

Illiquidity Contagion and Liquidity Crashes*

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Abstract

Liquidity providers in a security often use prices of other securities as a source of information to set their quotes. As a result, liquidity is higher when prices are more informative. In turn, prices are more informative when liquidity is higher. We show that this self-reinforcing relationship between price informativeness and liquidity is a source of contagion and fragility: a small drop in the liquidity of one security propagates to other securities and can, through a feedback loop, result in a large drop in market liquidity. Furthermore, this relationship also generates multiple equilibria characterized either by high illiquidity and low price informativeness or low illiquidity and high price informativeness. A switch from the latter to the former type of equilibrium generates a liquidity crash. We use the model to interpret the Flash Crash of May 6, 2010.

Keywords: Liquidity spillovers, Contagion, Illiquidity Crashes, Multiple equilibria, Rational expectations

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1 Introduction

In the early afternoon of May 6, 2010, the prices of thousands of individual equity securities and exchange traded funds (ETFs) fell by significant amounts (sometimes more than 60%). This drop occurred without obvious changes in economic fundamentals and, in less than twenty minutes, prices reverted to their pre crash level (see the joint report of the Commodities Futures Trading Commissions (CFTC) and the Securities and Exchange Commission (SEC)).¹ This “Flash Crash” raises many questions and concerns about the current structure of securities markets.

The CFTC-SEC report identifies a large sell order in the E-mini S&P500 futures as the spark of the crash. However, many aspects of the Flash Crash remain mysterious.² In particular, the Flash Crash is also a liquidity crash with contagious illiquidity: a sharp increase in the illiquidity of the E-mini S&P500 futures that instantaneously “infected” other asset classes. As an illustration, Figure 1 (reproduced from the CFTC-SEC report) shows the evolution of the cumulative depth on the buy side of the limit order book of various assets affected by the Flash Crash. Clearly, there is a meltdown in the buy-side depth for all these assets from 2:30 p.m. (the beginning of the crash) until 2:47 p.m. The CFTC-SEC report also shows that the same phenomenon happened for the cumulative depth on the sell side of the market for many stocks and ETFs. Thus, this evaporation of liquidity reflects liquidity suppliers’ decision to curtail their liquidity provision (e.g., by cancelling limit orders), and not only a mechanical consumption of liquidity due to the arrival of sell market orders. Furthermore, Borkovec et al. (2010) show that the liquidity dry-up for ETFs and their underlying securities precedes the decline in prices for these securities, suggesting that the evaporation of liquidity might be a cause of the price crash rather than the other way round.³

[Figure 1 about here.]

How can illiquidity contagion and liquidity crashes happen? Is there something new in securities market structure that makes liquidity crashes more likely? In response to these questions, we propose the following hypothesis: markets for different assets have become more interconnected as market makers in one asset increasingly rely on the information contained in the prices of other securities to set their quotes. The reason is that prices contain information and progress in information technology has considerably increased liquidity providers’ ability to access and process this information in real-time. Using a rational expectations model of trading, we show that this evolution can make securities markets more liquid but also more

¹See “Findings regarding the market events of May 6, 2010,” CFTC-SEC joint report available at <http://www.sec.gov/news/studies/2010/marketevents-report.pdf>. For instance, this reports mentions that: “Over 20,000 trades across more than 300 securities were executed at prices more than 60% away from their values just moments before.” (on page 1).

²Kirilenko et al. (2011) provide a detailed empirical analysis of liquidity provision and price changes in the E.Mini S&P500 futures during the Flash Crash. Madhavan (2011) shows that the decline in price during the Flash Crash was stronger for stocks with a greater index of quote fragmentation.

³This is also suggested by the fact that in some stocks, buy market orders executed a very high prices, reflecting the lack of liquidity on the buy side of the limit order book.

fragile. For instance, a small exogenous increase in the illiquidity of one asset can trigger, through a multiplier mechanism, a large drop in the liquidity of other assets.

To see this intuitively, consider a liquidity provider (henceforth “dealer”) in one security, X , who uses the price of another security, Y , as a source of information. Fluctuations in the price of security Y are informative because they reflect information about fundamentals known to dealers in security Y . However, this signal is noisy since price fluctuations also reflect transient price pressures due to uninformed trades. These transient price pressures account for a larger fraction of price volatility when the cost of liquidity provision for dealers in security Y is higher.⁴ For this reason, the informativeness of the price of security Y for dealers in security X is smaller when security Y is less liquid.⁵

Now suppose that a shock *specific* to security Y increases the cost of liquidity provision for dealers in this security (e.g., dealers’s risk appetite in security Y declines). Thus, security Y becomes less liquid and, for this reason, the price of security Y becomes less informative for dealers in security X . As a result, inventory risk for dealers in security X is higher and the cost of liquidity provision for these dealers increases as well. In this way, the drop in liquidity for security Y propagates to security X , as shown in Figure 2. This spillover makes the price of security X less informative for dealers in security Y , sparking a chain reaction amplifying the initial shock.

[Figure 2 about here.]

We formalize this spillover mechanism in a two-securities rational expectations model. Securities have a two-factor structure. In the baseline version of our model, dealers in one security are specialized: they operate in only one asset. This assumption enables us to better identify the sources of the effects in our model and it reflects the fact that market-making trading desks are often specialized in trading specific types of risks. Dealers in one security are well-informed on one factor but not on the other factor. However, they can learn information about the risk factor on which they have no information by watching the price of the other security. This information structure generates the spillover mechanism and the vicious feedback loop which is portrayed in Figure 2.

In this feedback loop, liquidity and price informativeness reinforce each other. This complementarity gives rise to multiple equilibria with differing levels of liquidity and price informativeness. For instance, if dealers expect the prices of other assets to contain little information, they primarily rely on their own information. As a result, dealers perceive their inventory risk as being high. Liquidity is then low in all assets and prices are very noisy, which validates

⁴For stocks listed on the NYSE, Hendershott, Li, Menkveld and Seasholes (2010) show that 25% of the monthly return variance is due to transitory price changes. Interestingly, they also find that transient price pressures are stronger when market-makers’ inventories are relatively large. This finding implies that price movements are less informative when dealers’ cost of liquidity provision is higher, in line with our model.

⁵We measure liquidity by the sensitivity of the price of a security to market order imbalances, as in Kyle (1985) for instance. Illiquidity is higher when this sensitivity is high. This will be the case if the limit order book of a security is thin. Empirically, this sensitivity is often measured by regressing price changes on order imbalances (see for instance Glosten and Harris (1988) or Korajczyk and Sadka (2008)).

dealers' expectations. Thus, dealers' beliefs about the informativeness of the prices and liquidity of other securities are *self-fulfilling*. Consequently, there exist cases in which, for the same parameter values, liquidity and price informativeness are either relatively high for *all* assets or relatively low for all assets.

This multiplicity is another form of fragility. Indeed, it implies that liquidity can suddenly evaporate without any apparent reason. This happens when there is a switch from the equilibrium with high liquidity to the equilibrium with low liquidity. We interpret such a jump as a liquidity crash.⁶ If this liquidity crash precedes the arrival of a relatively large sell order in one security then large price drops can happen in all securities. Thus, the model can generate a simultaneous drop in liquidity and prices of multiple securities, very much as in the Flash Crash.

We show that multiple equilibria arises only when the fraction of “pricewatchers” (i.e., dealers using the prices of other assets as a source of information) is sufficiently large. Moreover, in a given equilibrium, the illiquidity multiplier becomes greater as the fraction of pricewatchers becomes larger. Yet, in a given equilibrium, market liquidity is always maximal when the fraction of pricewatchers is maximal. These findings vindicate our claim that progress in information technologies enabling dealers to condition their actions on a wide array of prices should have increased the liquidity of asset markets while making this liquidity more fragile.

As explained previously, in the baseline model, we assume that dealers operate only in one asset. Of course, in reality, there also exist liquidity providers who simultaneously operate in multiple assets. The CFTC-SEC report on the Flash Crash refer to these traders as “cross-market arbitrageurs” since they often respond to a demand for liquidity in one asset by hedging their position in another asset, attempting to profit from transient price divergences between closely related assets. These cross-asset arbitrageurs have played a role in the propagation of the price decline in the E.mini S&P500 futures to other assets (See Ben David, Franzoni and Moussavi (2011) and Sections I.5 of the CFTC-SEC report). In the last part of the paper, we incorporate cross-market arbitrageurs in our model and we study how their presence affects the findings in the baseline model. We show that the set of parameters for which multiple equilibria are obtained is exactly the same with and without cross-market arbitrageurs. Moreover, cross-market arbitrageurs dampen the illiquidity multiplier but do not suppress it. Hence, they do not fundamentally alter the conclusions obtained in the baseline model.

The model has two specific empirical implications. First, it implies that liquidity crashes should be more likely and relatively stronger for assets that are close substitutes. Indeed, multiple equilibria are more likely when the correlation in asset payoffs is high in our model. Moreover, when the two assets are close substitutes, the price system is very informative in the low illiquidity equilibrium. As a result, dealers face little uncertainty and are willing to provide liquidity at very good terms. For this reason, hedging costs for cross-market arbitrageurs are very low, which strengthen their contribution to liquidity provision. Overall, the combination of these two effects contributes to make the market very liquid in the equilibrium in which

⁶Exchange rate crises have often been interpreted as the result of a jump from one equilibrium to another in models with multiple equilibria (see for instance Jeanne (1997)).

illiquidity is relatively low. In the high illiquidity equilibrium, the price system is much less informative. Accordingly, dealers drastically curtail their liquidity provision, which raises hedging costs for cross-market arbitrageurs who therefore reduce their liquidity provision as well. Thus, the illiquidity differential between the low and the high illiquidity equilibria is relatively higher when assets become more similar. The equilibrium with high illiquidity can be associated with large deviations in the prices of the two assets because the channels for market integration (cross-asset learning and cross-market trading) broke down in the high illiquidity equilibrium. In line with these implications, analysts have observed that ETFs have been relatively more affected than other securities in the Flash Crash.

Second, our model implies that the size of illiquidity spillovers (measured by the covariation between the illiquidity of two securities) increases in the fraction of pricewatchers.⁷ Indeed, the size of the illiquidity multiplier is higher when the fraction of pricewatchers increases. Hence, random shocks on the illiquidity of one security have a greater effect, in the same direction, on the illiquidity of the other security when the fraction of pricewatchers becomes higher. One way to identify this effect empirically consists in studying changes in market structures that facilitate access to price information and testing whether co-movements in liquidity increase for stocks affected by these changes.⁸

Our analysis is related to models of financial crises (Gennotte and Leland (1990), Barlevy and Veronesi (2003), and Yuan (2005)) and financial contagion (e.g., King and Wadhvani (1990), Kodres and Pritsker (2002), Yuan (2005), or Pasquariello (2007)). As these models, we use a rational expectations model of trading with asymmetric information.⁹ For this reason, the channels highlighted by these models for the propagation of a price decline from one asset to another are present in our model. In particular, a drop in the price of one asset triggers a drop in the price of another asset because (i) price changes in one asset provide information on the value of another asset and (ii) cross-market arbitrageurs absorb the order imbalance in one market by hedging their position in the other asset. Kodres and Pritsker (2002) refer to these channels for contagion in price changes as the correlated information channel and the cross-market rebalancing channel, respectively.¹⁰ Our interest is not in explaining the propagation of price shocks, however. Rather, we focus on the propagation of a change in the illiquidity of one asset (its sensitivity to order imbalances) to other assets. To our knowledge, none of the existing models in the literature on contagion has this focus. We highlight the possibility of illiquidity multipliers and multiple equilibrium levels of illiquidity.

Most of these models feature a unique linear rational expectations equilibrium. Gennotte

⁷Evidence of co-variations in liquidity are provided in Chordia et al. (2000), Hasbrouck and Seppi (2001), or Korajczyk and Sadka (2008). None of these papers however relate the size of co-variations in liquidity to the extent to which dealers in one asset observe the prices of other assets.

⁸For instance, Easley, Hendershott, and Ramadorai (2009) analyze the effect on prices of a technological change improving the dissemination of quote information on the floor of the NYSE in 1980. Our model predicts that such a change should also affect the covariation in liquidity of different stocks.

⁹Models of contagion build upon the multi-asset pricing models of Admati (1985) or Caballe and Krishnan (1994).

¹⁰Several papers empirically study mechanisms through which price pressures in one security propagates to other securities. See, for instance, Andrade, Chang and Seasholes (2008) or Boulatov, Hendershott and Livdan (2010).

and Leland (1990), Barlevy and Veronesi (2003), and Yuan (2005) consider models in which there might be multiple clearing prices for given realizations of investors' signals and the liquidity supply because investors' aggregate demand curve is backward bending over some price ranges.¹¹ The source of multiplicity in our model is different since, in our model, the aggregate demand curve is everywhere linearly decreasing in the price in equilibrium.

Market wide declines in liquidity are often explained by wealth effects due to decreasing absolute risk aversion (e.g., Kyle and Xiong (2001)), tighter financing constraints (e.g., Gromb and Vayanos (2002), or Brunneimeier and Pedersen (2008)), or through the self-reinforcing effect that an adverse liquidity shock has on the number of liquidity providers which in turn feeds back into lower liquidity (as in Rahi and Zigrand (2011)). Our model describes a different mechanism based on the fact that the informativeness of the price of different assets and their liquidity reinforce each other. In contrast to models based on financing constraints or wealth effects, this mechanism can generate simultaneous shifts in the liquidity of several securities even if these securities do not share common liquidity suppliers.

The rest of the paper is organized as follows. Section 2 describes the model. In Section 3, we consider the the baseline case in which liquidity providers are specialized in one asset and have all access to price information. We show how cross-asset learning generates and magnifies illiquidity spillovers and is a source of multiple equilibria. In Section 4, we consider two extensions: (i) the case in which only a fraction of dealers have access to price information and (b) the case in which some liquidity providers ("cross-market arbitrageurs") can operate in both securities. In Section 5, we discuss some policy implications of the model. Proofs are collected in the Appendix.

2 The model

We consider two assets, denoted D and F . These assets pay-off at date 2 and their payoffs, v_D and v_F , are given by a factor model with two risk factors δ_D and δ_F , i.e.,

$$v_D = \delta_D + d_D \times \delta_F + \eta, \quad (1)$$

$$v_F = d_F \times \delta_D + \delta_F + \nu. \quad (2)$$

We also make the following additional parametric assumptions. First, to simplify the exposition, we assume that there is no idiosyncratic risk for security F (i.e., $\nu = 0$). Second, we assume that the factors affecting the asset payoffs are normally distributed:

$$\begin{pmatrix} \delta_D \\ \delta_F \\ \eta \end{pmatrix} \sim N \left(\begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & \sigma_\eta^2 \end{pmatrix} \right).$$

Third, we assume that $d_F = 1$ and $d_D \in [0, 1]$, so that the payoffs of the two securities are positively correlated (for instance security F could be viewed as a derivative written on security

¹¹This property of the aggregate demand curve arises for different reasons in each of these models. For instance, in Yuan (2005), it arises when informed investors face borrowing constraints while in Gennote and Leland (1990), it follows from the fact that some investors follow hedging strategies that call for selling the asset when the price is falling.

D). To simplify notations, we therefore denote d_D by d . The correlation between the payoff of the two securities increases in d and decreases in σ_η^2 . We denote by p_j the price of asset j . This price is determined at date 1.

The CFTC-SEC report on the Flash Crash distinguishes two types of liquidity providers: (i) market-makers, who often are specialized in one asset class and (ii) cross-market arbitrageurs. Cross-market arbitrageurs seek to take advantage of price discrepancies between asset classes by taking a position in one asset while hedging this position in another asset. In contrast, as noted by the CFTC-SEC report (see Section II.2.), market-making trading desks “*attempt to profit primarily from trading passively by submitting non marketable resting limit orders and capturing the bid-ask spread*” and do not systematically hedge their positions (see CFTC-SEC (2011), p.37-38).¹² Hence, we assume that there are two types of liquidity providers in securities D and F : (i) a continuum of dealers who only take position in one the two assets and (ii) a continuum of cross-market arbitrageurs.

