:

$$\hat{x}(n|n) = \hat{x}(n|n-1) + \gamma_n K(n) [y(n) - C\hat{x}(n|n-1)] \quad (3)$$

$$\hat{x}(n|n-1) = \Phi \hat{x}(n-1|n-1) + \Gamma u(n-1) \quad (4)$$

$$K(n) = P(n|n-1) C^T [C P(n|n-1) C^T + R]^{-1} \quad (5)$$

$$P(n|n-1) = \Phi P(n-1|n-1) \Phi^T + Q \quad (6)$$

$$P(n|n) = [I - \gamma_n K(n) C] P(n|n-1) \quad (7)$$

Then, the optimal estimated state $\hat{x}(k|Y_k)$ is computed based on $Y_k = \{\gamma_{n+1} y_{n+1}, \dots, \gamma_k y_k\}$.

Since this packet dropping policy makes the observer applicable against network uncertainties, in fact, it transforms network uncertainties into the problem of packet arrival probability. Our aim is to find an optimal threshold T_{hold} so as to estimate states based on a T_{hold} -based arrival process and guarantee the estimator convergence.

4. CONVERGENCE ANALYSIS

To evaluate the convergence of the estimator using the proposed policy, we can establish the relation between packet arrival probability and convergence. Then, the choice of threshold will be demonstrated in the light of this relation.

4.1 Modelling of the relation between packet arrival probability and threshold

Following (2), we can present the probability mass function of this distribution and its mathematical expectation:

$$\Pr[\tau < T_{hold} | \tau = k - t] = \lambda \quad (8)$$

$$E[\gamma] = \lim_{k \rightarrow \infty} E[\gamma_k] = \gamma \Pr + (1 - \gamma)(1 - \Pr) = \lambda \quad (9)$$

where t is the timestamp of the arrival packet and the difference τ indicates its delay.

Herein, we may assume that it is simulated by a discrete Poisson arrival process. Its T_{hold} -based packet arrival probability can be defined by following (8) and Poisson arrival probability density function of delays as follows:

$$\lambda = 1 - \int_{T_{hold}}^{\infty} \frac{e^{-D} D^\tau}{\tau!} d\tau, \quad \tau \in \{0, T_s, 2T_s, \dots\} \quad (10)$$

where D is the distribution parameter given beforehand.

According to Poisson distribution with packet loss rate and choice of D , legends to simulate CAN is presented (Fig. 2a) as well as the threshold backward arrival process (Fig. 2b).

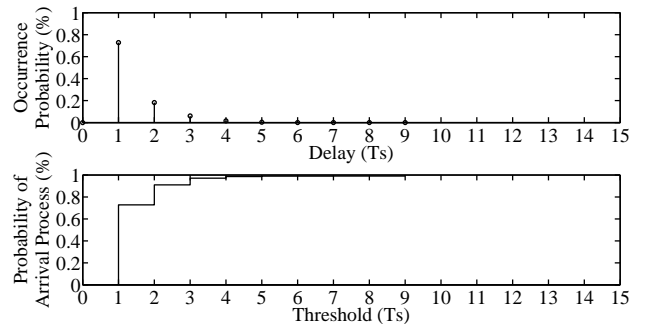


Fig. 2a. Poisson distribution with $D=1$ for CAN: original packet loss rate 1%.

Fig. 2b. Probability function of arrival process with respect to thresholds for CAN.

4.2 Modelling of relation between packet arrival probability and convergence

According to the modified Kalman filter in (3)~(7), we can rewrite its error estimation equation by the definition (11).

$$\begin{cases} x_{n+1} = \Phi x_n + \Gamma u_n + \omega_n \\ \hat{x}_{n+1} = \Phi \hat{x}_n + \Gamma u_n + \gamma_{n+1} K_{n+1} [y_{n+1} - C(\Phi \hat{x}_n + \Gamma u_n)] \\ e_{n+1} = x_{n+1} - \hat{x}_{n+1} \\ = \Phi e_n + \omega_n - \gamma_{n+1} K_{n+1} (C \Phi e_n + C \omega_n + v_{n+1}) \\ = (1 - \gamma_{n+1}) (\Phi e_n + \omega_n) + \gamma_{n+1} [(I - K_{n+1} C)(\Phi e_n + \omega_n) - K_{n+1} v_{n+1}] \end{cases} \quad (11)$$

Given the below transition model of state estimation error-covariance, we now study the corresponding modified Riccati equation subject to incomplete observations.

