Generation of Optical Vortex Beams by Nonlinear Wave Mixing

Alon Bahabad and Ady Arie

School of Electrical Engineering, Wolfson Faculty of Engineering, Tel Aviv University
Tel Aviv 69978, Israel

ady@eng.tau.ac.il

Abstract: It is shown that optical vortex beams can be generated from a non-vortex fundamental beam by an optical frequency conversion process taking place within a twisted nonlinear photonic crystal. This is done without any first-order (linear) refractive optics. Through such a proposed structure, all-optical switching of vortices with different helicities is made possible, as well as the simultaneous application of counter-rotating vortex beams of different frequencies.

OCIS codes: (190.0190) Nonlinear optics; (190.2620) Harmonic generation and mixing optical vortex beams; (190.4223) Nonlinear wave mixing.

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Optical vortices are light waves possessing a phase singularity[1]. The Poynting vector of such beams contains an azimuthal component, causing energy flow around the singularity[2] and a ring-shaped intensity profile. The electric field of a vortex beam can be written in cylindrical coordinates as $\mathbf{E}(r,z,\theta) = \mathbf{u}(r,z)e^{il\theta}e^{-ikz}$ where $\mathbf{k} = k\hat{z}$ is the beam's wave vector. l is called the topological charge, indicating, for integer values, phase fronts which are l intertwined helical surfaces and an $l\hbar$ orbital angular momentum carried by each of the beam's photons. For non-integer values the resulting beam contains a chain of alternating charge vortices[3]. Varied applications are found for vortex beams, for example, optical tweezers[4] and actuators for micro-electro-mechanical systems[5], multi-dimensional quantum entanglement[6] and extra-solar planet detection[7]. Although vortex beams occur naturally as higher order modes of laser cavities and optical fibers, beam shaping techniques are needed to control their properties. The current solutions are based on linear optics by employing diffractive optical elements[2], spatial light modulators[8] and spiral phase plates[9]. We show that vortex beams can also be created by using nonlinear optics, specifically by frequency conversion processes in which one or two beams with different frequencies give rise to a nonlinear material polarization term. We would like to stress that extensive work went into the research of vortex beams interacting within a nonlinear medium[10, 11] where a vortex beam could be generated by another (already-present) vortex beam of a different frequency. However, we show that using a special-tailored nonlinear medium a vortex beam can be generated from a fundamental beam that contains no singularity.

Let us consider the prototype process for three wave mixing - second harmonic generation within a three dimensional nonlinear material. For quasi-plane waves under the slowly varying envelope approximation, were all the interacting beams are collinear along z, the evolution of the second-harmonic field amplitude is given in M.K.S units by:

$$k^{2\omega}\frac{\partial E^{2\omega}(\mathbf{r})}{\partial z} = \frac{-2i\omega^2}{c^2}(E^{\omega}(\mathbf{r}))^2 d_{eff,ij}(\mathbf{r})e^{-i\Delta kz}, \tag{1}$$

where $E^{2\omega}$ and E^{ω} are the second harmonic and fundamental (pump) beam amplitudes respectively, $\mathbf{k}^{2\omega}$ is the second harmonic wave vector, ω is the pump beam angular frequency, c is the speed of light, $d_{eff,ij}$ is the space-dependent coupling component of the nonlinear susceptibility tensor and $\Delta\mathbf{k} = \mathbf{k}^{2\omega} - 2\mathbf{k}^{\omega}$ is the phase mismatch vector of the interacting waves. Collinearity implies: $\mathbf{k}^{\omega} = k^{\omega}\hat{\mathbf{z}}$, $\mathbf{k}^{2\omega} = k^{2\omega}\hat{\mathbf{z}}$, and $\Delta\mathbf{k} = \Delta k\hat{\mathbf{z}}$.

