Spatially continuous six degree of freedom position and orientation sensor

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ABSTRACT

This paper describes SHAPE TAPE™, a thin array of fiber optic curvature sensors laminated on a ribbon substrate, arranged to sense bend and twist. The resulting signals are used to build a three dimensional computer model containing six degree of freedom position and orientation information for any location along the ribbon. The tape can be used to derive dynamic or static shape information from objects to which it is attached or scanned over. This is particularly useful where attachment is only partial, since shape tape "knows where it is" relative to a starting location. Measurements can be performed where cameras cannot see, without the use of magnetic fields. Applications include simulation, film animation, computer aided design, robotics, biomechanics, and crash testing.

Keywords: Shape, curvature, fiber, bend, sensor.

1. INTRODUCTION

In previous publications, fiber optic curvature sensors have been described (1-6). Current forms of these sensors use plastic fibers formed into tight loops that have been processed to lose light along short, narrow "loss zones" on one side of the fiber. As the loop is bent out of plane, it either loses more light, or less light, depending on the direction of the bend; according to the subtention of modes by the loss zones. This modulates the intensity of the light passing through the loop, so that throughput varies linearly with curvature. Additional light is lost at the apex of the loop. This is not detrimental to the modulation, since modes that would simply add to the throughput as an offset are preferentially removed. The result is a fiber optic curvature sensor that is linear, bipolar, and requires minimal illumination. Sensors with a large linear range (4.5 V-cm/rad) have a noise floor of 70µVrms/Hz$^{-1/2}$, at an LED current of less than 1 mA. Much larger signal to noise ratios are easily obtained by raising the LED current.

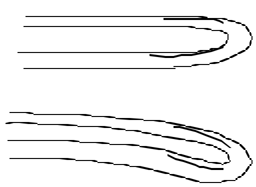

Figure 1 A looped fiber optic sensor with loss zones on one side.

A typical looped curvature sensor is shown in Figure 1, in unbent and bent states.

Curvature sensors have been used to make a variety of single degree of freedom measurements including displacement, force, flow, acceleration, and magnetic field strength. In this paper, we describe arrays of these sensors used to measure six degree of freedom (DOF) position and orientation (POSOR), continuously along a flexible substrate (flexure).

2. 3D SHAPE SENSING

The measurement of surface strain with gauges is well known. However, strain gauges do not lend themselves well to measurements on highly compliant flexures, because of problems with delamination, and stiffness of the leads. They are rormally used to infer material distension rather than shape, or applied forces rather than deflections, although two dimensional shape sensing where large forces are available to move the substrate has been reported using strain gauges on a thick metal band (7).

The availability of linear curvature sensors made with highly flexible fibers raises the possibility of measuring three dimensional (3D) shape of a flexure, and therefore the shape of objects to which the flexure conforms. Measuring curvature is conversely related to measuring strain. Curvature sensors require no transfer of strain to the gauge. Rather, they are designed to be uncoupled from the substrate. If they are near or in the neutral axis (where there is no strain), they may simply be held loosely to the surface by tapes or flexible adhesives, even though the curvatures may be very large.

It can be shown (8) that any n DOF inextensible surface may be fitted with curvature sensors sufficient in number to sample the n DOFs , and calibrated to yield its shape within a range that does not exceed the spatial resolution of the sensor population. In fact, if distension is also measured or known, ANY surface may be fitted in this way. Since the sensors must sample all the DOFs, they must in general be placed at a variety of orientations on the surface. Calibration consists of placing the surface into m known poses so that all the sensors (if they are linear) experience at least two different values of each DOF. If the sensors are not linear, more poses along each DOF are required. For linear sensors, the calibration procedure involves the solution of n equations in n unknowns, to find n linear equations relating output of each sensor to change in a curvature DOF. It can be further shown that the calibration method does not require knowledge of the exact sensor orientations or the transfer function of each sensor (its change in output for a given change in curvature).