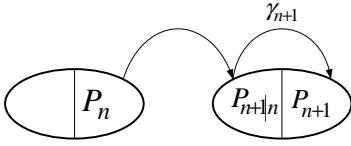


Fig. 3. Transition model of state error-covariance.

According to the definition of state error-covariance (12), we can transform its transition relation (Fig. 3) into a discrete mathematical expression.

$$P_n = E \left[e_n e_n^T \mid y_n, \gamma_n \right] \quad (12)$$

$$P_{n+1|n} = \Phi P_n \Phi^T + Q \quad (13)$$

Thanks to the orthogonality among $\{x_k\}$, $\{\omega_k\}$, and $\{v_k\}$, we have:

$$\begin{aligned} P_{n+1} &= E \left[1 - \gamma_{n+1} \right] P_{n+1|n} + E \left[\gamma_{n+1} \right] \left[(I - K_{n+1} C) P_{n+1|n} \right. \\ &\quad \left. \cdot (I - K_{n+1} C)^T + K_{n+1} R K_{n+1}^T \right] \\ &= E \left[1 - \gamma_{n+1} \right] (\Phi P_n \Phi^T + Q) + E \left[\gamma_{n+1} \right] \left[(I - K_{n+1} C) \right. \\ &\quad \left. \cdot (\Phi P_n \Phi^T + Q) (I - K_{n+1} C)^T + K_{n+1} R K_{n+1}^T \right] \end{aligned} \quad (14)$$

Let us denote the corresponding steady-state filter covariance by satisfying (14) and $\lim_{n \rightarrow \infty} P_n = p$.

$$\begin{aligned} \lim_{n \rightarrow \infty} P_{n+1} &= \lim_{n \rightarrow \infty} \left\{ E \left[1 - \gamma_{n+1} \right] (\Phi p \Phi^T + Q) \right\} + \lim_{n \rightarrow \infty} \left\{ E \left[\gamma_{n+1} \right] \right. \\ &\quad \left. \cdot \left[(I - K_{n+1} C) (\Phi p \Phi^T + Q) (I - K_{n+1} C)^T + K_{n+1} R K_{n+1}^T \right] \right\} \end{aligned}$$

$$\begin{aligned} \lim_{n \rightarrow \infty} P_{n+1} &= (1 - \lambda) (\Phi p \Phi^T + Q) + \lambda \left[(I - KC) (\Phi p \Phi^T + Q) \right. \\ &\quad \left. \cdot (I - KC)^T + KRK^T \right] \end{aligned} \quad (15)$$

Now, for the sake of simplicity, the function Ω is defined by:

$$\begin{aligned} \Omega(p, \lambda, K) &= (1 - \lambda) (\Phi p \Phi^T + Q) + \lambda \left[(I - KC) (\Phi p \Phi^T + Q) \right. \\ &\quad \left. \cdot (I - KC)^T + KRK^T \right] \end{aligned} \quad (16)$$

Lemma 1: Consider the operator (16) and the properties $p > 0$, $Q > 0$, $R > 0$, $0 \leq \lambda \leq 1$, there exists a critical value λ_c by solving (17) under the guarantee of convergence.

$$\left\{ \lambda > \lambda_c \mid \Omega(p, \lambda, \underline{K}) - p < 0 \right\} \quad (17)$$

■Proof: Since $p > 0$, $Q > 0$, $R > 0$, $0 \leq \lambda \leq 1$, the function $\Omega(p, \lambda, K)$ is quadratic and convex with respect to K . Its minimum can be found by solving $\partial \Omega(p, \lambda, K) / \partial K = 0$, which gives the same result as the gain of basic Kalman filter:

$$\underline{K} = (\Phi p \Phi^T + Q) C^T \left[C (\Phi p \Phi^T + Q) C^T + R \right]^{-1} \quad (18)$$

$$\begin{aligned} \Omega(p, \lambda, \underline{K}) &= \Phi p \Phi^T + Q - \lambda (\Phi p \Phi^T + Q) C^T \\ &\quad \cdot \left[C (\Phi p \Phi^T + Q) C^T + R \right]^{-1} C (\Phi p \Phi^T + Q) \end{aligned} \quad (19)$$