Assuming an undepleted pump beam with a constant cross-section: $E^{\omega}(\mathbf{r}) = E^{\omega}(z=0) = E^{\omega}$ (as for example given by a plane wave or approximated by a Gaussian beam with a wide enough waist) and a binary modulation for the nonlinear coupling coefficient: $d_{eff,ij}(\mathbf{r}) = d_{ij}g(\mathbf{r})$ with $g(\mathbf{r}) = \pm 1$, we can integrate the last equation over an interaction length L to get:

$$E^{2\omega}(z=L) = \kappa\int_0^L g(\mathbf{r})e^{-i\Delta kz}dz, \tag{2}$$

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where $\kappa = -i\omega(E^{\omega})^2 d_{ij}/cn_{2\omega}L$. We further require that the binary modulation of the nonlinear coupling coefficient is of the form $g(\mathbf{r}) = g(z + f(x,y))$, in which case the transverse function $f(x,y)$ acts as a translation factor under the one-dimensional Fourier transform $\mathscr{F}\{g(z)\} = G(\Delta k)$. If a phase matching condition is satisfied, resulting in a significant build-up of the second harmonic amplitude, $G(\Delta k)$ behaves as a Dirac delta function for an infinite integration range. In this case:

$$E^{2\omega}(z = L) = \kappa G_{\Delta k} e^{if(x,y)\Delta k}, \tag{3}$$

where $G_{\Delta k}$ is just the Fourier coefficient at frequency Δk in the Fourier series expansion of the function $g(z)$. For $f(x,y) = l\theta/\Delta k$ the generated second harmonic is a vortex beam with a topological charge of value l. The nonlinear coefficient modulation yielding the greatest efficiency is the 50% duty cycle rectangular wave with period equal to $2\pi m/\Delta k$ (m being the phase matching order) [12] which is modified in our case to be:

$$g(\mathbf{r}) = \mathrm{sign}\{\cos[\Delta k/m(z + l\theta/\Delta k)]\}, \tag{4}$$

where $\mathrm{sign}(x) = x/|x|$ for nonzero values (0 otherwise).

The axial component of this structure function is responsible to the breaking of an otherwise infinitesimal translational symmetry into a finite translational symmetry. The consequence is that momentum need to be conserved only to some discrete quantities, known as quasi-momenta. This is essentially regular quasi-phase-matching[13, 12, 14]. Similarly, the transverse component of the structure function breaks the infinitesimal rotational symmetry, so angular momentum needs to be conserved up to some quasi-angular momentum related to the structure. Put otherwise, the outcome of a nonlinear wave mixing in such a structured material can result in a radiation which possesses orbital angular momentum which is different than the sum of angular momenta of the other beams (unlike with a rotationally invariant setup[15]). This allows the generation of a vortex beam from a non-vortex beam. All of the above symmetry considerations only apply when all the beams are collinear along z. Note that quasi-angular momentum is naturally also used in linear optics as with the optical vortex lens[16, 9] and using a twisted optical fiber[17]. Also, planar transverse modulations of quasi-phase-matching structures were already used for different beam manipulations[18, 19] but not for generating a vortex beam from a non-vortex beam.

To verify the above result we used numerical simulations employing split step Fourier method[20], where a non-depleted Gaussian pump beam was used as an input to a nonlinear photonic crystal whose nonlinear coefficient is modulated according to Eq. 4. The results are depicted in Fig. 1. In each panel a specific spatial modulation is represented by giving the three dimensional shape of the positively modulated nonlinear coefficient, where the background is of the opposite polarization. The relevant scale along the propagation direction is indicated by the coherence length $l_c = \pi/\Delta k$. The simulation was carried over an interaction length of $600 \cdot l_c$ for a Gaussian pump beam with a Rayleigh range of about $z_0 \simeq 2800 \cdot l_c$. The linear (first-order) dielectric coefficient is assumed to be spatially independent. Along with the nonlinear modulation the resulting second-harmonic normalized field amplitude and field phase are presented. Panels $(a) - (c)$ exhibit creation of vortex beams with integer topological charge of $l = 1, 2$ and $l = 5$ respectively. In panel (d) a vortex beam with a fractional topological charge of $l = 1.5$ is created. For all of these cases the phase matching order is the fundamental $m = 1$. It is interesting to note that unlike with phase plates[21], generation of vortex beams with larger topological charge is actually easier in our case. For example when the topological charge is $l = 3$ but the phase matching order is also $m = 3$ we get a rescaling of the basic $l = 1$ and $m = 1$ structure as can be seen in panel (e). The penalty is a reduced efficiency in the generated field with a factor of $(1/m)^2$[12]. We also note that to employ this rescaling to even orders we should use a pattern with a different duty cycle than the 50% example (in which only the odd orders are needed to