We also assume that specialization in market-making is associated with specialized information on risk factors. This assumption is consistent with Schultz (2003), who show empirically that dealers specialize in stocks in which they have an informational advantage. Moreover, dealers often learn information from orders they receive and these orders are likely to be more informative about the risk factor to which the asset (or asset class) in which they are active is the most exposed (see Baruch and Saar (2009)). To formalize the idea that dealers in different assets are informed on different risk factors, we assume that dealers specialized in security j have perfect information on factor δ_j and no information on factor δ_{-j} .

Prices in each security will reflect dealers’ information (see below). Hence, dealers in one asset have an incentive to monitor the price in the other market even though they do not trade this asset. We denote by μ_j the fraction of dealers specialized in security j who monitor the price of security $-j$.¹³ We refer to these dealers as being *pricewatchers*. We use W to index the decisions made by pricewatchers and I to index the decisions made by other dealers (index “ I ” stands for “Inattentive”).

Each dealer in market j has a CARA utility function with risk tolerance γ_j . Thus, if dealer i in market j holds x_{ij} shares of the risky security, her expected utility is

$$E [U(\pi_{ij}) | \delta_j, \mathcal{P}_j^k] = E [-\exp \{-\gamma_j^{-1} \pi_{ij}\} | \delta_j, \mathcal{P}_j^k], \quad (3)$$

where $\pi_{ij} = (v_j - p_j)x_{ij}$ and \mathcal{P}_j^k is the *price information* available to a dealer with type

¹²In reality, dealer firms are active in multiple securities. However, these firms delegate trade-related decisions to individuals who operate on specialized trading desks. Naik and Yadav (2003) show empirically that the decision-making of these trading desks is largely decentralized (e.g., dealers’ trading decisions within a firm are mainly driven by their own inventory exposure rather than the aggregate inventory exposure of the dealer firm to which they belong). Their results suggest that there is no direct centralized information sharing between dealers within these firms.

¹³For instance, the SEC-CFTC report on the Flash crash notes that “*some ETF market-makers and liquidity providers treat ETFs as if they were the same as corporate stocks and do not track the prices of the individual securities underlying the ETFs Others heavily depend upon the tracking of underlying securities as part of their ETF pricing ...and yet others trade in individual securities at the same time they trade ETFs.*” (see page 39, SEC-CFTC (2011)). The first group corresponds to our inattentive dealers, the second to pricewatchers and the third group to cross-market arbitrageurs.

$k \in \{W, I\}$ operating in security, that is, $\mathcal{P}_j^W = \{p_j, p_{-j}\}$ and $\mathcal{P}_j^I = \{p_j\}$. Pricewatchers in asset j can condition their optimal position in this asset, $x_j^W(\delta_j, p_j, p_{-j})$, on their risk-factor information, δ_j , the price of the asset in which they make the market, and the price of the other asset. The optimal position of inattentive dealers is denoted $x_j^I(\delta_j, p_j)$.

Cross-market arbitrageurs have no direct risk-factor information on assets D and F but they can take a position in each security. Each arbitrageur has a CARA utility function with risk tolerance γ_H .¹⁴ Hence, if arbitrageur i holds x_{ij} shares of security j , her expected utility is

$$E[U(\pi_i) | \{p_j, p_{-j}\}] = E[-\exp\{-\gamma_H^{-1}\pi_{ij}\} | \{p_j, p_{-j}\}], \quad (4)$$

where $\pi_i = (v_j - p_j)x_{iF} + (v_j - p_j)x_{iD}$. We denote by $x_j^H(p_j, p_{-j})$ the optimal position of cross-market arbitrageurs in asset j . It depends on prices since cross-market arbitrageurs, like dealers, can submit price-contingent orders. As usual in rational expectations model, they account for the information conveyed by prices in determining their optimal strategies.

Liquidity traders constitute the last group of participants at date 1. The aggregate demand of liquidity traders for asset j is $u_j \sim N(0, \sigma_{u_j}^2)$. We refer to u_j as the size of the *demand shock* in market j . These shocks are independent and they are also independent from the risk factors. In equilibrium, the demand shock in a given security is absorbed by dealers in this security and cross-market arbitrageurs. Hence, the clearing price of security j is such that

$$\mu_j x_j^W(\delta_j, p_j, p_{-j}) + (1 - \mu_j) x_j^I(\delta_j, p_j) + \lambda x_j^H(p_j, p_{-j}) + u_j = 0, \quad \text{for } j \in \{D, F\}, \quad (5)$$

where λ is the mass of arbitrageurs relative to the mass of dealers. Thus, λ is an index of the amount of capital committed to cross-market arbitrage. As in many other papers (e.g., Kyle (1985)), we measure the level of illiquidity in security j by the sensitivity of the clearing price to the demand shock in this security (that is, $\partial p_j / \partial u_j$).

In order to better isolate the role of cross-asset learning, we first solve the model when $\lambda = 0$ and $\mu_D = \mu_F = 1$ (Section 3). All the main and novel results of the model can be obtained in this particular case. In Section 4, we show that these results are robust when we relax these two assumptions.

3 Cross-asset learning and liquidity spillovers

3.1 Benchmark: Fully segmented markets

As a benchmark, it is useful to consider the case in which both markets are fully segmented: $\mu_D = \mu_F = 0$ and $\lambda = 0$.

Lemma 1. (*Benchmark*) *When $\mu_F = \mu_D = 0$ and $\lambda = 0$, the equilibrium price in market j is:*

$$p_j = \delta_j + B_{j0} u_j, \quad (6)$$

with $B_{D0} = \gamma_D^{-1}(\sigma_\eta^2 + d^2)$ and $B_{F0} = \gamma_F^{-1}$.

¹⁴Arbitrageurs can also be seen as the sophisticated traders in Goldstein, Li, and Yang (2011), with the difference that in our case arbitrageurs *do not* possess private information on asset factors.

In this case, the illiquidity of security D , B_{j0} , is determined by parameters σ_η^2 , d , and γ_D . We refer to these parameters as being the “liquidity fundamentals” of security D . Similarly, we refer to γ_F as a liquidity fundamental of security F since it only affects the illiquidity of security F . Illiquidity decreases with dealers’ risk appetite (γ_j) and increases in uncertainty on the securities’ payoffs (B_{D0} increases in σ_η^2 and d).

In the benchmark case, the equilibrium is unique and there are no liquidity spillovers: a change in the illiquidity fundamental of one market does not affect the illiquidity of the other market. For instance, an increase in the risk tolerance of dealers in security D makes this security more liquid without affecting the illiquidity of the other security.¹⁵

3.2 Interconnected liquidity and Multiple Equilibria

We now consider the case in which all dealers are pricewatchers ($\mu_D = \mu_F = 1$) while maintaining the assumption that $\lambda = 0$. The analysis is more complex than in the benchmark case as dealers in one security extract information about the risk-factor that is unknown to them from the price of the other security. To solve this signal extraction problem, dealers must form beliefs on the relationship between clearing prices and risk factors. We focus on equilibria in which these beliefs are correct, that is, the rational expectations equilibria of the model.

A linear rational expectations equilibrium is a set of prices $\{p_{j1}^*\}_{j \in \{D,F\}}$ such that

$$p_{j1}^* = R_{j1}\delta_j + B_{j1}u_j + A_{j1}\delta_{-j} + C_{j1}u_{-j}, \quad (7)$$

and p_{j1}^* clears the market of asset j for each realization of $\{u_j, \delta_j, u_{-j}, \delta_{-j}\}$ when dealers anticipate that clearing prices satisfy equation (7) and choose their trading strategy to maximize their expected utility (given in equation (3)). We say that the equilibrium is non-fully revealing if dealers in security j cannot infer perfectly the realization of risk factor δ_{-j} from the price of security $-j$. The illiquidity of market j is measured by B_{j1} . Index “1” is used to refer to the equilibrium when $\mu_D = \mu_F = 1$.

Proposition 1. *When $\mu_D = \mu_F = 1$, $\lambda = 0$ and $\sigma_\eta^2 > 0$, there always exists one or three non-fully revealing linear rational expectations equilibrium. At any non-fully revealing equilibrium, $B_{j1} > 0$, $R_{j1} = 1$ and the coefficients, A_{j1} , and C_{j1} can be expressed as functions of B_{j1} and B_{-j1} . Moreover*

$$B_{D1} = f_1(B_{F1}; \gamma_D, \sigma_\eta^2, d, \sigma_{u_F}^2) = \frac{\sigma_\eta^2}{\gamma_D} + \frac{d^2 B_{F1}^2 \sigma_{u_F}^2}{\gamma_D(1 + B_{F1}^2 \sigma_{u_F}^2)}, \quad (8)$$

$$B_{F1} = g_1(B_{D1}; \gamma_F, \sigma_{u_D}^2) = \frac{B_{D1}^2 \sigma_{u_D}^2}{\gamma_F(1 + B_{D1}^2 \sigma_{u_D}^2)}. \quad (9)$$

In contrast to the benchmark case, the illiquidity of security D and the illiquidity of security F are now interconnected: B_{D1} is a function of B_{F1} and vice versa. Moreover, all coefficients

¹⁵In our model, a variation in risk tolerance of dealers in one security is just one way to vary the cost of liquidity provision for dealers in one asset class. In reality variations in this cost may be due to variations in risk tolerance, risk limits or financing constraints for dealers in this asset class. The important point is that they do not directly affect dealers in other asset classes.

in the equilibrium price function can be expressed as functions of the illiquidity of securities D and F . Thus, the number of non-fully revealing linear rational expectations equilibria is equal to the number of pairs $\{B_{D1}, B_{F1}\}$ solving the system of equations (8) and (9). In general, we cannot characterize these solutions analytically and therefore cannot solve for the equilibria in closed-form. However, we can find these solutions numerically. In Figure 3 we illustrate the determination of the equilibrium levels of illiquidity by plotting the functions $f_1(\cdot)$ and $g_1(\cdot)$ for specific values of the parameters. The three equilibria are the values of B_{D1} and B_{F1} at which the curves representing the functions $f_1(\cdot)$ and $g_1(\cdot)$ intersect.

[Figure 3 about here.]

Corollary 1. *If $4d^2 < \sigma_\eta^2$ and $\mu_D = \mu_F = 1$ then there is a unique non-fully revealing rational expectations equilibrium.*

Hence, multiple equilibria are more likely when the correlation in the payoffs of the two assets is high (d high and σ_η^2 low).

Proposition 1 focuses on the case $\sigma_\eta^2 > 0$. The case in which $\sigma_\eta^2 = 0$ is slightly different for the following reason. In this case, dealers in both markets have collectively all information on securities D and F 's payoffs. Cross-asset learning is potentially a way to share this information and suppress all uncertainty about the final payoffs of the two securities. For this reason, there always exists a fully revealing rational expectations equilibrium in this case. That is, an equilibrium in which the prices of each asset is equal to its final payoff and the market of each asset is infinitely deep, that is, $B_{D1} = B_{F1} = 0$. The second point can easily be seen by setting $\sigma_\eta^2 = 0$ in equations (8) and (9). As when $\sigma_\eta^2 > 0$, this fully revealing equilibrium is not necessarily unique and the non-fully revealing equilibria are obtained by solving equations (8) and (9) for B_{D1} and B_{F1} .

Multiple equilibria arise because liquidity and price informativeness are self-reinforcing in our model, as shown by Corollaries 2 and 3 below. The following lemma is useful to derive and understand these corollaries.

Lemma 2. *When $\mu_D = \mu_F = 1$ and $\lambda = 0$, in any non-fully revealing linear rational expectations equilibrium,*

$$p_j^* = (1 - A_{j1}A_{-j1})\omega_j + A_{j1}p_{-j}^* \text{ for } j \in \{F, D\}. \quad (10)$$

where $\omega_j \equiv \delta_j + B_{j1}u_j$ for $j \in \{D, F\}$. Hence, ω_{-j} is a sufficient statistic for the price information, $\mathcal{P}_j^W = \{p_j^*, p_{-j}^*\}$, available to pricewatchers operating in security j .

Thus, ω_{-j} is the signal about the risk factor δ_{-j} that dealers operating in security j extract from prices. Hence the precision of the forecast formed by dealers in security j about the payoff of security j is¹⁶

$$\text{Var}[v_j|\delta_j, \omega_{-j}]^{-1} = (\text{Var}[v_j|\delta_j] (1 - \rho_{j1}^2))^{-1}, \quad (11)$$

¹⁶This result follows from the fact that if X and Y are two random variables with normal distribution then $\text{Var}[X|Y] = \text{Var}[X] - \text{Cov}^2[X, Y]/\text{Var}[Y]$ and the fact that $E[\omega_{-j}|\delta_j] = 0$.

where

$$\rho_{j1}^2 \stackrel{\text{def}}{=} \frac{E[v_j \omega_{-j} | \delta_j]^2}{\text{Var}[v_j | \delta_j] \text{Var}[\omega_{-j}]} \quad (12)$$

The higher ρ_{j1}^2 is, the greater the precision of the signal conveyed by the price of security $-j$ to dealers in security j . For this reason, we refer to ρ_{j1}^2 as the informativeness of the price of security $-j$ about the payoff of security j for dealers operating in security j . When $\rho_{j1}^2 = 0$, the price of security $-j$ is completely uninformative about the payoff of security j . For instance, when $d = 0$, the payoff of security D does not depend on factor δ_F . Thus, dealers in security D have nothing to learn from the price of security F and $\rho_{D1}^2 = 0$. Using the definition of ω_j , we obtain

$$\rho_{D1}^2 = \frac{d^2}{(\sigma_\eta^2 + d^2)(1 + B_{F1}^2 \sigma_{u_F}^2)}, \quad (13)$$

$$\rho_{F1}^2 = \frac{1}{1 + B_{D1}^2 \sigma_{u_D}^2}. \quad (14)$$

We deduce from Proposition 1 that

$$B_{j1} = B_{j0}(1 - \rho_{j1}^2), \quad (15)$$

and the following result.

Corollary 2. *When $\mu_D = \mu_F = 1$ and $\lambda = 0$, an increase in the informativeness of the price of security $-j$ for dealers in security j makes security j more liquid, i.e.,*

$$\frac{\partial B_{j1}}{\partial \rho_{j1}^2} \leq 0. \quad (16)$$

The intuition for this result is straightforward. By watching the price of another security, dealers learn information. Hence, they face less uncertainty about the payoff of the security in which they are active, which makes the liquidity of this security higher.

This relationship also works in the other way round: an increase in the liquidity of one security makes the price of this security more informative for dealers in other securities. Indeed, the contribution of demand shocks to price variations becomes relatively smaller when security j becomes more liquid. As a consequence, the price of security j is more informative for dealers in other markets when security j is more liquid. More formally, remember that the signal about factor δ_j conveyed by the price of security j to dealers in security $-j$ is $\omega_j = \delta_j + B_{j1} \times u_j$. Clearly, this signal is noisier when B_{j1} is higher, which yields the following result.

Corollary 3. *When $\mu_D = \mu_F = 1$ and $\lambda = 0$, an increase in the illiquidity of security j makes its price less informative for dealers in security $-j$:*

$$\frac{\partial \rho_{-j1}^2}{\partial B_{j1}} \leq 0. \quad (17)$$

Corollaries 2 and 3 explain why the illiquidity of securities D and F are interconnected when dealers in each security learn from each other's prices. Indeed, the illiquidity of security $-j$

determines the informativeness of the price of this security for dealers in security j (Corollary 3) and as a result the illiquidity of security j (Corollary 2).

Corollaries 2 and 3 also show that price informativeness and liquidity are self-reinforcing: an increase in the liquidity of one asset makes its price more informative, which makes the liquidity of the other asset higher etc... This self-reinforcing relationship explains why multiple equilibria are possible in our model. Suppose that dealers expect other assets to be illiquid and therefore prices to be relatively uninformative. Then dealers perceive their positions as being relatively risky since they have relatively little information on the payoff of the asset in which they make the market. As a result, liquidity is low in all assets and prices are not very informative, which validates dealers' beliefs. For the same reason however, beliefs that other assets are very liquid and prices very informative can be self-fulfilling as well. Thus, for the same parameter values, various levels of liquidity and price informativeness can be sustained in equilibrium.¹⁷

When d is low relative to σ_η^2 , security D is not much exposed to factor δ_F . Thus, the beliefs of dealers in security D about the liquidity of security F play a relatively minor role in the determination of the liquidity of security D and, for this reason, the equilibrium is unique, as shown in Corollary 1. For instance, consider the extreme case in which $d = 0$. In this case, dealers in security D have no information to learn from the price of security F . Thus, the illiquidity of security D is uniquely pinned down by its "fundamentals" (γ_D and σ_η^2) and, as a result, the beliefs of dealers in security F regarding the liquidity of security D are uniquely defined as well (since dealers' expectations about the illiquidity of the other security must be correct in equilibrium). As a result, the equilibrium is unique.