Note that for any K_0 ,

$$\Omega(p, \lambda, \underline{K}) = \min_K \Omega(p, \lambda, K) \leq \Omega(p, \lambda, K_0) \quad (20)$$

Obviously, if $\lambda_1 \leq \lambda_2$, $\Omega(p, \lambda_1, \underline{K}) \geq \Omega(p, \lambda_2, \underline{K})$. Relatively, if we can find a critical value λ_c such that $\Omega(p, \lambda, \underline{K}) \leq \Omega(p, \lambda_c, \underline{K}) \leq p$ for $\lambda \geq \lambda_c$, the filter is convergent. That means:

$$\forall 1 \geq \lambda \geq \lambda_c : \Omega(p, \lambda, \underline{K}) \leq \Omega(p, \lambda_c, \underline{K}) \leq p \quad (21)$$

$$\forall 0 \leq \lambda < \lambda_c : \Omega(p, \lambda, \underline{K}) > \Omega(p, \lambda_c, \underline{K}) > p \quad (22)$$

Proof done. ■

Lemma 2(The Schur complement lemma (Zhang, 2005, and Haynsworth, 1968)): Given constant matrices s_1, s_2, s_3 of appropriate dimensions where s_1 and s_2 are symmetric, then $s_2 > 0$ and $s_1 + s_3^T s_2^{-1} s_3 < 0$ if and only if

$$\begin{bmatrix} s_1 & s_3^T \\ s_3 & -s_2 \end{bmatrix} < 0, \text{ or equivalently } \begin{bmatrix} -s_2 & s_3 \\ s_3^T & s_1 \end{bmatrix} < 0.$$

Lemma 3: λ_0 is a feasible value for λ if and only if $\Omega(p, \lambda_0, \underline{K}) < p$ is solvable. The condition is equivalent to: there exists a minimum for X such that:

$$\begin{bmatrix} X & -\sqrt{1-\lambda_0} X \Phi & \sqrt{\lambda_0} (Y C \Phi - X \Phi) & -\sqrt{1-\lambda_0} X & \sqrt{\lambda_0} (Y C - X) & -\sqrt{\lambda_0} Y \\ -\sqrt{1-\lambda_0} (X \Phi)^T & X & 0 & 0 & 0 & 0 \\ \sqrt{\lambda_0} (Y C \Phi - X \Phi)^T & 0 & X & 0 & 0 & 0 \\ -\sqrt{1-\lambda_0} X^T & 0 & 0 & -Q^{-1} & 0 & 0 \\ \sqrt{\lambda_0} (Y C - X)^T & 0 & 0 & 0 & -Q^{-1} & 0 \\ -\sqrt{\lambda_0} Y^T & 0 & 0 & 0 & 0 & -R^{-1} \end{bmatrix} < 0, X < 0 \quad (23)$$

where $p=-X^{-1}$, $K=X^{-1}Y$.

■Proof: Lemma 1 gives the convergence condition of the filter, $\Omega(p,\lambda,K)<p$. Because $\Omega(p,\lambda,K)$ is a positive-convex function, $\partial\Omega(p,\lambda,K)/\partial K=0$ equivalently means to minimize the objective under LMI constraints. Suppose λ exists:

$$(1-\lambda)(\Phi p\Phi^T+Q)-p+\lambda\left[(I-KC)(\Phi p\Phi^T+Q)(I-KC)^T+KRK^T\right]<0 \quad (24)$$