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expand the rectangular wave). If the beam waist is not wide enough (that is - the interaction length is larger than the Rayleigh range) the phase pattern is modified to include a quadratic radial contribution from the Gaussian pump $\theta \rightarrow \theta + \alpha r^2$ (where α is some constant) as can be seen in panel (f) where a Rayleigh range of $z_0 \simeq 20 \cdot l_c$ was used (while the interaction length was still $600 \cdot l_c$). Note that a rigorous treatment that takes into account the nature of focused beams results in a different evolution equation than the one we started with[22].

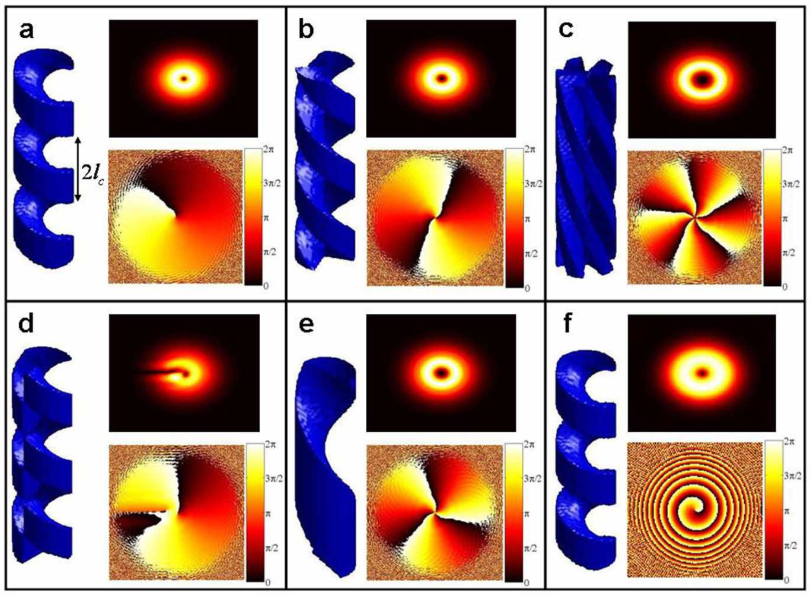

Fig. 1. Simulations of positively modulated nonlinear coefficient (left in each panel, while the background represents negatively modulated nonlinear coefficient) and the generated second harmonic amplitude (top right) and phase (bottom right) for a Gaussian pump beam. The linear coefficient (index of refraction) is assumed to be homogeneous. l_c is the coherence length along the fields propagation direction. m is the phase-matching order. l is the topological charge. (a) $m = 1$, $l = 1$ (b) $m = 1$, $l = 2$ (c) $m = 1$, $l = 5$ (d) $m = 1$, $l = 1.5$ (e) $m = 3$, $l = 3$ (f) $m = 1$, $l = 1$ with a narrow beam waist.