When there are multiple equilibria, the three equilibria can be ranked in terms of liquidity because liquidity and price informativeness are self-reinforcing. To see this, let index each equilibrium by L , M and H (like Low, Medium and High) and let B_{D1}^k denote the level of illiquidity in security D in the equilibrium of type $k \in \{L, M, H\}$. Suppose that $B_{j1}^L < B_{j1}^M < B_{j1}^H$. Then, Corollary 3 implies that the informativeness of the price of security j is relatively high in the equilibrium of type L , medium in the equilibrium of type M , and relatively low in the equilibrium of type H . In turn, Corollary 2 implies that $B_{-j1}^L < B_{-j1}^M < B_{-j1}^H$. We deduce that $B_{D1}^L < B_{D1}^M < B_{D1}^H$ if and only if $B_{F1}^L < B_{F1}^M < B_{F1}^H$. That is, if the level of illiquidity is relatively low in one security (equilibrium of type L), it must also be relatively low in the other security. Similarly, if the level of illiquidity is relatively high in one security (equilibrium of type H), it must also be relatively high in the other security. Thus, a switch from a Low to a High (or Medium) illiquidity equilibrium in *one* security will affect *all* securities in the same way, as if illiquidity was contagious.

This is a manifestation of a more general property: when $\mu_D = \mu_F = 1$, an exogenous change in the illiquidity of one market (due for instance to an increase in dealers' risk tolerance

¹⁷Ganguli and Yang (2009) consider a single security model of price formation similar to Grossman and Stiglitz (1980). They show that multiple non-fully revealing linear rational expectations equilibria arise when investors have private information both on the asset payoff and the aggregate supply of the security. The source of multiplicity here is different since dealers have no supply information in our model.

in this market) affects the illiquidity of the other market. We call this effect a *liquidity spillover*.

3.3 Contagion and amplification: Liquidity spillovers and the illiquidity multiplier

To see how liquidity spillovers arise, consider the effect of an increase in the risk tolerance of dealers in security D . The immediate effect of this increase is to make security D more liquid. Hence, its price becomes more informative for dealers in security F (Corollary 3), which then becomes more liquid (Corollary 2) because inventory risk for dealers in security F is smaller when they are all better informed. Thus, in contrast to the benchmark case, the improvement in the liquidity of security D spreads to liquidity F although security F experiences no change in its liquidity fundamentals.

To analyze this formally, consider the system of equations (8) and (9). Other things equal, an increase in the risk tolerance of dealers in security D makes this security more liquid since $\partial f_1/\partial \gamma_D < 0$. In turn this improvement spreads to security F because $\partial g_1/\partial B_{D1} \neq 0$. More generally, any exogenous change in the illiquidity of security D will spill over to security F because $\partial g_1/\partial B_{D1} \neq 0$. Similarly, an exogenous change in the illiquidity of security F will spill over to security D when $\partial f_1/\partial B_{F1} \neq 0$. The direction (positive/negative) of these liquidity spillovers is determined by the signs of $\partial g_1/\partial B_{D1}$ and $\partial f_1/\partial B_{F1}$.

Corollary 4. *When $\mu_D = \mu_F = 1$ and $\lambda = 0$, liquidity spillovers are always positive, i.e., $\partial f_1/\partial B_{F1} > 0$ and $\partial g_1/\partial B_{D1} > 0$. Furthermore, when $d = 0$, there is no spillover from security F to security D because the price of security F conveys no information to dealers in security D . In contrast, when $d > 0$, liquidity spillovers operate in both directions.*

When liquidity spillovers operate in both directions ($d > 0$), the total effect of a small change in the illiquidity fundamentals of one security is higher than the direct effect of these changes. To see this, suppose that the equilibrium is unique and denote the equilibrium liquidity levels by B_{D1}^* and B_{F1}^* . Now consider the chain of effects that follows a small reduction, denoted by $\Delta \gamma_D < 0$, in the risk tolerance of dealers in security D . The *direct effect* of this reduction is to increase the illiquidity of security D by $(\partial f_1/\partial \gamma_D)\Delta \gamma_D > 0$. As a consequence, the price of this security becomes less informative. Hence, dealers in security F face more uncertainty and security F becomes less liquid as well, although its liquidity fundamental (γ_F) is unchanged. The immediate increase in the illiquidity of security F is equal to $(\partial g_1/\partial B_{D1})(\partial f_1/\partial \gamma_D)\Delta \gamma_D > 0$. When $d > 0$, this increase in illiquidity for security F leads to an even greater increase in the illiquidity of security D , starting a new vicious loop: the increase in illiquidity for security D leads to a further increase in illiquidity for security F etc... As a result, the total effect of the initial decrease in the risk tolerance of dealers in security D is an order of magnitude larger than its direct effect on the illiquidity of both securities. The next corollary formalizes this discussion.

Corollary 5. *Let*

$$\kappa(B_{D1}, B_{F1}) \equiv \frac{1}{(1 - (\partial g_1/\partial B_{D1})(\partial f_1/\partial B_{F1}))}. \quad (18)$$

Suppose that parameters are such that the equilibrium is unique. In this equilibrium, $\kappa(B_{D1}^*, B_{F1}^*) > 1$ when $d > 0$ and $\kappa(B_{D1}^*, B_{F1}^*) = 1$ if $d = 0$. Moreover, a small increase in the risk tolerance of dealers in security D change the illiquidity of securities D and F in equilibrium by

$$\underbrace{\frac{dB_{D1}}{d\gamma_D}}_{\text{Total Effect}} = \kappa \underbrace{\frac{\partial f_1}{\partial \gamma_D}}_{\text{Direct Effect}} < 0,$$

$$\underbrace{\frac{dB_{F1}}{d\gamma_D}}_{\text{Total Effect}} = \kappa \underbrace{\frac{\partial g_1}{\partial B_{D1}} \frac{\partial f_1}{\partial \gamma_D}}_{\text{Direct Effect}} < 0.$$

The corollary focuses on the effect of an increase in the risk tolerance of dealers in security D but the initial effects of a change in the other exogenous parameters of the model (γ_F and σ_η^2) are also amplified by a factor κ , for exactly the same reasons. Thus, cross-asset learning is a source of fragility: it amplifies the effect of a drop in the liquidity of one asset.¹⁸ This amplification effect can be quite large. Figure 4 illustrates this point for specific values of the parameters (in all our numerical examples we choose the parameter values such that there is a unique rational expectations equilibrium, except otherwise stated). It shows the value of κ for various values of the idiosyncratic risk of security D (σ_η) and the resulting values for the direct and total effects of a change in this risk tolerance on the illiquidity of securities D and F , as a function of σ_η . In this example, the total drop in illiquidity of each security after a decrease in risk tolerance for dealers in security D can be up to ten times bigger than the direct effect of this drop.

[Figure 4 about here.]

Table 1 provides another perspective on the illiquidity multiplier by showing the elasticity, denoted $\mathcal{E}_{B_{j1}, \gamma_D}$, of illiquidity in each security to the risk tolerance of dealers in security D , i.e., the percentage change in illiquidity in each security for a one percent increase in the risk tolerance of dealers in security D . The table also shows the elasticity that would be obtained ($\hat{\mathcal{E}}_{B_{j1}, \gamma_D}$) in the absence of the illiquidity multiplier (i.e., by assuming $\kappa = 1$). For instance, when $\gamma_D = 1.8$, a drop of 1% in the risk tolerance of dealers in security D triggers an increase of 9% in the illiquidity of security D and 14.9% in the illiquidity of security F . This is much higher than the direct effects of this shock ($\hat{\mathcal{E}}_{B_{j1}, \gamma_D}$) since in this case the illiquidity of securities D and F would increase by only 1% and 1.5% respectively.

[Table 1 about here.]

As explained previously, when there are multiple equilibria, they can be ranked in terms of illiquidity. Corollary 5 remains valid for the two extreme equilibria: the equilibrium in which the illiquidity of both securities is relatively Low and the equilibrium in which the illiquidity of both securities is relatively High (see the proof of the corollary). The equilibrium with a medium level

¹⁸Allen and Gale (2004) define a financial market as being fragile if “small shocks have disproportionately large effects.” (Allen and Gale (2004), page 1015).

of illiquidity is such that $\kappa < 0$ and delivers “unintuitive” comparative statics. For instance, in this equilibrium, a reduction in the risk tolerance of dealers in, say, security D *increases* the liquidity of both securities. Such an equilibrium may exist because, as explained previously, the illiquidity of each security is in part determined by dealers’ beliefs about the illiquidity of the other market. These beliefs may be disconnected from the illiquidity fundamentals of each security and yet be self-fulfilling.

This finding is closely linked to the notion of stability. In our setting, an equilibrium can be viewed as stable if, when we shock the illiquidity of one of the two securities by a small amount and trace the evolution of illiquidity using the system of equations (8) and (9) (as shown on Figure 3 for instance), then we are brought back to the same equilibrium point. For instance, suppose that the low illiquidity equilibrium is obtained and increase the illiquidity of security D from B_{D1}^L to $B_{D1}^L + \Delta$ where Δ is small. Then using the system of equations (8) and (9), we can compute the new value of B_{F1} and then compute the resulting value of B_{D1} and so and so forth. If, following this tâtonnement process, the illiquidity of both markets eventually converges to (B_{D1}^L, B_{F1}^L) then the Low illiquidity equilibrium is stable. It can be shown that stability in this sense is obtained if and only if at the equilibrium point $\kappa(B_{D1}^*, B_{F1}^*) > 1$.¹⁹ Thus, only the two extreme equilibria are stable.

3.4 Illiquidity crashes: An example

The purpose of this section is to illustrate how the model can simultaneously generate illiquidity crashes and price crashes. To this end, we consider the following numerical values for the parameters: $\gamma_j = d = 1$, $\sigma_{u_j} = 2$ and $\sigma_\eta = 0.2$. In this case we obtain three rational expectations equilibria (see Figure 3). We focus on the equilibria with a low and a high level of illiquidity since the other equilibrium is unstable.

In the low illiquidity equilibrium, the equilibrium price functions in securities D and F are:

$$\begin{aligned} p_D &= \delta_D + 0.04 \times u_D + 0.99 \times \delta_F + 0.006 \times u_F, \\ p_F &= \delta_F + 0.006 \times u_F + 0.99 \times \delta_D + 0.04 \times u_D, \end{aligned}$$

and in the high illiquidity equilibrium, equilibrium price functions are:

$$\begin{aligned} p_D &= \delta_D + 0.65 \times u_D + 0.38 \times \delta_F + 0.24 \times u_F, \\ p_F &= \delta_F + 0.63 \times u_F + 0.36 \times \delta_D + 0.23 \times u_D, \end{aligned}$$

Hence, in the low illiquidity equilibrium, the standard deviation of prices is 1.47 for both securities. Now, suppose that a sell order arrives in security F . Specifically suppose that, $u_F = -5$. This order is large since u_F is normally distributed with mean zero and standard deviation equal to 2. Hence, an order of this size has less than a one percent chance to happen. We set other variables at their mean values. In particular, the large sell order in security F does not occur simultaneously with a change in fundamentals ($\delta_D = \delta_F = 0$).

¹⁹If one views the system of equations (8) and (9) as a system of reaction functions, our notion of stability and the resulting condition on the slopes of the reaction functions are standard in game theory (see Fudenberg and Tirole (1991), Chapter 1, Section 1.2.5).

In the low illiquidity equilibrium, the sell order in security F triggers a drop in the price of this security equal to -0.0279 . Dealers in security D cannot tell whether the drop in the price of security F is due to a change in fundamentals (a drop in δ_F) or a sell order in security F . Hence, they mark down their estimate of the payoff of the security and as a result the price of security D falls as well by -0.0276 . These are minor changes in prices since they are just equal to about 2.1% of the standard deviation of prices in both markets in the low illiquidity equilibrium.

Now consider a different scenario. Suppose that just before the arrival of the large sell order, dealers switch from the low illiquidity equilibrium to the high illiquidity equilibrium. This switch generates an illiquidity crash: the illiquidity of both markets increases sharply. The ratios of illiquidity in the high and low illiquidity equilibria for securities D and F are respectively 16 and 105. As a result, the large sell order does trigger a price crash: the price of security F falls to -1.4 and precipitates a fall to -1.15 in the price of security D . These are much larger price changes since they represent respectively 223% and 99% of the standard deviations of prices in securities F and D in the low illiquidity equilibrium. Thus, they will be perceived as abnormal by participants if, in normal times, participants coordinate on the low illiquidity equilibrium.

This example fits well with the sequence of events in the Flash Crash (see Borkovec et al. (2011)): (i) limit order books thin out precipitously in the absence of clear changes in liquidity fundamentals (the large increase in B_{D1} and B_{F1}), (ii) a large sell order arrives in the E.mini S&P500 futures (security F), and (iii) this order triggers a large drop in prices because limit order books are thin. Existing models of financial contagion explain well (ii) and (iii) but do not study (i). Our analysis shows that a sudden drop in illiquidity is possible when liquidity providers active in different assets learn from each other prices. In Section ??, we revisit this example when cross-market arbitrageurs are active in addition to specialized dealers.

4 Extensions

In this section, we extend the model in two directions. First, in Section 4.1, we study the effect of increasing the fraction of dealers in an asset with information on the price of the other asset. This analysis is interesting as technological improvements have considerably eased access to price information in recent years. We use the model to study how this evolution affects the mechanisms described in our baseline case. Then, in Section 4.2, we consider the role of cross-market arbitrageurs by relaxing the assumption that $\lambda = 0$.

4.1 Illiquidity contagion and the scope of access to price information

We now generalize the baseline model to an arbitrary fractions, μ_D and μ_F of pricewatchers in each market while maintaining the assumption that $\lambda = 0$. In this way we can analyze how a change in these fractions affects the mechanism for fragility and contagion uncovered in the previous section. As when $\mu_D = \mu_F = 1$, a linear rational expectations equilibrium is a set of

prices $\{p_j^*\}_{j \in \{D, F\}}$ such that

$$p_j^* = R_j \delta_j + B_j u_j + A_j \delta_{-j} + C_j u_{-j}, \quad (19)$$

and p_j^* clears the market of asset j for each realizations of $\{u_j, \delta_j, u_{-j}, \delta_{-j}\}$ when dealers anticipate that clearing prices satisfy equation (19) and choose their trading strategies to maximize their expected utility. The next proposition generalizes Proposition 1 for arbitrary values of μ_D and μ_F .

Proposition 2. *Suppose $\sigma_\eta^2 > 0$ and $\lambda = 0$. For each value of value of μ_D and μ_F , there always exist one or three non-fully revealing linear rational expectations equilibrium. At any non-fully revealing equilibrium, $B_j > 0$, $R_j = 1$ and the coefficients A_j , C_j , and E_j can be expressed as functions of B_j and B_{-j} . Moreover*

$$B_j = B_{j0}(1 - \rho_j^2) \times \frac{\gamma_j^2 \mu_j \rho_j^2 + \sigma_{u_j}^2 \text{Var}[v_j | \delta_j](1 - \rho_j^2)}{\gamma_j^2 \mu_j^2 \rho_j^2 + \sigma_{u_j}^2 \text{Var}[v_j | \delta_j](1 - \rho_j^2)(1 - \rho_j^2(1 - \mu_j))}, \quad (20)$$

where $\rho_D^2 \equiv d^2 / ((\sigma_\eta^2 + d^2)(1 + B_F^2 \sigma_{u_F}^2))$ and $\rho_F^2 \equiv 1 / (1 + B_D^2 \sigma_{u_D}^2)$.