3D shape sensing is done most conveniently on substrates that require a minimal number of sensors. Useful substrates are rods and tapes (thin ribbons). These particular flexures can conform to a wide variety of 3D shapes, yet have limited numbers of DOFs. Rods can bend in two DOFs and twist in a third. Tapes have only 2 DOFs, if we discount cupping along the long axis (like a pre-formed steel measuring tape), yet can conform to a wide variety of shapes. Tapes are also very convenient for mounting fiber optic curvature sensors.

3. 3D SHAPE TAPE DESIGN AND MATHEMATICS

3.1 Sensors on tape

An example of sensor population on a tape illustrates the general method of 3D shape sensing:

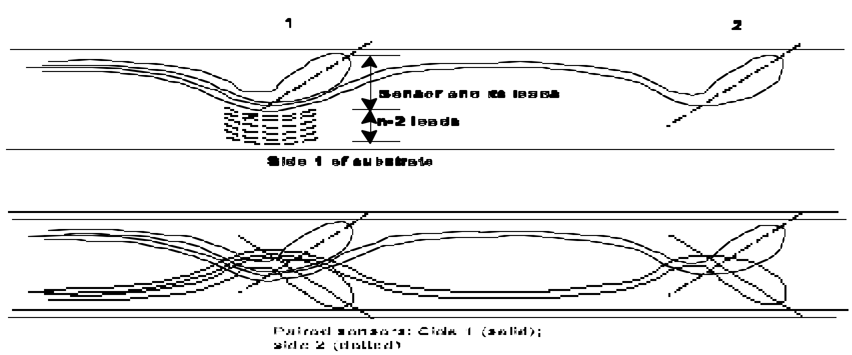

Figure 2 Typical sensor loop arrangements for SHAPE TAPE™.

Figure 2 shows sensor loops on a tape.They are arranged at approximately 45 degrees to the long axis, so that each sensor experiences a bend and twist component. In this example, the loops are in crossed pairs on opposite sides of a thin spring steel tape, at known spacings along the axis (along the arc length s). However, the method does not require paired, or regular spacing (but does require that distance along s be known). Each sensor has a single output proportional to its curvature, but the curvature has bend and twist components. We could calibrate by applying mixed bends and twists (e.g. by using helical poses). However, it is easiest to think in terms of pure bends and twists. Our objective is to obtain calibration coefficients so that if we know all the sensor signals, we can calculate bends and twists for each sensor (for each sensor pair in this example).

3.2 Calibration

If we collect data for three poses:

? flat
? circular hoop
? uniform twist from end to end

there will be sufficient information to calibrate the sensorized flexure. It turns out that the calibrated signals may be

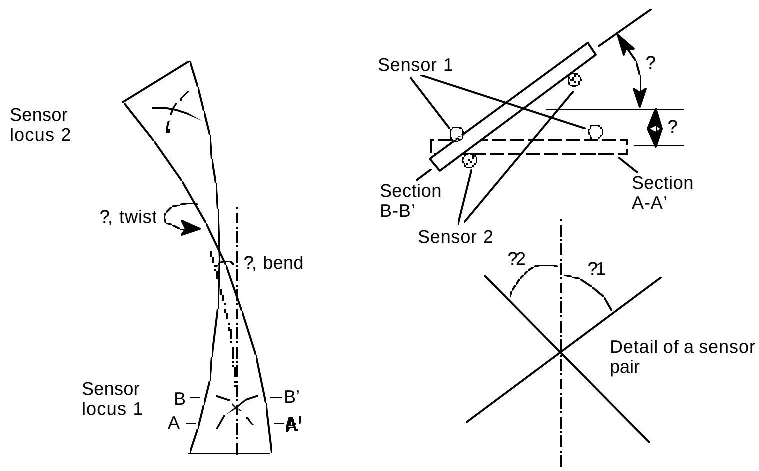

Figure 3 Crossed sensor pairs for sensing bend and twist on a tape.