Using three-wave mixing to create vortex beams suggests all-optical switching schemes involving some parameter of the vortex. For example, let us consider an interaction involving three different frequencies such that $\omega_3 = \omega_1 + \omega_2$. Given a device to phase-match a generation of a vortex beam at the sum frequency of ω_3 where the input beams are at frequencies of ω_1 and ω_2 it would also phase-match a generation of the difference frequency ω_1 where the input beams are at frequencies of ω_3 and ω_2. However, the resulting helicity of the vortex beam would be different for the two cases. This is due to the different dependencies of the field amplitudes evolution on the phase mismatch value $\Delta\mathbf{k} = \mathbf{k}_1 + \mathbf{k}_2 - \mathbf{k}_3$ for the two cases. Whereas for the sum-frequency generation process the phase mismatch term appears as $e^{-i\Delta\mathbf{k}\cdot\mathbf{r}}$ (as in Eq. 1) for the difference-frequency generation process it appears as $e^{+i\Delta\mathbf{k}\cdot\mathbf{r}}$. Integrating the equations and using the same spatial modulation of the nonlinear coefficient ($g(\mathbf{r}) = g(z + f(x,y))$) would yield a phase of $e^{if(x,y)\Delta k}$ for the sum-frequency process but $e^{-if(x,y)\Delta k}$ for the difference frequency process. This way optical helicity switching is achieved (the vortex frequency also changes unless we use two different processes with the same output frequency and the same phase mismatch value). Another option is given by concatenating two nonlinear crystals - each with a spatial modulation for phase matching a different process, for example - two second-harmonic generation processes of $\omega_1 + \omega_1 \rightarrow 2\omega_1$ and $\omega_2 + \omega_2 \rightarrow 2\omega_2$. If the difference between the two phase-mismatch values is much larger than π/L (where L is shortest crystal length) then using as input, say ω_1, would generate a vortex beam at frequency $2\omega_1$ through one of the crystals while the other one would have only negligible effect. Thus this device is able to optically switch between vortex beams of different frequencies. Such a device, when pumped simultaneously with ω_1 and ω_2 would yield two vortex beams (at different frequencies) with the same helicity or with an opposing helicity depending on the helicities of the poling patterns.

A possible solution for constructing such devices is by electric-field poling of ferroelectric materials into thin $\chi^{(2)}$-modulated planar plates and stacking them together. The stacking could be made in the material polarization direction, as depicted in Fig. 2.a (an X-sampled structure). This way we may assume that the pump field is linearly polarized in the same direction as the material polarization, utilizing the largest available nonlinear tensor component d_{33}, characteristic of ferroelectric materials such as $LiNbO_3$ or $KTiOPO_4$. Another option would be to stack thin ferroelectric plates at the beams propagation direction (a Z-sampled structure). For a topological charge which is equal to the phase matching order $m = l$, all the plates would have the same poling pattern - where half of the plain is positively poled and they would be stacked together with a relative angular shift as depicted in Fig. 2.b. Here, however, the material and field polarizations would be perpendicular, utilizing the tensor component d_{22} (which exists, for example, in $LiNbO_3$). Both options would be challenging to implement considering that the typical coherence length in the optical regime is about $10\mu m$ and a typical beam waist is about $50\mu m$, meaning that each plate should have a thickness of just a few microns. However, a possible strategy could be bonding at each construction stage a relatively thick plate and polishing it down to an adequate thickness, as was recently demonstrated for a thickness of about $6\mu m$[23].

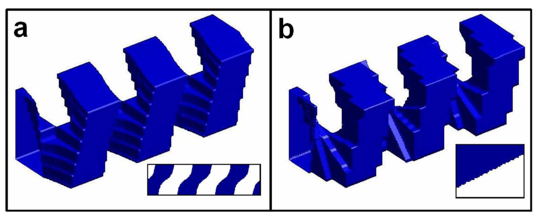

Fig. 2. Planar fabrication sampling for a vortex-generating ferroelectric nonlinear photonic crystal. The insets show a typical plate. (a) Stacking plates at the material polarization direction. (b) Stacking plates at the beams propagation direction.