As in the baseline case, pricewatchers in security j extract a signal $\omega_{-j} = \delta_{-j} + B_{-j} u_{-j}$ from the price of security $-j$ and variable ρ_j^2 is the informativeness of this signal. As the pricewatchers' trading strategy depends on this signal (i.e., ω_{-j}), the price of security j partially reveals pricewatchers' private information. Equation (19) implies that observing the price of security j and risk factor δ_j is informationally equivalent to observing $\hat{\omega}_j \equiv A_j \delta_{-j} + C_j u_{-j} + B_j u_j$. Thus, in equilibrium, the information set of inattentive dealers, $\{\delta_j, p_j\}$, is informationally equivalent to $\{\delta_j, \hat{\omega}_j\}$. In what follows, we refer to $\hat{\omega}_j$ as inattentive dealers' price signal. Using the expressions for A_j and C_j (given in the proof of Proposition 2), we obtain that $\hat{\omega}_j = A_j \omega_{-j} + B_j u_j$. When $B_j > 0$, inattentive dealers' price signal is less precise than pricewatchers' price signal, which means that inattentive dealers in security j are at an informational disadvantage compared to pricewatchers.

This disadvantage creates an adverse selection problem for the inattentive dealers. Indeed, pricewatchers will bid aggressively when the price of security $-j$ indicates that the realization of the risk factor δ_{-j} is high and conservatively when the price of security $-j$ indicates that the realization of the risk factor δ_{-j} is low. As a consequence, inattentive dealers in one security will tend to have relatively large holdings of the security when its value is low and relatively small holdings of the security when its value is large. This winner's curse is absent when all dealers are pricewatchers.

Substituting ρ_D^2 and ρ_F^2 by their expressions in equation (20), we can express B_j as a function of B_{-j} . Formally, we obtain:

$$B_D = f(B_F; \mu_D, \gamma_D, \sigma_\eta^2, d, \sigma_{u_F}^2), \quad (21)$$

$$B_F = g(B_D; \mu_F, \gamma_F, \sigma_{u_D}^2), \quad (22)$$

where the expressions for the functions $f(\cdot)$ and $g(\cdot)$ are given in the Appendix (see equations (A.26) and (A.28)). The linear rational expectations equilibria are completely characterized by

the solution(s) of this system of equations. As in the baseline case and for the same reason, there are either one or three possible equilibria. Of course, when $\mu_D = \mu_F = 1$, the solutions to the previous system of equations are identical to those obtained in the baseline model, since the latter is just a special case of the model in this section.

Corollary 6. *There exists a threshold $\hat{\mu}_j$ such that if $\mu_j < \hat{\mu}_j$ then the model has a unique rational expectations equilibrium. However, liquidity spillovers arise from security F to security D if $\mu_D > 0$ and $d > 0$ and from security D to security F if $\mu_F > 0$.*

This corollary suggests that improvements in access to price information might be a cause of greater instability in market illiquidity. Indeed, when $\mu_j < \hat{\mu}_j$, the equilibrium is unique. Hence, coordination problems cannot trigger a sudden switch from a low to a high illiquidity equilibrium as in the baseline case. However, the liquidity of the two assets remains interconnected as long as the fraction of pricewatchers is not zero (second part of the corollary).

In contrast to the baseline case, liquidity spillovers are not necessarily positive when only a fraction of dealers in one asset are well informed on the price of the other asset. That is, there might exist cases in which $\partial g / \partial B_D < 0$ or $\partial f / \partial B_F < 0$. To see why, consider a decrease in the risk tolerance of the dealers operating in security D (γ_D decreases) and suppose that $\mu_F < 1$. Security D becomes less liquid and therefore less informative for pricewatchers in security F . Thus, uncertainty about the payoff of security F increases. This “uncertainty effect” increases the illiquidity of security F as when $\mu_F = 1$. However, when $\mu_F < 1$, there is a countervailing “adverse selection effect.” Indeed, as pricewatchers’ private information is less precise, their informational advantage is smaller. As a consequence, inattentive dealers are less exposed to adverse selection. This effect reduces the illiquidity of security F . The net effect of an increase in the illiquidity of security D on the illiquidity of security F is positive only if the uncertainty effect dominates. The next corollary provides a sufficient condition for this to be the case. For this corollary, we define

$$\mathcal{R}_j = \frac{\gamma_j^2}{\sigma_{u_j}^2 \text{Var}[v_j | \delta_j]}. \quad (23)$$

This ratio is a measure of the risk bearing capacity of dealers in asset j : indeed, this is a measure of dealers’ risk tolerance per unit of risk taken by the dealers in the aggregate.²⁰

Corollary 7. *Let*

$$\bar{\mu}_j = \max \left\{ 0, \frac{\mathcal{R}_j - 1}{\mathcal{R}_j} \right\}, \text{ for } j \in \{D, F\}. \quad (24)$$

If $\mu_D \in [\bar{\mu}_D, 1]$ and $\mu_F \in [\bar{\mu}_F, 1]$ then liquidity spillovers from security D to security F and vice versa are positive for all values of $d > 0$.

Hence, when dealers’ risk bearing capacity is not too large ($\mathcal{R}_j \leq 1$), $\bar{\mu}_j = 0$. In this case, liquidity spillovers are positive. In this case, the uncertainty effect prevails because risk

²⁰In equilibrium, the aggregate inventory position of dealers in security j after trading at date 1 is $-u_j$ and the total dollar value of this position at date 1 is $-u_j \times v_j$. The risk associated with this position for dealers in security j is measured by its variance conditional on information on risk factor δ_j , i.e., $\sigma_{u_j}^2 \text{Var}[v_j | \delta_j]$.

considerations are first order relative to informational asymmetries among dealers in the same market. Otherwise ($\mathcal{R}_j > 1$ for $j = D$ or $j = F$), liquidity spillovers are positive only if the fraction of pricewatchers in both markets is high enough since $\bar{\mu}_D > 0$ or/and $\bar{\mu}_F > 0$. When liquidity spillovers are positive, the illiquidity multiplier effect works as in the baseline model. The size of the illiquidity multiplier is smaller however since markets are less interconnected when the fraction of pricewatchers is smaller. Interestingly, an increase in the fraction of pricewatchers is itself a source of illiquidity spillovers, as shown by the next corollary.

Corollary 8. *Suppose $\mathcal{R}_j \leq 1$ for $j \in \{D, F\}$ and that parameters are such that the equilibrium is unique. Let κ be the illiquidity multiplier defined in Corollary 5. In this equilibrium, a small increase in the fraction of pricewatchers in security D decreases the illiquidity of securities D and F in equilibrium by $dB_D/d\mu_D = \kappa(\partial f/\partial\mu_D) < 0$ and $dB_F/d\mu_D = \kappa(\partial g/\partial B_D)(\partial f/\partial\mu_D) < 0$. Similarly, a small increase in the fraction of pricewatchers in security F decreases the illiquidity of securities D and F in equilibrium by $(dB_D/d\mu_F) = \kappa(\partial f/\partial B_F)(\partial g/\partial\mu_F) < 0$ and $dB_F/d\mu_F = \kappa(\partial g/\partial\mu_F) < 0$.*

The proof of this result is similar to that of Corollary 5. Thus, for brevity, we provide it in the Internet Appendix for this paper. Moreover, as for Corollary 5, this result also holds for the two stable equilibria when there are multiple equilibria. An immediate implication is that when $\mathcal{R}_j \leq 1$, an increase in the fraction of pricewatchers in either market makes both securities more liquid. Hence, in a given equilibrium, illiquidity is minimal when the fraction of pricewatchers is maximal ($\mu_D = \mu_F = 1$) when $\mathcal{R}_j \leq 1$.²¹ Thus, restricting access to price information can make markets less fragile (Corollary 6) at the cost of a less liquid market (Corollary 8).

Empirically, positive liquidity spillovers should translate into positive co-movement in liquidity. Intuitively, if pricewatchers contribute to these spillovers, the size of co-movement in liquidity should increase in the fraction of pricewatchers. We illustrate this testable implication of the model with the following experiment. For a given value of μ_F , we compute the illiquidity of securities D and F assuming that γ_D is uniformly distributed in $[0.5, 1]$ for $\sigma_{u_F} = \sigma_{u_D} = 1/2$, $\sigma_\eta = 2$, $\gamma_F = 1/2$. For these values of the parameters $\bar{\mu}_j = 0$ and liquidity spillovers are therefore positive. We then compute the covariance between the resulting equilibrium values for B_D and B_F . Figure 5, Panel (a) and Panel (b) show this covariance as a function of μ_F when $d = 0$ and $d = 0.9$, respectively (for $\mu_D = 0.1$ and $\mu_D = 0.9$). In both cases, the covariance between the illiquidity of securities D and F is positive because illiquidity spillovers are positive. Moreover, as expected, covariations in illiquidity increases in the fraction of pricewatchers.

[Figure 5 about here.]

²¹It is also the case that, in a given equilibrium, illiquidity is minimal in both markets when the fraction of pricewatchers is maximal when $\mathcal{R}_j > 1$. However, in this case, a small increase in the fraction of pricewatchers in one market can make the other market less liquid when the fraction of pricewatchers is small since illiquidity spillovers can then be negative. Thus, in this case, we cannot state that illiquidity is uniformly decreasing in both markets when the fraction of pricewatchers increases in one market.

4.2 Illiquidity crashes with cross-market arbitrageurs

When $\mu_D = \mu_F$, the markets for the two assets are informationally integrated since the price of security D reflects the information contained in the price of security F and vice versa. However, they remain segmented in the sense that no cross-market arbitrageurs simultaneously trade in both markets. We now relax this assumption. To better isolate the role of cross-market arbitrageurs and how they interact with dealers in supplying liquidity, we first solve for the equilibrium in absence of dealers. Comparison of this case with the baseline model also helps to understand what is new in the mechanism for liquidity spillovers described in our paper.

4.2.1 A benchmark: no dealers

As shown in Appendix B, when there are no specialized dealers, arbitrageurs' optimal positions in securities D and F are given by:

$$x_D^H(p_D, p_F) = \frac{\gamma_H}{2\sigma_\eta^2 + (1-d)^2} (-2p_D + (1+d)p_F) \quad (25)$$

$$x_F^H(p_D, p_F) = \frac{\gamma_H}{2\sigma_\eta^2 + (1-d)^2} (-(1+d^2 + \sigma_\eta^2)p_F + (1+d)p_D), \quad (26)$$

As a result, there is a unique equilibrium and in this equilibrium, the prices in securities D and F are respectively:

$$p_D = \frac{1+d^2 + \sigma_\eta^2}{\lambda\gamma_H} u_D + \frac{1+d}{\lambda\gamma_H} u_F \quad (27)$$

$$p_F = \frac{2}{\lambda\gamma_H} u_F + \frac{1+d}{\lambda\gamma_H} u_D. \quad (28)$$

Hence, the difference in the price of the two assets is given by

$$p_D - p_F = \frac{(1-d)}{\lambda\gamma_H} (d \times u_D + u_F) + \frac{\sigma_\eta^2}{\lambda\gamma_H} u_D.$$

If both assets are perfect substitutes, $d = 1$ and $\sigma_\eta^2 = 0$, the prices of both assets are identical and equal to

$$p_D = p_F = \frac{2}{\lambda\gamma_H} (u_D + u_F).$$

If $u_D + u_F < 0$, the price of each asset is smaller than the expected payoff of both assets (zero) because cross-market arbitrageurs must eventually be the counterparty of liquidity demanders and their portfolio is risky. The price of both assets however must be equal as otherwise a riskless arbitrage opportunity would exist.

In contrast, when the two assets are not perfect substitutes ($d < 1$ or $\sigma_\eta^2 > 0$), the prices of the two assets are equal only on average. For specific realizations of liquidity demands in both assets, they will in general be different.

In any case, cross-market arbitrageurs propagate the price effects of liquidity demand shocks in one market to the other market. Consider for instance a sell order in security F ($u_F < 0$), holding the liquidity demand for security D at its mean level ($u_D = 0$). The sell order in

security F triggers a drop in prices for security F (equation (28)) and security D (equation (27)). The mechanism is as follows. Cross-market arbitrageurs collectively absorb the sell order in security F , while hedging their position by selling shares of security D . As the assets are not perfect substitutes, the hedge is imperfect. Thus, the price of security F must drop to induce cross-market arbitrageurs to buy the shares in asset F . As cross-market arbitrageurs sell shares in security D to hedge their position in security F , the price of the security D drops as well.

This propagation mechanism implies that an order imbalance in one security will trigger a drop in prices of other related securities as in other models of contagion. The baseline model with no cross-market arbitrageurs delivers a similar prediction but for a different reason. To see this consider equation (7) in the baseline model. This equation implies that if liquidity traders sell u_F shares of security F then the price of security F will drop by $C_{F1} \times u_F$, other things equal. Thus, in the baseline model as well, a drop in the price of security F (due to for instance to a large liquidity sell order in this security) will propagate to security D . The propagation however is due to cross-asset learning. Dealers in security D observe the drop in price for security F and revise downward their estimate of δ_F and therefore the payoff of security D . When both dealers and cross-market arbitrageurs co-exist, the cross-asset learning channel and the cross-market hedging channel will work together and in the same direction to transmit a price pressure from one asset to another. These channels for the propagation of prices declines from one market to another have been analyzed in other models of contagion (see Kodres and Pritsker (2002)). David, Franzoni, and Moussawi (2011) (Section 5) show empirically that both transmission channels have played a role during the Flash Crash.

In the absence of dealers, the illiquidity of securities D and F are given by

$$\frac{\partial p_D}{\partial u_D} = \frac{1 + d^2 + \sigma_\eta^2}{\lambda \gamma_H}, \quad (29)$$

and

$$\frac{\partial p_F}{\partial u_F} = \frac{2}{\lambda \gamma_H}. \quad (30)$$

In this case, due to the absence of cross-asset learning, the illiquidity of securities D and F are not interconnected. That is, a change in the illiquidity of security F for instance does not affect per se the illiquidity of security D and vice versa. Thus, there is no illiquidity multiplier and no illiquidity spillovers.

4.2.2 Arbitrageurs and dealers

We now consider the case in which arbitrageurs coexist with dealers and all dealers are price-watchers ($\mu_F = \mu_D = 1$). As in the baseline model, we focus on the linear rational expectations equilibria of the model. The following proposition extends Proposition 1 to the case in which $\lambda > 0$.

Proposition 3. *When $\mu_D = \mu_F = 1$, $\lambda > 0$ and $\sigma_\eta^2 > 0$, there always exists one or three non-fully revealing linear rational expectations equilibrium. An equilibrium is such that*

$$p_{j1}^* = R_j^H \delta_j + R_j^H B_{j1} u_j + A_j^H \delta_{-j} + A_j^H B_{-j1}^H u_{-j}. \quad (31)$$

where the coefficients, A_j^H , and R_j^H can be expressed as functions of B_{j1} and B_{-j1} , which are defined as in Proposition 1 (that is solve (8) and (9)).

Thus, the presence of arbitrageurs neither mitigates, nor aggravates the possibility of multiple equilibria. In fact, the set of parameters for which multiple equilibria obtain is exactly the same as in the baseline model since the coefficients B_{j1} s solve the same system of equations and determine all other coefficients in the price functions in equilibrium.

The reason for multiplicity is exactly the same. In fact, the informativeness of the price system is the same with and without arbitrageurs in the model. This is intuitive since cross-market arbitrageurs have no private information and their orders are completely predictable. Hence, dealers can extract exactly the same information from prices with or without cross-market arbitrageurs. This is easily seen by rewriting equation (31) as

$$p_{j1}^* = R_j^H \omega_j + A_j^H \omega_{-j}, \quad (32)$$

with $\omega_j = \delta_j + B_{j1}u_j$, as in the baseline model. Hence, by observing transaction prices, traders obtain noisy signals ω_D and ω_F about the payoffs of securities D and F and the informational content of these signals is the same as in the baseline model. As in the baseline model, the information set of dealers in security j is $\Omega_j^W = \{\delta_j, \mathcal{P}^W\} = \{\delta_j, \omega_{-j}\}$ and dealers behave therefore exactly in the same way (that is, their demand functions for the security is as in the baseline model). Cross-market arbitrageurs' information set is $\Omega^H = \{p_D, p_F\} = \{\omega_D, \omega_F\}$.

Using equation (31), we obtain that the illiquidity of market j is given by

$$\frac{\partial p_j^*}{\partial u_j} \equiv R_j^H B_{j1}. \quad (33)$$

For $\lambda = 0$, $R_j^H = 1$ and the level of illiquidity in equilibrium is exactly as in the baseline model. Otherwise, for $\lambda > 0$, $R_j^H < 1$ and therefore illiquidity in the model with arbitrageurs is lower than in the baseline model. This is intuitive since arbitrageurs add to the risk bearing capacity of the market. Thus, their presence works to dampen the price pressures exerted by liquidity demand shocks relative to the case $\lambda = 0$. To illustrate this point, Figure 6 shows the evolution of illiquidity in securities F and D for various values of λ .