used in sum and difference relation to yield calibrated twist and bend, respectively, if the sensor loss zones face outward on each side of the flexure. A typical configuration for a tape flexure fitted with sensors is shown in Figure 3. The tape is fitted with sensor pairs at loci 1 and 2. The tape is free to bend in one DOF and twist in another, so the sensor pairs must each resolve two DOFs. The sensors are attached at unknown angles ? and ? (not shown) relative to the long axis, and their transfer functions are unknown. Linear equations relate bend and twist of the tape to calibration coefficients a_{ij} and sensor outputs V_j:

$V_1 = a_{11}?c + a_{21}?_c$
$V_2 = a_{12}?c + a_{22}?_c$

where $?_c$ and $?_c$ are pure bends and twists applied during calibration. Simultaneous solution of these equations yields the calibration coefficients, which may then be used to find bend ? and twist ? from voltage, for any applied shape:

$? = (a_{22}V_1 - a_{21}V_2) / (a_{11}a_{22} - a_{21}a_{12})$

$? = (a_{11}V_2 - a_{12}V_1) / (a_{11}a_{22} - a_{21}a_{12})$

When affixing sensors to a tape substrate, we can choose to make one sensor have a negative response with respect to the other (by applying it "upside down"). If we choose the definition of calibration constants a little differently, we can rearrange the signs so that ? and ? can be found from sums and differences of calibrated voltages.

3.3 Interpolation

If we can measure bend and twist at discrete locations on a flexure, we can take advantage of its mechanical properties to interpolate bend and twist values between the sensors. Valid interpolations can be made for curvatures that are sufficiently gradual within a given sensor density. More exact criteria are given in (8). For discrete sensor pairs, we require that at least two pairs sample the largest monotonic curvature within a DOF. For distributed sensors (i.e. sensors with a long gauge length, or sensors with series strings of sensitive zones), the spatial resolution is improved.

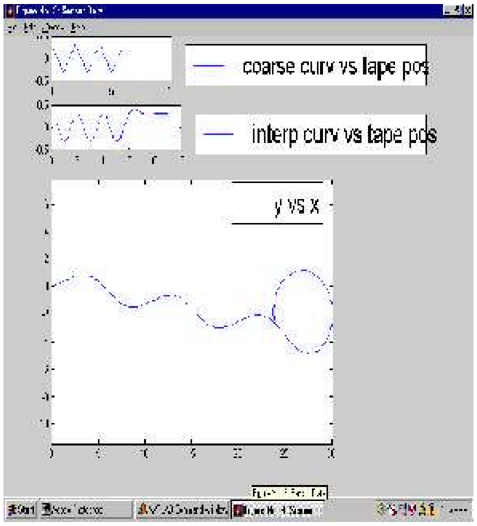

Figure 4 Matlab screen, showing, from top, raw curvature, interpolated curvature, and cartesian shape for 2D bends.

Flexures tend to have bends and twists that change gradually along their lengths. This suggests they be modeled as spatial filters, or splines. Cubic spline interpolation can be used to obtain accurate values of bend and twist everywhere along the flexure. This is shown in figure 4, which shows plots of discrete and interpolated values obtained with the Matlab (9) "interp.m" function, as well as shape data obtained from the curvatures. The figure is for simulated two dimensional (2D) data from a flexure undergoing only bending, but similar 3Dplots are obtained for flexures experiencing more degrees of freedom.

End conditions may or may not be known. Usually, one end of a tape (the base) is affixed to a surface, so that bend and twist are zero. If the other end (the tip) is not held in a fixed shape, then the first derivatives of bend and twist may be set to zero as a boundary condition, so that interpolation near the tip assumes an unbent virtual extension beyond the tip.