According to lemma 2, the matrix inequality is rewritten as:

$$\begin{bmatrix} -p & -\sqrt{1-\lambda}\Phi & \sqrt{\lambda}(\Phi-KC\Phi) & \sqrt{1-\lambda} & \sqrt{\lambda}(I-KC) & \sqrt{\lambda}K \\ \sqrt{1-\lambda}\Phi^T & -p^{-1} & 0 & 0 & 0 & 0 \\ \sqrt{\lambda}(\Phi-KC\Phi)^T & 0 & -p^{-1} & 0 & 0 & 0 \\ \sqrt{1-\lambda} & 0 & 0 & -Q^{-1} & 0 & 0 \\ \sqrt{\lambda}(I-KC)^T & 0 & 0 & 0 & -Q^{-1} & 0 \\ \sqrt{\lambda}K^T & 0 & 0 & 0 & 0 & -R^{-1} \end{bmatrix} < 0 \quad (25)$$

Pre- and post-multiplying by the block-diagonal matrix $diag\{p^{-1},I,I,I,I\}$ and denoting $X=-p^{-1}$, $Y=-p^{-1}K$, one obtains LMI (23). Hence, if we can find its minimum for X under (23) constraints with λ_0 , λ_0 is a feasible solution for λ to guarantee the filter stable. Proof done.■

Substituting (19) into (17), we can obtain Riccati inequality:

$$\Phi p\Phi^T+Q-p-\lambda(\Phi p\Phi^T+Q)C^T\left[C(\Phi p\Phi^T+Q)\cdot C^T+R\right]^{-1}C(\Phi p\Phi^T+Q)<0 \quad (26)$$

Using the matrix inversion lemma,

$$\begin{aligned} \Rightarrow(1-\lambda)(\Phi p\Phi^T+Q)-p+\lambda\left[(\Phi p\Phi^T+Q)^{-1}+C^TR^{-1}C\right]^{-1}<0 \\ \Rightarrow(1-\lambda)\Phi p\Phi^T-p<(\lambda-1)Q-\lambda\left[(\Phi p\Phi^T+Q)^{-1}+C^TR^{-1}C\right]^{-1}<0 \end{aligned}$$

Theorem 1: If a critical value for λ is found so as to satisfy

$$(1-\lambda)\Phi p\Phi^T-p<0 \quad (27)$$

λ_{sys} can be called its ideal infimum as

$$\lambda_{\text{sys}}=1-\frac{1}{\max_i^2|\sigma_i(\Phi)|} \quad (28)$$

■Proof: Similar to a part of Sinopoli (2004) and Schenato (2008), suppose $(1-\lambda)\Phi p\Phi^T-p<0$ for all λ , all the eigenvalues of $\sqrt{1-\lambda}\Phi$ should be located in the unit cycle.

$$\begin{aligned} \sqrt{1-\lambda}\max_i|\sigma_i(\Phi)|<1 \\ \Rightarrow\lambda>1-\frac{1}{\max_i^2|\sigma_i(\Phi)|}=\lambda_{\text{sys}} \end{aligned}$$

Proof done.■

Theorem 2: If Φ is invertible, then there exists a value λ_c following the below algorithm for $\lambda>\lambda_c$. The mean state covariance of the filter is bounded for all initial conditions.

$$\lambda_c=\arg\inf_{\lambda}\left[\Omega(p,\lambda,K)-p\right] \quad (29)$$

subject to lemma 3 is satisfied such that $\Omega(p,\lambda,K)-p<0$ with initialization $p>0$, $\lambda_{\text{sys}}<\lambda\leq 1$.

■Proof: According to lemma 1, (17) is a quasi-convex optimization problem. The stability condition for any $\lambda_c<\lambda\leq 1$ is strictly true if and only if the minimum of the arrival probability of the observation exists so as to satisfy:

$$\Omega(p,\lambda_c,K)-p<0 \quad (30)$$

Proof done.■

Table 1. Program realization of theorem 2

Algorithm 1 Approximation of a critical probability λ_c

Inputs: initial $Q>0$, $R>0$ with Gaussian distribution, $\lambda=1$, and approximate accuracy i.e. $\eta=1e-4$.