[Figure 6 about here.]

The presence of arbitrageurs however does not alter the conclusions of the baseline model. First, as already explained, the model features multiple equilibria. Second, a small shock to the illiquidity of one market can have a large effect on the illiquidity of both markets due to the multiplier effect described in Section 3.3. To illustrate this point, in Figure 7 we plot the total effect of an increase in dealers' risk appetite in market D on the illiquidity of security D (panel (a)) and the illiquidity of security F (panel (b)) as a function of σ_η and $\lambda \in \{0, 1, 2\}$.²² As

²²The total effect of an increase in the risk appetite of dealer D on the illiquidity of security D is given by $dR_D B_{D1}/d\gamma_D = (\partial R_{D1}/\partial \gamma_D) B_{D1} + R_D \kappa (\partial f_1/\partial \gamma_D)$. Similarly the total effect of an increase in the risk appetite of dealer D on the illiquidity of security F is given by $dR_F B_{F1}/d\gamma_D = (\partial R_{F1}/\partial \gamma_D) B_{F1} + R_F \kappa (\partial g_1/\partial B_{D1})(\partial f_1/\partial \gamma_D)$.

can be seen in Figure 7, an increase in arbitrage capital (λ) tends to attenuate the illiquidity multiplier. However, the latter remains high even when λ is high. Thus, the presence of arbitrageurs does not remove the source of fragility and exposure to illiquidity crashes due to cross-asset learning.

[Figure 7 about here.]

Finally, we investigate the effect of arbitrageurs on the comovement in illiquidity for securities D and F . To this end, we compute the illiquidity of securities F and D assuming that $d = 0.9$, $\sigma_{u_F} = \sigma_{u_D} = 1/2$, $\gamma_F = \gamma_D = \gamma_H = 1/2$. Then, for different values of λ , we compute numerically the equilibrium values for the illiquidity of securities F and D (that is, $(\partial p_D^*/\partial u_D)$ and $(\partial p_F^*/\partial u_F)$) for each value of σ_η in $[0.01, 2.5]$. Finally, we plot the resulting covariance for the illiquidity of securities D and F against λ assuming that σ_η has a uniform distribution over $[0.01, 2.5]$ (see Figure 10). When only arbitrageurs are present in the market, this covariance is zero since a change in σ_η has no effect on the illiquidity of security F (see equation (30) in Section 4.2.1). When arbitrageurs and dealers co-exist, this covariance is in general strictly positive but it decreases as the amount of capital dedicated to arbitrage increases. Thus, the presence of arbitrageurs attenuates, without eliminating, illiquidity spillovers due to cross-asset learning.

[Figure 8 about here.]

4.2.3 Liquidity crashes for close substitutes

Our model predicts that when assets have close substitutes ($d = 1$ and σ_η^2 small), they are more exposed to an illiquidity crash since multiple equilibria are more likely when σ_η^2 is small. Moreover, illiquidity crashes will appear to be stronger in this case.

Indeed, when σ_η^2 is close to zero, assets are very liquid in the low illiquidity equilibrium. Indeed, prices in this case are almost fully revealing and for this reason the sensitivity of prices to order imbalances in each asset is close to zero. Hence, dealers face very little uncertainty about the payoff of the security in which they market. For the same reason, the risk of cross-market arbitrageurs' portfolios is very low. Moreover, arbitrageurs can hedge their position in one asset at a very low cost since dealers in this asset face little inventory risk. As a result, the markets for securities D and F appear very liquid.

In this situation, a sudden switch from the low to the high illiquidity equilibrium is associated with a breakdown of price discovery: suddenly, prices switch from being very informative to being relatively very uninformative. Thus, dealers perceive risk as being much higher and demand a larger compensation for risk bearing, especially if the risk bearing capacity of the market-making sector is low (that is, if γ_D and γ_F are low). As a result, hedging is more costly for cross-market arbitrageurs and they will therefore provide liquidity less efficiently as well. Thus, in relative terms, the drop in liquidity is much more pronounced when σ_η^2 is low than when σ_η^2 is high.

[Table 2 about here.]

We illustrate this point with a numerical example. We set $d = 1$ and market participants' risk tolerance at $\gamma_j = \gamma^H = 1$. Then, in Table 2, we show the sensitivity of the price of securities D and F to a unit demand shock in this market in the low and the high illiquidity equilibria and the ratio between these two measures of illiquidity for values of σ_η^2 varying between 0.25 and 0.1 (as $d = 1$, the correlation in the payoff of both assets varies between 0.94 to 0.97). As expected the ratio of the illiquidity of a given asset in the high illiquidity equilibrium to the low illiquidity equilibrium increases when σ_η^2 decreases. For instance, when $\lambda = 1$, the ratio goes from 26.3 when $\sigma_\eta^2 = 0.25$ to 943.63 when $\sigma_\eta^2 = 0.1$. Thus, illiquidity crashes will appear relatively stronger in assets that are close substitutes (and which in normal times appear very liquid).

This implication of the model is in line with the fact that, during the Flash Crash, ETFs have experienced a particularly severe liquidity meltdown (see the CFTC-SEC report, Section II.2.c). Furthermore, Borokovec et al. (2010) show that ETFs prices deviated wildly from the value of the underlying portfolio of securities. At first glance this finding is surprising since the basket of securities underlying an ETF and the ETF itself are close substitutes. Hence one would expect cross-market arbitrageurs to closely integrate the markets for the ETF and the underlying securities.

Our model suggests a potential explanation. Cross-market arbitrageurs are more willing to countervail liquidity demand shocks in one market when they can hedge their positions at a low cost in the other market. To do so, they benefit from the presence of dealers. To see this, suppose that a sell order arrives in security F . Arbitrageurs will absorb part of this demand and hedge their long position in security F by selling shares of security D to dealers in this market. In this sense, dealers in security D provide liquidity to arbitrageurs and ultimately enhance their ability to provide liquidity in security F . As explained previously, when the equilibrium switches from the low illiquidity regime to the high illiquidity regime, dealers suddenly scale back on their liquidity provision. As a result, arbitrageurs' hedging costs increase considerably and they cut back on their liquidity provision in both markets. Hence, in the high illiquidity equilibrium, the markets for both assets appear less integrated than in the low illiquidity equilibrium since (i) prices are less informative so that the cross-asset learning channel does not work very well and (ii) cross-market hedging is more costly.

As an example, in table 2, we report the variance of the price differential between assets, $\text{Var}[p_D - p_F]$ in the low and the high illiquidity equilibria for each value of σ_η^2 considered in this table. Markets for assets F and D are more integrated if this variance is smaller. As can be seen, the market for both assets is much less integrated in the high illiquidity equilibrium than in the low illiquidity equilibrium. For instance, if $\sigma_\eta^2 = 0.25$ and $\lambda = 1$, the variance of the price differential between securities D and F is 15000 bigger in the high illiquidity equilibrium. Moreover, the increase in this variance between the low and the high illiquidity equilibrium becomes relatively higher when σ_η^2 declines. Thus, the dislocation of prices between related assets associated with an illiquidity crash should appear relatively bigger for assets that are

close substitutes according to the model.

5 Discussion: Implications of the model

Improvements in information technology and inter-market linkages enable dealers in a security to incorporate in their strategies the information contained in the prices of other securities. Our model shows that this evolution should make the liquidity of various securities more interconnected and fragile: (i) a small change in the illiquidity of one security can have relatively large effects on the illiquidity of all securities and (ii) multiple equilibria with very different levels of illiquidity arise when the fraction of dealers with access to price information increases.

As explained previously, we think that the mechanism described in our paper provides a possible explanation for the Flash Crash. The CFTC-SEC report on the Flash crash notes that (on page 39): *“market makers that track the prices of securities that are underlying components of an ETF are more likely to pause their trading if there are price-driven or data-driven integrity questions about those prices. Moreover extreme volatility in component stocks makes it very difficult to accurately value an ETF in real-time. When this happens, market participants who would otherwise provide liquidity for such ETFs may widen their quotes or stop providing liquidity [...]”* This is consistent with our model in which the liquidity of a security drops when prices of other securities become less reliable as a source of information.

In light of the Flash Crash, the SEC has encouraged exchanges to redesign their circuit-breakers. Circuit-breakers stop trading when price changes exceed a specific threshold. Our model suggests to adopt a complementary approach. Indeed, in our model, a price crash is a consequence of a market wide evaporation in liquidity due to a switch from one equilibrium to another. This evaporation may find its origin in one market first, which then through a chain reaction drags the other markets from a low to the high illiquidity equilibrium. Hence, one way to block the occurrence of a crash at its very early stage consists in defining trigger points for a trading halts in reference to illiquidity indicators. The advantage of this approach is that it stops the chain reaction that leads to an illiquidity crash at its starting point. The exact design of such a circuit-breaker is an interesting topic for future research.

Our model has three implications that one could use to test its explanatory power. First, as shown in Section 4.1, an increase in the fraction of pricewatchers should contribute to increase co-movements in the liquidity across securities. One way to test this prediction consists in using technological changes that made the access to information on the price of other securities easier for liquidity providers in one asset class. Second, our model has two predictions specific to the Flash Crash. First, the drop in illiquidity during the Flash Crash should occur jointly with a massive drop in price informativeness. Second, the drop in illiquidity should precede the drop in prices. Indeed, as shown by our numerical example in Section 3.4, it is the switch from a low to a high illiquidity equilibrium that exposes the market to destabilization by a large sell order. In line with our hypotheses, Borkovec et al. (2010) observe a breakdown of the price discovery process in ETFs during the Flash Crash and show that the evaporation of liquidity in ETFs precede the drop in prices for this class of assets. They note (on page 3)

“The sharp drop in liquidity and its correlation with the failure of the price discovery process is unambiguous.” Further tests require a more detailed investigation of the lead-lag relationships between the drop in asset prices and the drop in liquidity during the Flash Crash.

A Appendix A: No cross-market arbitrageurs.

Proof of Lemma 1

If $\mu_D = 0$ then all dealers in security D only observe factor δ_D when they choose their demand function. As dealers have a CARA utility function, it is immediate that their demand function in this case is

$$x_D^I(\delta_D) = \gamma_D \frac{E[v_D|\delta_D] - p_D}{\text{Var}[v_D|\delta_D]} = \gamma_D \left(\frac{\delta_D - p_D}{\sigma_\eta^2 + d^2} \right). \quad (\text{A.1})$$

Using the clearing condition for $\lambda = 0$ and $\mu_D = 0$, we deduce that the clearing price is such that:

$$p_D = \delta_D + \left(\frac{\sigma_\eta^2 + d^2}{\gamma_D} \right) u_D = \delta_D + B_{D0} u_D.$$

A similar reasoning yields the expression of the clearing price for security F . \square

Proof of Proposition 1 and 2

Proposition 1 is a special case of Proposition 2, which considers the more general case in which μ_j can take any value. Thus, we directly prove Proposition 2. The method that we follow to prove this result is standard. That is, (i) we conjecture that the equilibrium price is such that $p_j^* = R_j \delta_j + B_j u_j + A_j \delta_{-j} + C_j u_{-j}$ with $R_j = 1$, $C_j = A_j B_{-j}$, and $B_j > 0$ (ii) we compute dealers' demand functions given this conjecture, (iii) we deduce using these demand functions and the clearing condition that our conjecture is correct if B_j is given as in the system of equations defined by equation (20). When $\mu_D = \mu_F = 1$, this equation yields the system of equations (8) and (9). Last, we show that this system of equations always has a solution, which proves the existence of a linear rational expectations equilibrium.

Step 1. Let $\hat{\omega}_j = B_j u_j + A_j \omega_{-j}$ and $\omega_j = \delta_j + B_j u_j$. The conjectured equilibrium price in market j can be written $p_j^* = \omega_j + A_j \omega_{-j}$. As pricewatchers know the prices in both markets, they can deduce signals ω_j and ω_{-j} from these prices. For pricewatchers in security j , ω_j is not informative since they already know δ_j . In contrast ω_{-j} is informative about δ_{-j} . Thus, $\{\delta_j, \omega_{-j}\}$ is a sufficient statistic for pricewatchers' information set $\{\delta_j, p_{-j}^*, p_j^*\}$. Inattentive dealers in security j just observe the price in their market. As they know δ_j , they extract the signal $\hat{\omega}_j$ from p_j^* . Hence, $\{\delta_j, \hat{\omega}_j\}$ is a sufficient statistic for inattentive dealers' information set $\{\delta_j, p_j^*\}$.

Step 2. Equilibrium in market j .

Pricewatchers' demand function. A pricewatcher's demand function in market j , $x_j^W(\delta_j, p_j, p_{-j})$, maximizes

$$E \left[-\exp \left\{ - \left((v_j - p_j) x_j^W \right) / \gamma_j \right\} \mid \delta_j, p_j, p_{-j} \right].$$

We deduce that

$$\begin{aligned} x_j^W(\delta_j, p_j, p_{-j}) &= \gamma_j \left(\frac{E[v_j|\delta_j, p_{-j}, p_j] - p_j}{\text{Var}[v_j|\delta_j, p_{-j}]} \right) \\ &= a_j^W (E[v_j|\delta_j, p_{-j}, p_j] - p_j), \end{aligned} \quad (\text{A.2})$$

with $a_j^W = \gamma_j \text{Var}[v_j | \delta_j, p_{-j}]^{-1}$. As $\{\delta_D, \omega_F\}$ is a sufficient statistic for $\{\delta_D, p_F, p_D\}$, we deduce (using well-known properties of normal random variables) that

$$\begin{aligned} E[v_D | \delta_D, p_F, p_D] &= E[v_D | \delta_D, \omega_F] \\ &= \delta_D + \frac{d}{(1 + B_F^2 \sigma_{u_F}^2)} \omega_F, \end{aligned} \quad (\text{A.3})$$

and

$$a_D^W = \frac{\gamma_D}{\text{Var}[v_D | \delta_D, \omega_F]} \quad (\text{A.4})$$

$$= \gamma_D \left(\frac{1 + B_F^2 \sigma_{u_F}^2}{d^2 B_F^2 \sigma_{u_F}^2 + \sigma_\eta^2 (1 + B_F^2 \sigma_{u_F}^2)} \right) \quad (\text{A.5})$$

$$= \frac{\gamma_D}{\text{Var}[v_D | \delta_D] (1 - \rho_D^2)}, \quad (\text{A.6})$$

where $\rho_D^2 \equiv d^2 / ((\sigma_\eta^2 + d^2)(1 + B_F^2 \sigma_{u_F}^2))$. Thus,

$$x_D^W(\delta_D, \omega_F) = a_D^W(\delta_D - p_D) + b_D^W \omega_F,$$

where

$$\begin{aligned} b_D^W &= \frac{\gamma_D}{\text{Var}[v_D | \delta_D, \omega_F]} \frac{\text{Cov}[v_D, \omega_F]}{\text{Var}[\omega_F]} \\ &= da_D^W \left(\frac{1}{1 + B_F^2 \sigma_{u_F}^2} \right). \end{aligned} \quad (\text{A.7})$$

Similarly, for pricewatchers in security F we obtain

$$x_F^W(\delta_F, \omega_D) = a_F^W(\delta_F - p_F) + b_F^W \omega_D, \quad (\text{A.8})$$

where $\omega_D = \delta_D + B_D u_D$, and

$$a_F^W = \gamma_F \left(\frac{1 + B_D^2 \sigma_{u_D}^2}{B_D^2 \sigma_{u_D}^2} \right) = \frac{\gamma_F}{\text{Var}[v_F | \delta_F] (1 - \rho_F^2)}, \quad b_F^W = a_F^W \frac{1}{1 + B_D^2 \sigma_{u_D}^2}, \quad (\text{A.9})$$

where $\rho_F^2 \equiv (1 + B_D^2 \sigma_{u_D}^2)^{-1}$.