Flexures may be thought of as containing a 3D space curve along a central axis (or any other convenient part of the flexure). A space curve is shown in Figure 5. Each infinitesimal arc of the curve has

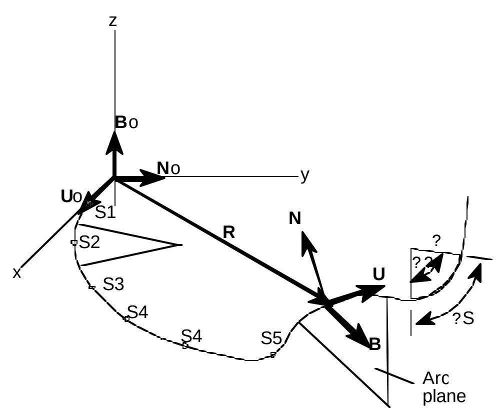

Figure 5 Space curve, showing arc length, arc plane, curvature, and **R, U, N, B** vector set.

a curvature c?/ds, which lies in a plane. The space curve has enough “structure” to allow a definition of vectors **R, U, N,** and **B**. The components of **R** are the x, y, and z coordinates of any point on the curve. **U** is a tangent vector in the arc plane, **N** is perpendicular to **U** and also in the arc plane, and **B** is orthogonal to both. All three are unit vectors. Thus, d**R**/ds = **U**

d**U**/ds = ?**N**

B = **U** X **N**

where ?= d?/ds is curvature. More specifically, it is ?, the bend component of curvature.

On a tape flexure, the arc plane is normal to the surface of the tape and contains the long axis. The arc plane angle changes relative to a fixed coordinate system according to the twist of the tape, and the curvature of the space curve corresponds to the bend. **R, U, N**, and **B** are shown on a tape flexure in Figure 6.

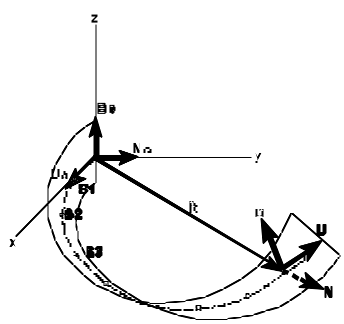

Figure 6 **R, U, N, B** vector set on a tape flexure.

3.4 Integration

The interpolation operation yields spatially quasi continuous bend and twist functions along the length of the tape, to the chosen interpolation resolution. This data set and the space curve equations can be used to solve for position and orientation of all parts of the tape in a cartesian coordinate frame. **R, U, N,** and **B** are found from interpolated bend and twist by an integration process, the integration step size being determined by the spatial resolution of the interpolation.

We already know from the space curve discussion that

d**U**/ds = ?**N**

so **U** may be found from

U = ? ?**N**ds

Similarly,

d**B**/ds = -?**N**

so **B** may be found from

B = -? ?**N**ds

Since **N** = **U** X **B**, we can find d**N**/ds:

d**N**/ds = ? **B** - ?**U**

These equations can be solved separately for each cartesian axis to determine a set of components

U_x, U_y, U_z; N_x, N_y, N_z; and B_x, B_y, and B_z.

We also know from the space curve equations presented earlier that

d**R**/ds = **U**

so R may be found from

R = ? **U**ds

3.5 Cartesian data set

Once **U, N**, and **B** are known along s, one can find orientation angles for them according to:

$\cos(rotx) = \mathrm{sqrt}(B_z N_y / U_x)$
$\cos(roty) = \mathrm{sqrt}(B_z U_x / U_y)$
$\cos(rotz) = \mathrm{sqrt}(N_y U_x / B_z)$

where U_x, N_y, and B_z are the x, y, and z components of any given **U, N**, and **B** respectively; and rotx, roty, and rotz are "roll", "pitch", and "yaw". More explicitly, at a given local point along the curve, rotx is the angle between the local xy plane and the xy plane at the origin; and roty and rotz are associated in a similar way with the yz and xz planes.

The R components can be combined with the cos(rot) components to form a cartesian set C:

$C = x_0, y_0, z_0, \cos(rotx_0), \cos(roty_0), \cos(rotz_0); x_1, y_1, z_1, \cos(rotx_1), \cos(roty_1), \cos(rotz_1), \ldots.$

which is a continuous set of data completely describing the position and orientation of every portion of the curve along s: a set of cartesian POSORs describing the complete shape of the flexure. A more detailed derivation appears in (5).