Outputs: A phase transition between state estimate covariance and packet arrival probability in scalar case.

for $\lambda=\lambda-\eta$ **do** Solve $[p,K,\text{copt}]=LMI_s(23)$.

if `isempty(copt)` **then** Verify Kalman gain (18);

else break; get λ_c ; **end if**

end for

5. SPECIAL CASES AND EXAMPLES

Consider a discrete linear time-invariant plant like (1) with sampling period $T_s=0.01s$, this system parameters are tuned identically to the example of Sinopoli (2004):

$$\Phi=\begin{bmatrix} 1.25 & 1 & 0 \\ 0 & 0.9 & 7 \\ 0 & 0 & 0.6 \end{bmatrix}, \Gamma=\begin{bmatrix} 2.2 \\ 1 \\ 0.1 \end{bmatrix}, C=[1 \ 0 \ 2];$$

$$\omega_{kT}\sim N(0,Q) \text{ with variance } Q=20\cdot I_3;$$

$$v_{kT}\sim N(0,R) \text{ with variance } R=2.5.$$

where I_m denotes the identity matrix of dimension m .

For this system, we have $\lambda_{\text{sys}}=0.36$ from theorem 1 and get a critical bound $\lambda_c=0.3619$ from theorem 2 by means of Matlab LMI toolbox. Both transitions clearly appear (Fig. 4), which shows the trace of the minimum of the steady state error-covariance tends to infinity as λ approaches λ_c for the optimal estimator with Kalman gain.

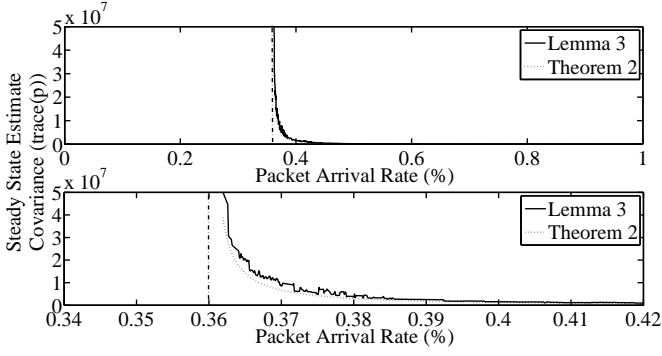


Fig. 4a. Phase transition between packet arrival probability and the trace of the steady state error covariance;
 Fig. 4b. Phase transition of Figure (4a) between [0.34, 0.42].

The solid line gives the result from lemma 3. The dash-dotted line shows the system stability subject to λ_{sys} . Relatively, the dashed line gives the result from theorem 2. It succeeds in verifying two points. Firstly, the problem of Kalman gain computation is solved by LMI minimization. Secondly, the trend curve of the performance of the designed linear optimal estimator is smooth and monotonically decreasing with the rise of packet arrival probability.

Further, this kind of discrete observer is established. The plant is operated in such working conditions showing in Figure (5). Taking into account three packet arrival rates (Tab. 2), output performances of this observer are shown in Figure (6) as well as state estimation error-covariances in Table (2).

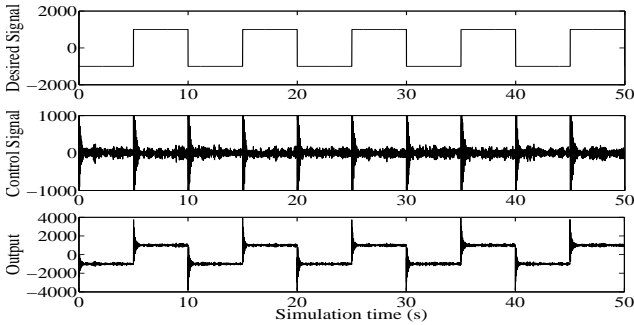


Fig. 5. Input and output of the plant.

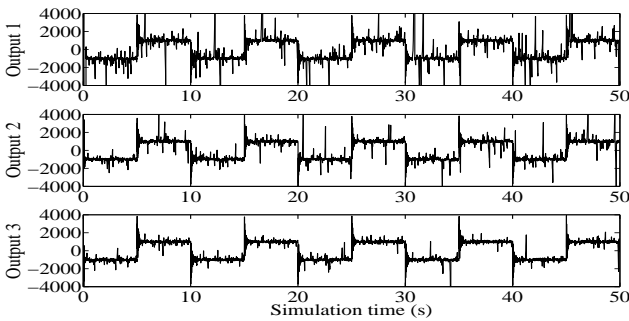


Fig. 6. Estimated outputs comparison in terms of packet arrival probability.