Inattentive Dealers. An inattentive dealers' demand function in market j , $x_j^I(\delta_j, p_j)$, maximizes:

$$E \left[-\exp \left\{ -((v_j - p_j) x_j^I) / \gamma_j \right\} | \delta_j, p_j \right].$$

We deduce that

$$\begin{aligned} x_j^I(\delta_j, p_j) &= \gamma_j \left(\frac{E[v_j | \delta_j, p_j] - p_j}{\text{Var}[v_j | \delta_j, p_{-j}]} \right) \\ &= a_j^I (E[v_j | \delta_j, p_j] - p_j), \end{aligned} \quad (\text{A.10})$$

with $a_j^I = \gamma_j \text{Var}[v_j | \delta_j, p_{-j}]^{-1}$. As $\{\delta_D, \hat{\omega}_D\}$ is a sufficient statistic for $\{\delta_D, p_D\}$, we deduce (using well-known properties of normal random variables) that

$$\begin{aligned} E[v_D | \delta_D, p_D] &= E[v_D | \delta_D, \hat{\omega}_D] \\ &= \delta_D + \frac{dA_D}{A_D^2 (1 + B_F^2 \sigma_{u_F}^2) + B_D^2 \sigma_{u_D}^2} \hat{\omega}_D, \end{aligned} \quad (\text{A.11})$$

and

$$a_D^I = \frac{\gamma_D}{\text{Var}[v_D | \delta_D, \hat{\omega}_D]} \quad (\text{A.12})$$

$$= \gamma_D \frac{A_D^2(1 + B_F^2 \sigma_{u_F}^2) + B_D^2 \sigma_{u_D}^2}{d^2(A_D^2 B_F^2 \sigma_{u_F}^2 + B_D^2 \sigma_{u_D}^2) + \sigma_\eta^2(A_D^2(1 + B_F^2 \sigma_{u_F}^2) + B_D^2 \sigma_{u_D}^2)}. \quad (\text{A.13})$$

Thus,

$$x_D^I(\delta_D, \hat{\omega}_D) = a_D^I(\delta_D - p_D) + b_D^I \hat{\omega}_D,$$

where

$$\begin{aligned} b_D^I &= \frac{\gamma_D}{\text{Var}[v_D | \delta_D, \hat{\omega}_D]} \frac{\text{Cov}[v_D, \hat{\omega}_D]}{\text{Var}[\hat{\omega}_D]} \\ &= a_D^I \frac{dA_D}{A_D^2(1 + B_F^2 \sigma_{u_F}^2) + B_D^2 \sigma_{u_D}^2}. \end{aligned} \quad (\text{A.14})$$

Similarly, for market F we obtain:

$$x_F^I(\delta_F, \hat{\omega}_F) = a_F^I(\delta_F - p_F) + b_F^I \hat{\omega}_F, \quad (\text{A.15})$$

where

$$a_F^I = \gamma_F \frac{A_F^2(1 + B_D^2 \sigma_{u_D}^2) + B_F^2 \sigma_{u_F}^2}{A_F^2 B_D^2 \sigma_{u_D}^2 + B_F^2 \sigma_{u_F}^2}, \quad b_F^I = a_F^I \frac{A_F}{A_F^2(1 + B_D^2 \sigma_{u_D}^2) + B_F^2 \sigma_{u_F}^2}. \quad (\text{A.16})$$

Clearing price in market j . When $\lambda = 0$, the clearing condition in market $j \in \{D, F\}$ imposes

$$\mu_j x_j^W(\delta_j, p_j, p_{-j}) + (1 - \mu_j) x_j^I(\delta_j, p_j) + u_j = 0.$$

Let $a_j = \mu_j a_j^W + (1 - \mu_j) a_j^I$. Using equations (A.2) and (A.10), we solve for the clearing price and we obtain

$$p_j^* = \delta_j + \left(\frac{\mu b_j^W + (1 - \mu_j) b_j^I A_j}{a_j} \right) \omega_{-j} + \left(\frac{(1 - \mu_j) b_j^I B_j + 1}{a_j} \right) u_j, \quad (\text{A.17})$$

Remember that we are searching for an equilibrium such that $p_j^* = R_j \delta_j + B_j u_j + A_j \delta_{-j} + C_j u_{-j}$. We deduce from equation (A.17) that in equilibrium, we must have $R_j = 1$,

$$B_j = \left(\frac{(1 - \mu_j) b_j^I B_j + 1}{a_j} \right), \quad A_j = \left(\frac{\mu b_j^W + (1 - \mu_j) b_j^I A_j}{a_j} \right), \quad \text{and } C_j = A_j B_{-j}.$$

Thus

$$B_j = \frac{1}{a_j - (1 - \mu_j) b_j^I}, \quad \text{for } j \in \{D, F\}, \quad (\text{A.18})$$

$$A_j = \mu_j B_j b_j^W, \quad \text{for } j \in \{D, F\}. \quad (\text{A.19})$$

We show below that coefficients a_j^W, a_j^I, b_j^W and b_j^I only depend on B_D and B_F . Hence, coefficients A_j and C_j can be expressed only in terms of $\{B_D, B_F\}$, which means that the equilibrium is fully characterized once B_D and B_F are known.

It is immediate from (A.6) a_D^W only depends on B_F . Moreover, substituting (A.6) in (A.7) and rearranging we obtain

$$b_D^W = \frac{d\gamma_D}{d^2 B_F^2 \sigma_{u_F}^2 + \sigma_\eta^2 (1 + B_F^2 \sigma_{u_F}^2)}. \quad (\text{A.20})$$

Using (A.19) (for $j = D$) and (A.20), we can rewrite (A.14) as

$$b_D^I = a_D^I \frac{d^2 \mu_D \gamma_D (d^2 B_F^2 \sigma_{u_F}^2 + \sigma_\eta^2 (1 + B_F^2 \sigma_{u_F}^2))}{B_D (\mu_D^2 d^2 \gamma_D^2 (1 + B_F^2 \sigma_{u_F}^2) + \sigma_{u_D}^2 (d^2 B_F^2 \sigma_{u_F}^2 + \sigma_\eta^2 (1 + B_F^2 \sigma_{u_F}^2))^2)}. \quad (\text{A.21})$$

Similarly, using (A.19) (for $j = D$) and (A.20), we can rewrite (A.13) as

$$a_D^I = \frac{\gamma_D (\mu_D^2 d^2 \gamma_D^2 (1 + B_F^2 \sigma_{u_F}^2) + \sigma_{u_D}^2 (d^2 B_F^2 \sigma_{u_F}^2 + \sigma_\eta^2 (1 + B_F^2 \sigma_{u_F}^2))^2)}{(d^2 B_F^2 \sigma_{u_F}^2 + \sigma_\eta^2 (1 + B_F^2 \sigma_{u_F}^2)) (\mu_D^2 d^2 \gamma_D^2 + \sigma_{u_D}^2 (\sigma_\eta^2 + d^2) (\sigma_\eta^2 (1 + B_F^2 \sigma_{u_F}^2) + d^2 B_F^2 \sigma_{u_F}^2))} \quad (\text{A.22})$$

$$= \frac{\gamma_D (\mu_D^2 \gamma_D^2 \rho_D^2 + \sigma_{u_D}^2 (d^2 + \sigma_\eta^2) (1 - \rho_D^2)^2)}{(d^2 + \sigma_\eta^2) (1 - \rho_D^2) (\mu_D^2 \gamma_D^2 \rho_D^2 + \sigma_{u_D}^2 (d^2 + \sigma_\eta^2) (1 - \rho_D^2))}. \quad (\text{A.23})$$

Inserting (A.23) in (A.21) yields after some algebra

$$b_D^I = \gamma_D^2 \frac{d^2 \mu_D}{B_D (\mu_D^2 d^2 \gamma_D^2 + \sigma_{u_D}^2 (\sigma_\eta^2 + d^2) (\sigma_\eta^2 (1 + B_F^2 \sigma_{u_F}^2) + d^2 B_F^2 \sigma_{u_F}^2))}. \quad (\text{A.24})$$

Hence, as claimed previously, coefficients a_D^W, a_D^I, b_D^W and b_D^I only depend on B_D and B_F . We can now replace (A.6), (A.23) and (A.24) in (A.18) and, after some tedious algebra, we obtain

$$B_D = f(B_F; \mu_D, \gamma_D, \sigma_\eta^2, d, \sigma_{u_F}^2), \quad (\text{A.25})$$

where

$$f(B_F; \mu_D, \gamma_D, \sigma_\eta^2, d, \sigma_{u_F}^2) = \frac{B_{D0} (1 - \rho_D^2) (\mu_D \gamma_D^2 \rho_D^2 + (\sigma_\eta^2 + d^2) \sigma_{u_D}^2 (1 - \rho_D^2))}{\rho_D^2 \mu_D^2 \gamma_D^2 + \sigma_{u_D}^2 (\sigma_\eta^2 + d^2) (1 - \rho_D^2) (1 - \rho_D^2 (1 - \mu_D))}, \quad (\text{A.26})$$

with $\rho_D^2 = d^2 / ((\sigma_\eta^2 + d^2) (1 + B_F^2 \sigma_{u_F}^2))$ and $B_{D0} = (\sigma_\eta^2 + d^2) / \gamma_D$. In a similar way we can express coefficients a_F^W, a_F^I, b_F^W and b_F^I in terms of B_D and B_F and obtain

$$B_F = g(B_D; \mu_F, \gamma_F, \sigma_{u_D}^2), \quad (\text{A.27})$$

where

$$g(B_D; \mu_F, \gamma_F, \sigma_{u_D}^2) = \frac{B_{F0} (1 - \rho_F^2) (\mu_F \gamma_F^2 \rho_F^2 + \sigma_{u_F}^2 (1 - \rho_F^2))}{\rho_F^2 \mu_F^2 \gamma_F^2 + \sigma_{u_F}^2 (1 - \rho_F^2) (1 - \rho_F^2 (1 - \mu_F))}, \quad (\text{A.28})$$

with $\rho_F^2 = (1 + B_D^2 \sigma_{u_D}^2)^{-1}$ and $B_{F0} = \gamma_F^{-1}$. Last, as $\text{Var}[v_D | \delta_D] = \sigma_\eta^2 + d^2$ and $\text{Var}[v_F | \delta_F] = 1$, we obtain that

$$B_j = B_{j0} (1 - \rho_j^2) \times \frac{\gamma_j^2 \mu_j \rho_j^2 + \sigma_{u_j}^2 \text{Var}[v_j | \delta_j] (1 - \rho_j^2)}{\gamma_j^2 \mu_j^2 \rho_j^2 + \sigma_{u_j}^2 \text{Var}[v_j | \delta_j] (1 - \rho_j^2) (1 - \rho_j^2 (1 - \mu_j))}, \quad (\text{A.29})$$

as claimed in Proposition 2.

Step 3. Existence and number of non-fully revealing equilibrium when $\mu_D = \mu_F = 1$.

Let $f_1(B_{F1}; \gamma_D, \sigma_\eta^2, d, \sigma_{u_F}^2) \equiv f(B_F; 1, \gamma_D, \sigma_\eta^2, d, \sigma_{u_F}^2)$ and $g_1(B_{D1}; \gamma_F, \sigma_{u_D}^2) \equiv g(B_{D1}; 1, \gamma_F, \sigma_{u_D}^2)$ where the functions $f(\cdot)$ and $g(\cdot)$ are defined in equations (A.26) and (A.27). When $\mu_D = \mu_F = 1$, we deduce from equations (A.25) and (A.27) that a non-fully rational expectations equilibrium exists if and only if the following system of equations has a strictly positive solution

$$B_{D1} = f_1(B_{F1}; \gamma_D, \sigma_\eta^2, d, \sigma_{u_F}^2) = \frac{\sigma_\eta^2}{\gamma_D} + \frac{d^2 B_{F1}^2 \sigma_{u_F}^2}{\gamma_D (1 + B_{F1}^2 \sigma_{u_F}^2)}, \quad (\text{A.30})$$

$$B_{F1} = g_1(B_{D1}; \gamma_F, \sigma_{u_D}^2) = \frac{B_{D1}^2 \sigma_{u_D}^2}{\gamma_F (1 + B_{D1}^2 \sigma_{u_D}^2)}. \quad (\text{A.31})$$

Note that $B_{F1} > 0$ if and only if $B_{D1} > 0$.

Let define the function $h_1(B_{D1}) = f_1(g_1(B_{D1}; \gamma_F, \sigma_{u_D}^2))$. The number of non-fully revealing equilibria is equal to the number of times $h_1(B_{D1})$ cuts the 45 degrees line for values of $B_{D1} > 0$. It is immediate that $h_1(0) = \sigma_\eta^2/\gamma_D > 0$ since $\sigma_\eta^2 > 0$ and $h_1(B_{D0}) < B_{D0} = (\sigma_\eta^2 + d^2)/\gamma_D$ since $B_{F1}^2 \sigma_{u_F}^2 / (1 + B_{F1}^2 \sigma_{u_F}^2) < 1$. Hence, $h_1(B_{D1})$ cuts the 45 degrees line at least once in the interval $(\sigma_\eta^2/\gamma_D, B_{D0})$. Thus, when $\sigma_\eta^2 > 0$, there always exists at least one non fully revealing rational expectations equilibrium. Moreover, any equilibrium is such that $\sigma_\eta^2/\gamma_D < B_{D1} < B_{D0}$.

We now show that there is either one or three equilibria. Calculations yield

$$h_1'(B_{D1}) = \frac{4B_D^3 d^2 \gamma_F^2 \sigma_{u_D}^4 \sigma_{u_F}^2 (1 + B_D^2 \sigma_{u_D}^2)}{\gamma_D (\gamma_F^2 (1 + B_D^2 \sigma_{u_D}^2)^2 + B_D^4 \sigma_{u_D}^4 \sigma_{u_F}^2)^2} > 0,$$

and

$$h_1''(B_{D1}) = -\frac{4B_D^2 d^2 \gamma_F^2 \sigma_{u_D}^4 \sigma_{u_F}^2 (3\gamma_F^2 (-1 + B_D^2 \sigma_{u_D}^2) (1 + B_{D1}^2 \sigma_{u_D}^2)^2 + B_{D1}^4 \sigma_{u_D}^4 \sigma_{u_F}^2 (5 + 3B_{D1}^2 \sigma_{u_D}^2))}{\gamma_D (\gamma_F^2 (1 + B_D^2 \sigma_{u_D}^2)^2 + B_D^4 \sigma_{u_D}^4 \sigma_{u_F}^2)^3}.$$

Now define $y = B_{D1}^2$. After some algebra, we obtain that the numerator of $h_1''(B_{D1})$ is proportional to:

$$-3y^3 \sigma_{u_D}^6 (\gamma_F^2 + \sigma_{u_F}^2) - y^2 \sigma_{u_D}^4 (3\gamma_F^2 + 5\sigma_{u_F}^2) + 3y \gamma_F^2 \sigma_{u_D}^2 + 3\gamma_F^2. \quad (\text{A.32})$$

This is a polynomial function of degree 3 in y and using Descartes rule of signs, we deduce that this polynomial has only one positive root. Thus, the curvature of $h_1(\cdot)$ (which is given by the sign of $h_1''(B_{D1})$) changes only once, which implies that $h_1(\cdot)$ cuts the 45° either one time or three times. Thus, either there is a unique linear non-fully revealing rational expectations equilibrium or there are three equilibria.

Let

$$\Psi_1(B_{D1}) = h_1(B_{D1}) - B_{D1},$$

When $\mu_D = \mu_F = 1$, there is a unique non-fully revealing equilibrium if and only if $\Psi_1'(B_{D1}) < 0$, $\forall B_{D1}$. Using the expression for $h_1'(\cdot)$, we can easily write $\Psi_1'(B_{D1})$ as a ratio between two polynomials in B_{D1} . The denominator is always positive while the numerator of $\Psi_1'(B_{D1})$ is

$$-\gamma_D \gamma_F^2 (1 + B_{D1}^2 \sigma_{u_D}^2)^2 + 4B_{D1} \sigma_{u_D}^2 (\sigma_\eta^2 - \gamma_D B_{D1}) (\gamma_F^2 (1 + B_{D1}^2 \sigma_{u_D}^2) + B_{D1}^2 \sigma_{u_D}^2 \sigma_{u_F}^2) + B_{D1}^3 \sigma_{u_D}^4 \sigma_{u_F}^2 \gamma_D (4\gamma_D^{-1} d^2 - B_{D1}).$$

Remember that when $\mu_D = \mu_F = 1$, $B_{D1} > \sigma_\eta^2/\gamma_D$. Hence, if $4d^2/\gamma_D \leq \sigma_\eta^2/\gamma_D$, the numerator of $\Psi_1'(B_{D1})$ is negative, which implies $\Psi_1'(B_{D1}) < 0$ since its denominator is positive.