This continuous data set (a SHAPE set) contains considerably more information than the tip POSOR alone. It may be used to form computer images of human limb shapes, or to input 3D curves or surfaces for CAD design, or to either measure real shapes (e.g. vehicle interiors and exteriors) or design them. The measurement of shape does not require that the flexure be in continuous contact with the surface. In general, such perfect conformation is not possible or desirable. For instance, tape used to measure arm shape can bypass difficult parts like the shoulder or elbow, since the tape "knows where it is." It still properly locates surfaces that are in contact, using information from ALL parts of the tape, whether in contact or not.

4. TAPE ARRAYS

Figure 7 Two SHAPE TAPEs™ for robotic control or motion capture.

Figure 8 SHAPE TAPE™ interface box.

Figures 7 and 8 show a system with two SHAPE TAPEs™ used to measure arm position for robotic control. Each tape has a cross section of 1.3 x 12.5 mm, with 0.125 mm spring steel as a substrate.

Standard tapes have sensors along a 48 cm long zone at the end of 1.5 m of fiber optic leads, but spacing and lead length may be changed. The sensors are made with 0.25 mm diameter polymethylmethacrylate fiber, with all leads and loops planar on the substrates (no crossing fibers). An interface box between the tape and a computer turns on LEDs that illuminate groups of fibers, and signals are read out 8 at a time from the same 8 photodiodes and transimpedance amplifiers. The interface can multiplex up to 48 sensors into an 8 channel 12 bit analog to digital converter (A/D). The A/D passes signals to a C++ program that performs calibration, interpolation, integration, transformation, and imaging functions. For 16 sensors, an update rate of 24-30 Hz is easily achieved with a Pentium PC, the speed being determined by the graphical speed of the computer, not the calculations of cartesian POSORs, which can be obtained much more rapidly, independent of the image.

Figure 9 shows the computer model based on calculation of spatially continuous POSORs in real time from 16 fiber optic curvature sensors on a tape affixed to an arm. Figures 10 and 11 show images of other poses.

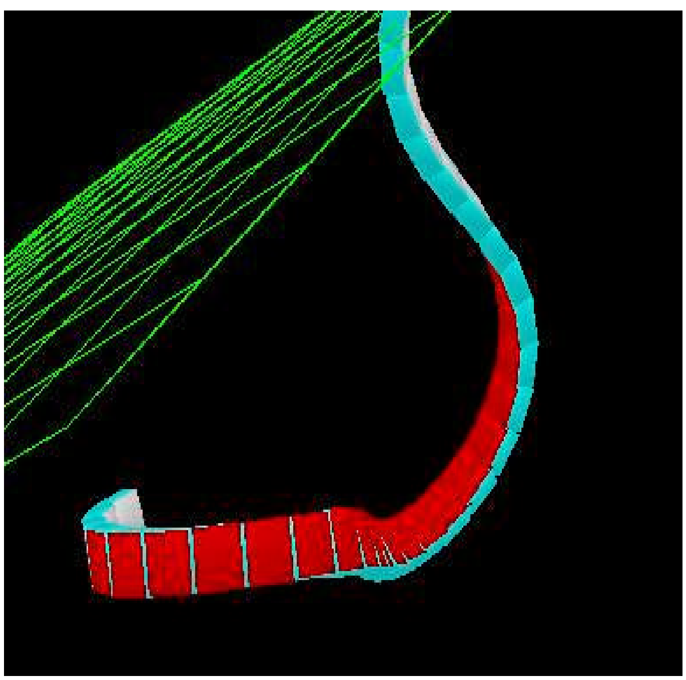

Figure 9 Computer image of arm shape.