When the arrival rate is 0.3 (output 1), the observer oscillates severely and leads to the great mass of performance loss. When the arrival rate is 0.364 (output 2) closely beyond the critical bound or 0.44 (output 3), the observer performs more and more smoothly.

Meanwhile, sequences of optimal filtering gains with respect to packet arrival rate are shown in Figure (7). It succeeds in verifying lemma 3 in simulation. It is effective to transform Kalman optimal approach into a LMI-based extreme problem.

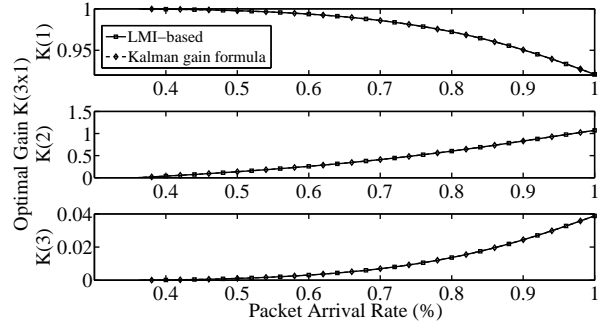


Fig. 7. Comparison of optimal filtering gain from between LMIs and Kalman gain formula for the initial model noise.

In addition, an observer (Chabir *et al.*, 2008) based on the use of an adaptive Kalman filter is introduced for comparison. Covariance of time uncertainties are certain as $Q=20I_3$ and $R=0.05$. In the same working conditions (Fig. 5), Figure 8 illustrates estimated outputs of the adaptive Kalman filter for three corresponding packet arrival rates (Tab. 2). It figures out this observer has the lower capability against delay uncertainties, even an unstable performance.

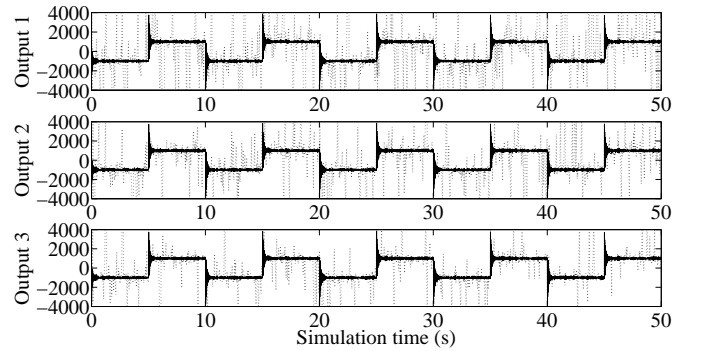


Figure 8. Comparison with the adaptive Kalman observer. (dashed line: kalman filter; solid line: system output)

Table2. State estimation error-covariance

	Es. Output1	Es. Output2	Es. Output3
λ	0.3	0.364	0.44
$trace(p)$	1.4155e6	3.5347e5	1.0081e5
	4.0276e9	3.4155e7	1.7773e7

Surely, results of Table (2) generally follow the phase transition. Results from simulation and error-covariance statistics show that the observer with a dropping over-delayed packets approach exhibits better performance than that with the adaptive Kalman filter. For a class of network like CAN (Fig. 2), we can find it is logic to meet the lowest requirement of packet arrival probability if T_{hold} is chosen as 1. The task of dropping strategy is always to find the suitable threshold for satisfying the critical bound of packet arrival probability.

6. CONCLUSION

As a result, a new dropping over-delayed packets approach has been presented. It is a specialized design for a robust linear estimator applying to discrete-time networked control systems with incomplete observations. According to statistics and probability theory, the contribution succeeds in illustrating the relation among threshold of buffer, arrival rate of measurement and stability.

Therefore, there is no need to care whether network-induced delay is short (shorter than one sampling time) or long. When the threshold is set to one, it is equivalent to the case of short delay and this result is also sound and solid. Through an adjustable buffering technique, main network-induced problems have been successfully unified as packet loss. Given network characteristics like CAN bus or real-time Ethernet, it is suitable for all the communication protocols.

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