Step 4. Existence of a non fully revealing equilibrium with limited attention ($\mu_j < 1$).

With limited attention, we deduce from equations (A.25) and (A.27) that a non-fully revealing equilibrium exists if and only if the following equation has one strictly positive solution

$$\Psi(B_D) \equiv f(g(B_D; \mu_F, \gamma_F, \sigma_{u_D}^2); \mu_D, \gamma_D, \sigma_\eta^2, d, \sigma_{u_F}^2) - B_D = 0.$$

Calculations show that $\Psi(\cdot)$ is an odd-degree polynomial in B_D with negative leading coefficient. Hence,

$$\lim_{B_D \rightarrow \infty} \Psi(B_D) = -\infty,$$

while, for $\sigma_\eta^2 > 0$,

$$\Psi(0) = \gamma_F^{12} \mu_F^8 \sigma_\eta^2 (d^2 \gamma_D^2 \mu_D + \sigma_\eta^2 \sigma_{u_D}^2 (d^2 + \sigma_\eta^2)) > 0.$$

Thus, there always exists a strictly positive value B_D^* , such that $\Psi(B_D^*) = 0$ when $\sigma_\eta^2 > 0$. \square

Proof of Corollary 1

See Step 3 in the proof of Propositions 1 and 2. \square

Proof of Lemma 2

Remember that the equilibrium price in market j can be written $p_j^* = \omega_j + A_j \omega_{-j}$ (see Step 1 in the proof of Propositions Proposition 1 and 2). As $\omega_{-j} = p_{-j}^* - A_{-j} \omega_j$, we can also write the equilibrium price in market j as

$$p_j^* = \omega_j + A_j (p_{-j}^* - A_{-j} \omega_j) = (1 - A_j A_{-j}) \omega_j + A_j p_{-j}^*.$$

\square

Proof of Corollary 2

The result follows immediately from equation (15). \square

Proof of Corollary 3

The result follows immediately from equations (13) and (14). \square

Proof of Corollary 4

The result follows immediately from the definition of functions $f_1(\cdot)$ and $g_1(\cdot)$ in Proposition 1. \square

Proof of Corollary 5

Step 1: The total effect of a change in γ_D on the illiquidity of security D is given by

$$\frac{dB_{D1}}{d\gamma_D} = \frac{\partial f_1}{\partial \gamma_D} + \frac{\partial f_1}{\partial B_F} \frac{dB_F}{d\gamma_D}.$$

As

$$\frac{dB_{F1}}{d\gamma_D} = \frac{\partial g_1}{\partial B_D} \frac{dB_{D1}}{d\gamma_D}.$$

We deduce that:

$$\begin{aligned} \frac{dB_{D1}}{d\gamma_D} &= \kappa \frac{\partial f_1}{\partial \gamma_D}, \\ \frac{dB_{F1}}{d\gamma_D} &= \kappa \left(\frac{\partial g_1}{\partial B_{D1}} \frac{\partial f_1}{\partial \gamma_D} \right), \end{aligned}$$

with $\kappa = 1 - ((\partial g_1/\partial B_{D1})(\partial f_1/\partial B_{F1}))$. If $d = 0$, we have $\partial f_1/\partial B_{F1} = 0$ and $\kappa = 1$. We now consider the case in which $d > 0$.

Step 2: Remember that by definition

$$h_1(B_{D1}) \equiv f_1(g_1(B_{D1}; \gamma_F, \sigma_{u_D}^2); \gamma_D, \sigma_\eta^2, d, \sigma_{u_F}^2).$$

Note that

$$\frac{\partial h_1}{\partial B_{D1}} = \frac{\partial f_1}{\partial B_{F1}} \frac{\partial g_1}{\partial B_{D1}}.$$

Hence, we have $\kappa > 1$ if and only if $h'_1(B_{D1}^*) < 1$ at an equilibrium value for B_{D1} . A necessary condition is $d > 0$ as otherwise $\frac{\partial f_1}{\partial B_{F1}} = 0$. Now remember that the equilibrium values for B_{D1} are obtained at the points where $h_1(B_{D1})$ crosses the 45° line. As explained in Step 3 of the proof of Propositions 1 and 2, there are either one such point or three.

Let consider first a case in which there are three equilibria and let call the equilibrium values of B_{D1} in this case, B_{D1}^L , B_{D1}^M and B_{D1}^H with $B_{D1}^L < B_{D1}^M < B_{D1}^H$. We have shown in Step 3 of the proof of Propositions 1 and 2 that $h''_1(B_{D1})$ was changing sign only once. Let \widehat{B}_{D1} be the value at which $h''_1(B_{D1})$ changes sign. Using the expression for $h''_1(B_{D1})$ in Step 3 of the proof of Propositions 1 and 2, it is easily seen that $h''_1(B_{D1}) > 0$ for B_{D1} small. Thus, $h''_1(B_{D1}) > 0$ for $B_{D1} < \widehat{B}_{D1}$. Moreover, $B_{D1}^m < \widehat{B}_{D1}$. Indeed, otherwise, $h''_1(B_{D1})$ would change sign before cutting the 45° line for the second time, which would imply that it never cuts the 45° line more than once. This is impossible in a case with three equilibria. Hence, at B_{D1}^L and B_{D1}^M , $h_1(\cdot)$ is convex. As $h_1(0) = \sigma_\eta^2/\gamma_D > 0$, it means that $h_1(\cdot)$ cuts the 45° line for the first time from above and the second time from below. That is, $h'_1(B_{D1}^L) < 1$ and $h'_1(B_{D1}^M) > 1$.

Moreover, we have $\widehat{B}_{D1} < B_{D1}^H$ since $h_1(\cdot)$ passes above the 45° line at B_{D1}^M and crosses it again at B_{D1}^H . Hence the curvature of $h_1(\cdot)$ must change in the interval (B_{D1}^M, B_{D1}^H) . At B_{D1}^H , the function $h_1(\cdot)$ cuts the 45 degree line from above since it cuts it from below at B_{D1}^m . Hence, $h'_1(B_{D1}^H) < 1$.

Now consider the case in which there is only one equilibrium. The analysis is identical except that $h_1(\cdot)$ cuts the 45° line only at one point B_{D1}^* . At this point $h_1(\cdot)$ is convex and as $h_1(0) = \sigma_\eta^2/\gamma_D > 0$, this means that $h_1(\cdot)$ cuts the 45° line from above. That is, $h'_1(B_{D1}^*) < 1$.

To sum up, if the equilibrium is unique then it must be such that $\kappa \geq 1$. If instead there are three equilibria, only the two extreme equilibria (those for which $B_{D1} = B_{D1}^L$ or $B_{D1} = B_{D1}^H$) are such that $\kappa > 1$. \square

Proof of Corollary 6 To be written.

Proof of Corollary 7

First observe that a change in B_{-j} only affects the illiquidity of security j through its effect on ρ_j^2 . As ρ_j^2 decline in B_{-j} , we deduce that liquidity spillovers from security j to security $-j$ are positive if and only if $(\partial B_j / \partial \rho_j^2) < 0$. Now we show that $\mu_j \geq \bar{\mu}_j$ is a sufficient condition for this to be the case. Observe that $B_j = B_{j0}(1 - \rho_j^2)G(\mu_j, \rho_j^2)$ with

$$G(\mu_j, \rho_j^2) \equiv \frac{\gamma_j^2 \mu_j \rho_j^2 + \sigma_{u_j}^2 \text{Var}[v_j | \delta_j](1 - \rho_j^2)}{\gamma_j^2 \mu_j^2 \rho_j^2 + \sigma_{u_j}^2 \text{Var}[v_j | \delta_j](1 - \rho_j^2)(1 - \rho_j^2(1 - \mu_j))}. \quad (\text{A.33})$$

Therefore, we have:

$$\frac{\partial B_j}{\partial \rho_j^2} = -B_{j0}G(\mu_j, \rho_j^2) + B_{j0}(1 - \rho_j^2) \frac{\partial G}{\partial \rho_j^2}. \quad (\text{A.34})$$

Now observe that

$$\begin{aligned} \frac{\partial G(\mu_D, \rho_D^2)}{\partial \rho_D^2} &= \\ &= \frac{(\sigma_\eta^2 + d^2)(1 - \mu_D)(1 - \rho_D^2)\sigma_{u_D}^2(\gamma_D^2 \mu_D(1 + \rho_D^2) + (\sigma_\eta^2 + d^2)(1 - \rho_D^2)\sigma_{u_D}^2)}{(\gamma_D^2 \mu_D^2 \rho_D^2 + \sigma_{u_D}^2 \text{Var}[v_D | \delta_D](1 - \rho_D^2)(1 - \rho_D^2(1 - \mu_D)))^2} > 0. \end{aligned}$$

Inserting this expression and the expression for $G(\mu_D, \rho_D^2)$ in equation (A.34), we obtain after some algebra

$$\begin{aligned} \frac{\partial B_D}{\partial \rho_D^2} &= - \frac{\text{Var}[v_D | \delta_D] \mu_D}{\gamma_D(\gamma_D^2 \mu_D^2 \rho_D^2 + \sigma_{u_D}^2 \text{Var}[v_D | \delta_D](1 - \rho_D^2)(1 - \rho_D^2(1 - \mu_D)))^2} \times \\ &= (\gamma_D^4 \mu_D^2 \rho_D^4 + \sigma_{u_D}^2 \text{Var}[v_D | \delta_D](1 - \rho_D^2)(\text{Var}[v_D | \delta_D](1 - \rho_D^2)\sigma_{u_D}^2 - \gamma_D^2(1 - \mu_D - \rho_D^2(1 + \mu_D))))). \end{aligned}$$

As $\rho_D^2 < 1$, we deduce that the sign of $(\partial B_D / \partial \rho_D^2)$ is the opposite of the sign of

$$\mu_D - \left(\frac{\mathcal{R}_D - 1}{\mathcal{R}_D} \right) \left(\frac{1 - \rho_D^2}{1 + \rho_D^2} \right),$$

which is positive if $\mu_D \geq \bar{\mu}_D$. We deduce that $(\partial f / \partial B_F) > 0$ if $\mu_D > \bar{\mu}_D$. A similar reasoning shows that $(\partial g / \partial B_D) > 0$ if $\mu_F > \bar{\mu}_F$. \square

B Appendix: Dealers and cross-market arbitrageurs

B.1 Benchmark: No dealers

We first analyze the case in which there are no dealers. Prices contain no information in this case since arbitrageurs have no information. Hence, an arbitrageur's optimal portfolio solves

$$\max_{\mathbf{x}^H} E \left[-\exp \left\{ -(1/\gamma_H)(\mathbf{x}^H)'(\mathbf{v} - \mathbf{p}) \right\} \right],$$

The FOC of this problem yields

$$\mathbf{x}^H = \gamma_H \text{Var}[\mathbf{v}]^{-1} (E[\mathbf{v}] - \mathbf{p}),$$

where \mathbf{x}^H is the 2×1 vector of arbitrageurs' positions in each asset and

$$\text{Var}[\mathbf{v}]^{-1} = \begin{pmatrix} 1 + d^2 + \sigma_\eta^2 & 1 + d \\ 1 + d & 2 \end{pmatrix}.$$

As $E[\mathbf{v}|\Omega^H] = E[\mathbf{v}] = \mathbf{0}$, we obtain

$$\mathbf{x}^H = \frac{\gamma_H}{2\sigma_\eta^2 + (1-d)^2} \begin{pmatrix} -2p_D + (1+d)p_F \\ -(1+d^2 + \sigma_\eta^2)p_F + (1+d)p_D \end{pmatrix},$$

as claimed in equations (25) and (26). Furthermore, using the clearing conditions in each market, we obtain the following equilibrium prices

$$p_D = \frac{1 + d^2 + \sigma_\eta^2}{\lambda\gamma_H} u_D + \frac{1 + d}{\lambda\gamma_H} u_F \quad (\text{B.1})$$

$$p_F = \frac{2}{\lambda\gamma_H} u_F + \frac{1 + d}{\lambda\gamma_H} u_D. \quad (\text{B.2})$$

B.2 Proof of Proposition 3

Step 1. In linear rational expectations equilibria, the demand functions of the dealers are linear functions of the prices of each asset and their private signals. Denote by $x_j^W(\delta_j, p_j, p_{-j}) = a_j^W \delta_j - \varphi_j^W(p_j, p_{-j})$ the demand function of asset j by dealers in this asset where $\varphi_j^W(\cdot)$ is linear in both prices. Similarly the demand functions for each asset of the cross-market arbitrageurs are linear in the prices of each asset. Denote by $x_j^H = \varphi_j^H(p_j, p_{-j})$ these demand functions where $\varphi_j^H(\cdot)$ is linear in both prices.

Let $B_D = 1/a_D^W$ and $B_F = 1/a_F^W$. The clearing conditions in each market imply

$$a_D^W \omega_D + \lambda \varphi_D^H(p_D, p_F) = \varphi_D^W(p_D, p_F) \quad (\text{B.3})$$

$$a_F^W \omega_F + \lambda \varphi_F^H(p_D, p_F) = \varphi_F^W(p_D, p_F). \quad (\text{B.4})$$

where

$$\omega_D = \delta_D + B_D u_D \quad (\text{B.5})$$

$$\omega_F = \delta_F + B_F u_F. \quad (\text{B.6})$$

As $\varphi_j^W(\cdot)$ and $\varphi_j^H(\cdot)$ are linear functions, we immediately deduce from the system of equations (B.3) and (B.4) that observing the prices of securities D and F is observationally equivalent to $\{\omega_D, \omega_F\}$. Signal ω_j does not contain new information for dealers in security j but signal ω_{-j} does. Hence $\Omega_j^W = \{\delta_j, \mathcal{P}_j^W\} = \{\delta_j, \omega_{-j}\}$. As cross-market arbitrageurs only observe prices, we also deduce that $\Omega^H = \{p_D, p_F\} = \{\omega_D, \omega_F\}$.