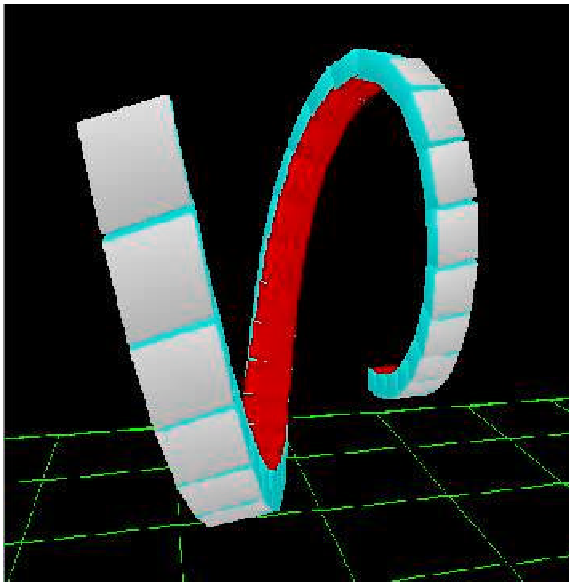

Figure 10 Shape image for helical pose.

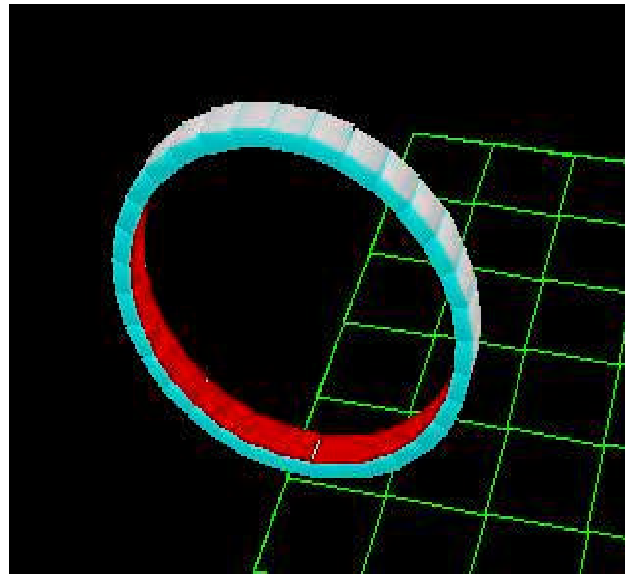

Figure 11 Shape image for hoop pose.

5. DISCUSSION

Shape sensing tapes and rods form a useful subset of the general class of deformable bodies fitted with sensors. We have shown that they may be calibrated and used to determine a spatially continuous set of cartesian POSORS for every part of the flexure. The data set is valid within a range determined by the sensor spacing and the linear range of the sensors.

Because of the inherent low noise characteristics of the individual sensors, shape sensing tape has resolution approximately that of the individual sensors, or about 5 orders of magnitude below range, for modest illuminations (LEDs running at 5% current rating). A typical range for fiber optic curvature sensors is ±4 cm radius.

Accuracy is a function of drift and hysteresis, and has not been studied in detail. However, the recent implementation of optical control loops on the LEDs has reduced thermal effects to negligibly small values, and improvements in construction have led to very small hysteresis. In future papers we will discuss tests of the overall accuracy of shape sensing tape. Initial tests indicate spatial resolution of typical tapes is of the order of 1 mm or better, at the tip.

6. CONCLUSIONS

? Shape sensing may be done in a very general way using at least n curvature sensors on an n degree of freedom surface.

? The use of calibration poses obviates knowing exact sensor orientation or slope (output vs. curvature).

? Rod and tape flexures are useful general purpose shape sensors requiring a minimum of sensors.

? Interpolation and integration lead to the determination of spatially continuous cartesian POSOR data from a sensorized flexure, valid over a wide geometric range.

? Tape flexures populated with fiber optic curvature sensors are practical, and can be used to measure complex shapes in real time, or as 6DOF computer input devices.

7. ACKNOWLEDGEMENTS

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SHAPE TAPE™ images in this paper were obtained with C++ software designed by Jeremy Prowse of Evans Computer Applications Ltd.

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