To asses the influence of such fabrication sampling over an ideal smooth structure we can use an estimation factor (η) such as the cross correlation between the normalized (intensity wise) field to a normalized field of an ideal vortex beam (generated through a smooth structure). Numerical simulations show that decreasing sampling resolution, for an X-sampled structure, reduces η almost linearly from a perfect value of 1 to 0.8 for a sampling period of about 40% of the beam radius (for a Rayleigh range of about $z_0 \simeq 2800 \cdot l_c$). The decreasing quality for a Z-sampled structure exhibit high η value around 0.98 for sampling periods up to a coherence length. However this quality behaves much more erratically as the sampling period is varied, while for sampling periods which are an integer division of twice the coherence length the generated beam is similar to ones generated by a step vortex lens[16]. To see this a closed analytical form can be derived for the generated field due to a sampled structure by using a straight-forward sampling theorem. For the Z-sampled structure the nonlinear coupling coefficient is of the form:

$$g_s(\mathbf{r}) = \left(g(\mathbf{r}) \cdot \sum \delta(z - n\Delta z)\right) \otimes \mathrm{rect}\left(z/\Delta z\right), \tag{5}$$

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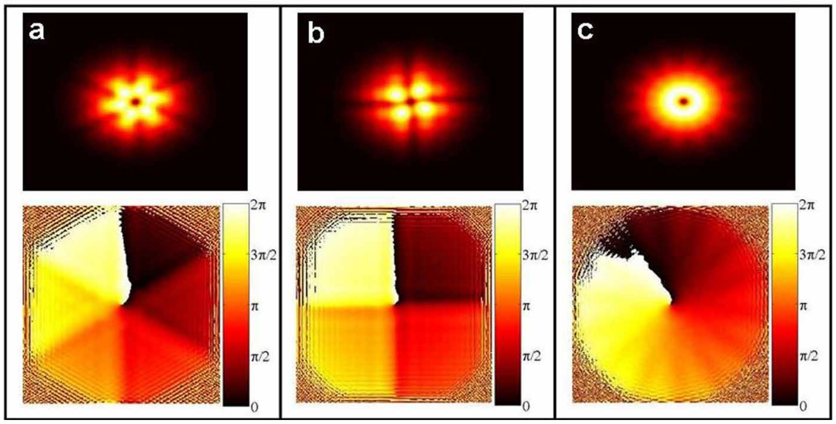

Fig. 3. Simulations of second harmonic amplitude (top) and phase (bottom) for a Gaussian pump beam within a Z-sampled structure. The phase matching order and topological charge are both 1. The sampling period is $\Delta z = (2\pi/\Delta k)/q$. (a) $q = 3$ (b) $q = 4$ (c) q=14.

where $g(\mathbf{r})$ is given by Eq. 4, Δz is the sampling period in the z direction, $\otimes$ represents convolution and $\mathrm{rect}(z) = \{1, |z| < 1/2; 0, else\}$. To simplify the analysis we examine an interaction over an idealized infinite length $L \to \infty$ to generate a vortex beam with a topological charge of $l = 1$ and phase matching order of $m = 1$. We are interested at the outcome for sampling periods of the type $\Delta z = \frac{2l_c}{q} = \frac{2\pi}{q\Delta k}$ where q is an integer. In this case, using Eq. 2 (where $g_s(\mathbf{r})$ replaces $g(\mathbf{r})$), Eq. 4 and Eq. 5, it can be shown that:

$$E^{2\omega} \propto \sum_{n=-\infty}^{\infty} \mathrm{sinc}\left(\frac{1-nq}{2}\right) e^{i\theta(1-nq)}, \tag{6}$$

where $\mathrm{sinc}(x) = \sin(\pi x)/(\pi x)$. Such an interference sequence accounts for a q-steps phase pattern for an even q and for a $2q$-steps phase pattern for an odd q value. This in turn leads to a q-fold or a $2q$-fold rotational symmetry for the field's amplitude. For large values of q the resulting field is very close to a vortex beam generated through a smooth structure, while for small values of q the small rotational symmetries lead to a shaped beam. These can be seen in Fig. 3 exhibiting the results of a simulation carried within a Z-sampled structure for different values of q. The simulation was carried over an interaction length of $600 \cdot l_c$ for a Gaussian pump beam with a Rayleigh range of about $z_0 \simeq 2800 \cdot l_c$.

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