Step 2. We now use these remarks to show that any linear rational expectations equilibrium has the following form

$$p_D = R_D^H \omega_D + A_D^H \omega_F, \quad (\text{B.7})$$

$$p_F = R_F^H \omega_F + A_F^H \omega_D, \quad (\text{B.8})$$

where A_j^H and R_j^H are equilibrium coefficients and ω_D and ω_F are as defined in equations (B.5) and (B.6). The problem of pricewatchers is exactly as in the baseline case. We deduce that the optimal demand of pricewatchers in security j is

$$\begin{aligned} x_j^W &= \gamma_j \frac{E[v_j | \Omega_j^W] - p_j}{\text{Var}[v_j | \Omega_j^W]} \\ &= \frac{1}{B_j} (\delta_j - p_j) + b_j^W \omega_{-j}, \end{aligned} \quad (\text{B.9})$$

where

$$B_j = \frac{\text{Var}[v_j | \{\delta_j, \omega_{-j}\}]}{\gamma_j}, \quad b_j^W = \frac{1}{B_j} \frac{\text{Cov}[v_j, \omega_{-j}]}{\text{Var}[\omega_{-j}]}.$$

An arbitrageurs' optimal portfolio, $\mathbf{x}^H = (x_D^H, x_F^H)^T$ (where superscript T is used to designate the transpose of a matrix) solves

$$\max_{\mathbf{x}^H} E \left[-\exp \left\{ -(1/\gamma_H) (\mathbf{x}^H)' (\mathbf{v} - \mathbf{p}) \right\} | \Omega^H \right],$$

which yields

$$\mathbf{x}^H = \mathbf{\Gamma}^H (E[\mathbf{v} | \Omega^H] - \mathbf{p}),$$

where $\mathbf{\Gamma}^H = \gamma_H \text{Var}[\mathbf{v} | \Omega^H]^{-1}$. As all random variables have a normal distribution, standard properties of conditional moments for these variables yield

$$E[\mathbf{v} | \Omega^H] = \begin{pmatrix} H_{DD} & H_{DF} \\ H_{FD} & H_{FF} \end{pmatrix} \begin{pmatrix} \omega_D \\ \omega_F \end{pmatrix},$$

with

$$\begin{aligned} \begin{pmatrix} H_{DD} & H_{DF} \\ H_{FD} & H_{FF} \end{pmatrix} &= \begin{pmatrix} \text{Cov}[v_D, \omega_D] / \text{Var}[\omega_D] & \text{Cov}[v_D, \omega_F] / \text{Var}[\omega_F] \\ \text{Cov}[v_F, \omega_D] / \text{Var}[\omega_D] & \text{Cov}[v_F, \omega_F] / \text{Var}[\omega_F] \end{pmatrix} \\ &= \begin{pmatrix} 1/\text{Var}[\omega_D] & d/\text{Var}[\omega_F] \\ 1/\text{Var}[\omega_D] & 1/\text{Var}[\omega_F] \end{pmatrix}. \end{aligned}$$

Furthermore

$$\begin{aligned}
\Gamma^H &= \gamma_H \text{Var}[\mathbf{v}|\Omega^H]^{-1} \\
&= \gamma_H (\text{Var}[\mathbf{v}] - \text{Cov}[\mathbf{v}, \{\omega_D, \omega_F\}]' \text{Var}[\{\omega_D, \omega_F\}]^{-1} \text{Cov}[\mathbf{v}, \{\omega_D, \omega_F\}])^{-1} \\
&= \gamma_H \left(\begin{pmatrix} 1 + d^2 + \sigma_\eta^2 & 1 + d \\ 1 + d & 2 \end{pmatrix} - \begin{pmatrix} 1 & d \\ 1 & 1 \end{pmatrix} \begin{pmatrix} (1 + B_D^2 \sigma_{u_D}^2)^{-1} & 0 \\ 0 & (1 + B_F^2 \sigma_{u_F}^2)^{-1} \end{pmatrix} \begin{pmatrix} 1 & d \\ 1 & 1 \end{pmatrix}' \right)^{-1} \\
&= \begin{pmatrix} a_{D1}^H & a_{D2}^H \\ a_{F2}^H & a_{F1}^H \end{pmatrix}.
\end{aligned}$$

We deduce that an arbitrageur's optimal positions in securities D and F are

$$x_D^H = a_{D1}^H (E[v_D|\Omega^H] - p_D) + a_{D2}^H (E[v_F|\Omega^H] - p_F) \quad (\text{B.11})$$

$$x_F^H = a_{F1}^H (E[v_F|\Omega^H] - p_F) + a_{F2}^H (E[v_D|\Omega^H] - p_D), \quad (\text{B.12})$$

where

$$a_{D1}^H = \gamma_H \frac{B_F^2 \sigma_{u_F}^2 + B_D^2 \sigma_{u_D}^2 (1 + 2B_F^2 \sigma_{u_F}^2)}{B_F^2 \sigma_{u_F}^2 \sigma_\eta^2 + B_D^2 \sigma_{u_D}^2 (\sigma_\eta^2 + B_F^2 \sigma_{u_F}^2 ((1-d)^2 + 2\sigma_\eta^2))} \quad (\text{B.13})$$

$$a_{D2}^H = a_{F2}^H = -\gamma_H \frac{B_D^2 \sigma_{u_D}^2 + B_F^2 \sigma_{u_F}^2 (B_D^2 \sigma_{u_D}^2 + d(1 + B_D^2 \sigma_{u_D}^2))}{B_F^2 \sigma_{u_F}^2 \sigma_\eta^2 + B_D^2 \sigma_{u_D}^2 (\sigma_\eta^2 + B_F^2 \sigma_{u_F}^2 ((1-d)^2 + 2\sigma_\eta^2))} < 0 \quad (\text{B.14})$$

$$a_{F1}^H = \gamma_H \frac{\sigma_\eta^2 B_F^2 \sigma_{u_F}^2 (d^2 + \sigma_\eta^2) + B_D^2 \sigma_{u_D}^2 (1 + \sigma_\eta^2 + B_F^2 \sigma_{u_F}^2 (1 + d^2 + \sigma_\eta^2))}{B_F^2 \sigma_{u_F}^2 \sigma_\eta^2 + B_D^2 \sigma_{u_D}^2 (\sigma_\eta^2 + B_F^2 \sigma_{u_F}^2 ((1-d)^2 + 2\sigma_\eta^2))}. \quad (\text{B.15})$$

The clearing conditions impose:

$$x_D^W + \lambda x_D^H + u_D = 0 \quad (\text{B.16})$$

$$x_F^W + \lambda x_F^H + u_F = 0, \quad (\text{B.17})$$

Substituting (B.9), (B.11), and (B.12) in (B.16) and (B.17) and collecting terms yields the following system of equations

$$\Phi_{D3} p_D = \Phi_{D1} \omega_D + \Phi_{D2} \omega_F - \Phi_{D4} p_F \quad (\text{B.18})$$

$$\Phi_{F3} p_F = \Phi_{F1} \omega_F + \Phi_{F2} \omega_D - \Phi_{F4} p_D, \quad (\text{B.19})$$

where

$$\begin{aligned}
\Phi_{D1} &= a_D^W + \lambda(a_{D1}^H H_{DD} + a_{D2}^H H_{FD}), & \Phi_{F1} &= a_F^W + \lambda(a_{F1}^H H_{FF} + a_{F2}^H H_{DF}) \\
\Phi_{D2} &= b_D^W + \lambda(a_{D1}^H H_{DF} + a_{D2}^H H_{FF}), & \Phi_{F2} &= b_F^W + \lambda(a_{F1}^H H_{FD} + a_{F2}^H H_{DD}) \\
\Phi_{D3} &= a_D^W + \lambda a_{D1}^H, & \Phi_{D4} &= \lambda a_{D2}^H, & \Phi_{F3} &= a_F^W + \lambda a_{F1}^H, & \Phi_{F4} &= \lambda a_{F2}^H.
\end{aligned}$$

Solving for the equilibrium prices yields

$$p_D = \frac{\Phi_{D1} \Phi_{F3} - \Phi_{D4} \Phi_{F2}}{\Phi_{D3} \Phi_{F3} - \Phi_{D4} \Phi_{F4}} \omega_D + \frac{\Phi_{D2} \Phi_{F3} - \Phi_{D4} \Phi_{F1}}{\Phi_{D3} \Phi_{F3} - \Phi_{D4} \Phi_{F4}} \omega_F \quad (\text{B.20})$$

$$p_F = \frac{\Phi_{D3} \Phi_{F1} - \Phi_{D2} \Phi_{F4}}{\Phi_{D3} \Phi_{F3} - \Phi_{D4} \Phi_{F4}} \omega_F + \frac{\Phi_{D3} \Phi_{F2} - \Phi_{D1} \Phi_{F4}}{\Phi_{D3} \Phi_{F3} - \Phi_{D4} \Phi_{F4}} \omega_D. \quad (\text{B.21})$$

Our conjecture on the form of the linear rational expectations is correct if and only if:

$$\begin{aligned} R_D^H &= \frac{\Phi_{D1}\Phi_{F3} - \Phi_{D4}\Phi_{F2}}{\Phi_{D3}\Phi_{F3} - \Phi_{D4}\Phi_{F4}}, & A_D^H &= \frac{\Phi_{D2}\Phi_{F3} - \Phi_{D4}\Phi_{F1}}{\Phi_{D3}\Phi_{F3} - \Phi_{D4}\Phi_{F4}}, \\ R_F^H &= \frac{\Phi_{D3}\Phi_{F1} - \Phi_{D2}\Phi_{F4}}{\Phi_{D3}\Phi_{F3} - \Phi_{D4}\Phi_{F4}}, & A_F^H &= \frac{\Phi_{D3}\Phi_{F2} - \Phi_{D1}\Phi_{F4}}{\Phi_{D3}\Phi_{F3} - \Phi_{D4}\Phi_{F4}}, \end{aligned} \quad (\text{B.22})$$

which proves our claim.

Step 3. Observe that the coefficients R_j^H and A_j^H given in equation (B.22) can be written ultimately only in terms of B_D and B_F since coefficients $a_{j1}^H, a_{j2}^H,$ and b_j^W are known once B_D and B_F are known. Moreover, using equation (B.10), we have:

$$B_j = \frac{\text{Var}[v_j|\{\delta_j, \omega_{-j}\}]}{\gamma_j}$$

That is:

$$\begin{aligned} B_D &= \frac{d^2 B_F^2 \sigma_{u_F}^2 + \sigma_\eta^2 (1 + B_F^2 \sigma_{u_F}^2)}{\gamma_D (1 + B_F^2 \sigma_{u_F}^2)} \\ &= \frac{\sigma_\eta^2}{\gamma_D} + \frac{d^2 B_F^2 \sigma_{u_F}^2}{\gamma_D (1 + B_F^2 \sigma_{u_F}^2)} \end{aligned}$$

and

$$B_F = \frac{B_D^2 \sigma_{u_D}^2}{\gamma_F (1 + B_D^2 \sigma_{u_D}^2)}$$

Thus, B_D and B_F solve the same system of equations as in the baseline case. Thus, as in the baseline case, there is either one of three non fully revealing linear rational expectations equilibria when $\sigma_\eta^2 > 0$.

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E-Mini, SPY, and S&P 500 Buy-Side Market Depth

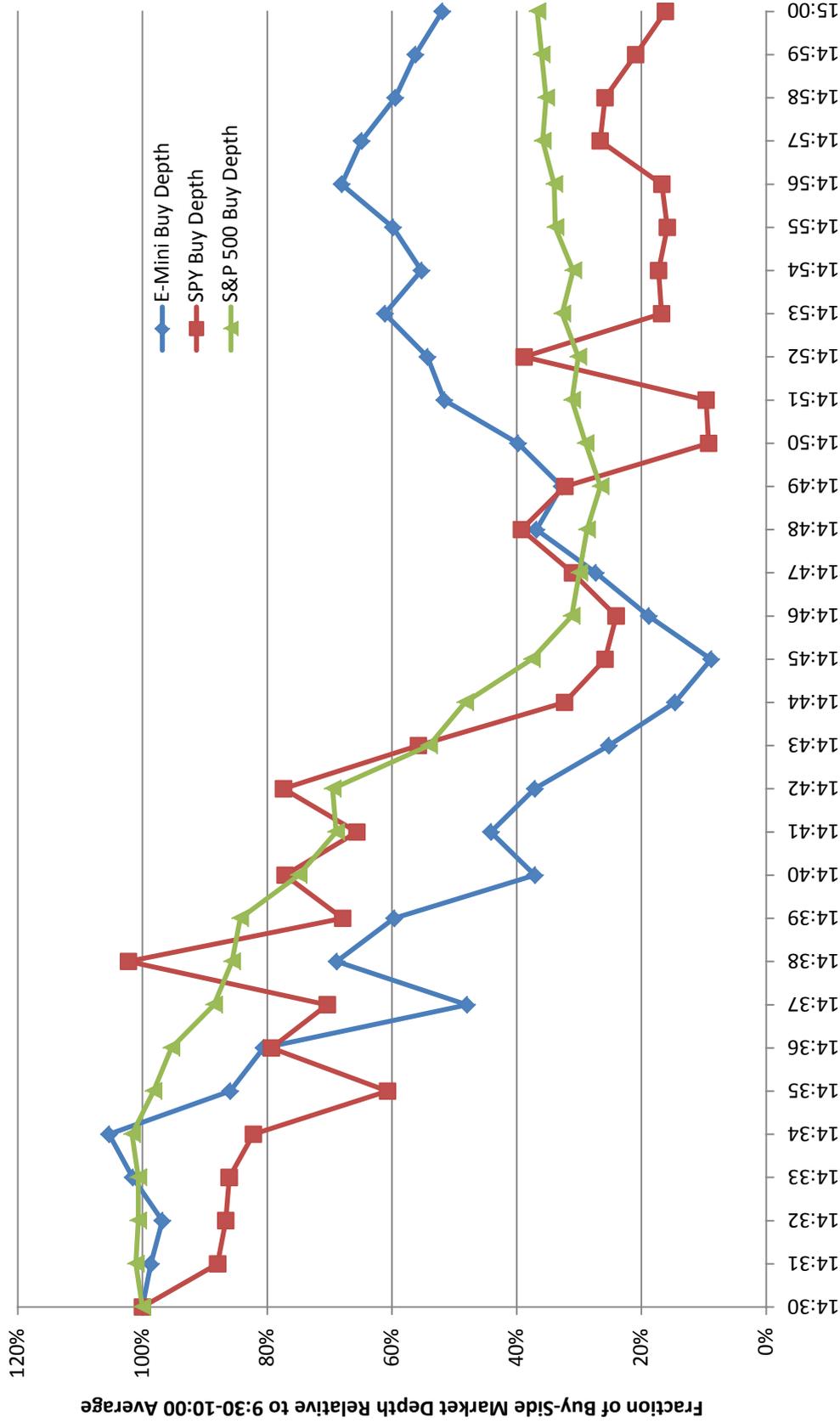


Figure 1: Comparison of Buy-side Market Depth for E-Mini (all quotes), and SPY and Aggregate S&P 500 (within 500 basis points of mid-quote). Source: “Findings regarding the market events of May 6, 2010,” *Reports of the staffs of the CFTC and SEC to the joint advisory committee on emerging regulatory issues*, Figure 1.12.

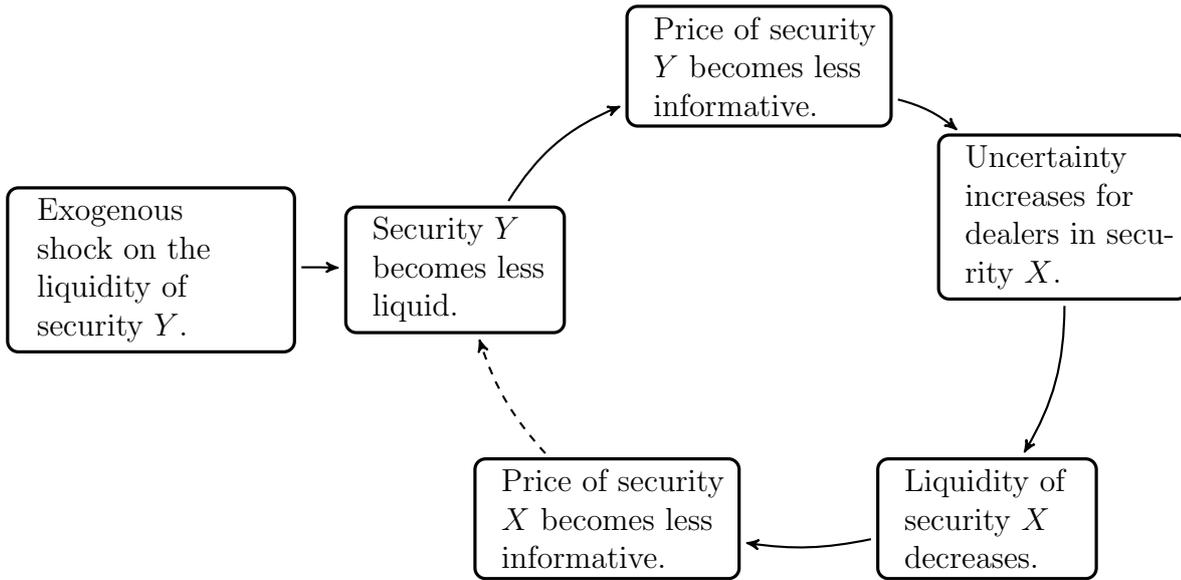


Figure 2: Cross-asset learning and liquidity spillovers.

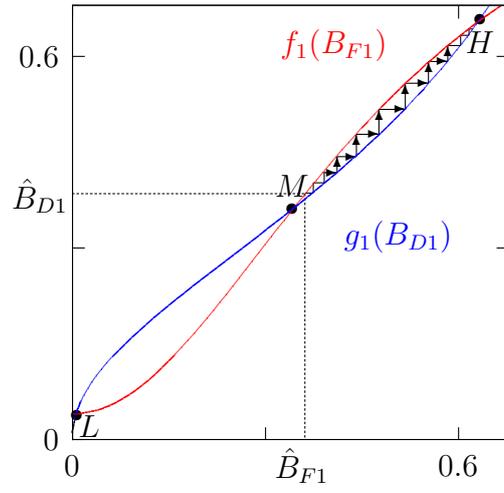


Figure 3: Equilibrium determination with full attention. Parameters' values are as follows: $\gamma_j = d = 1$, $\sigma_{u_j} = 2$, and $\sigma_\eta = .2$.

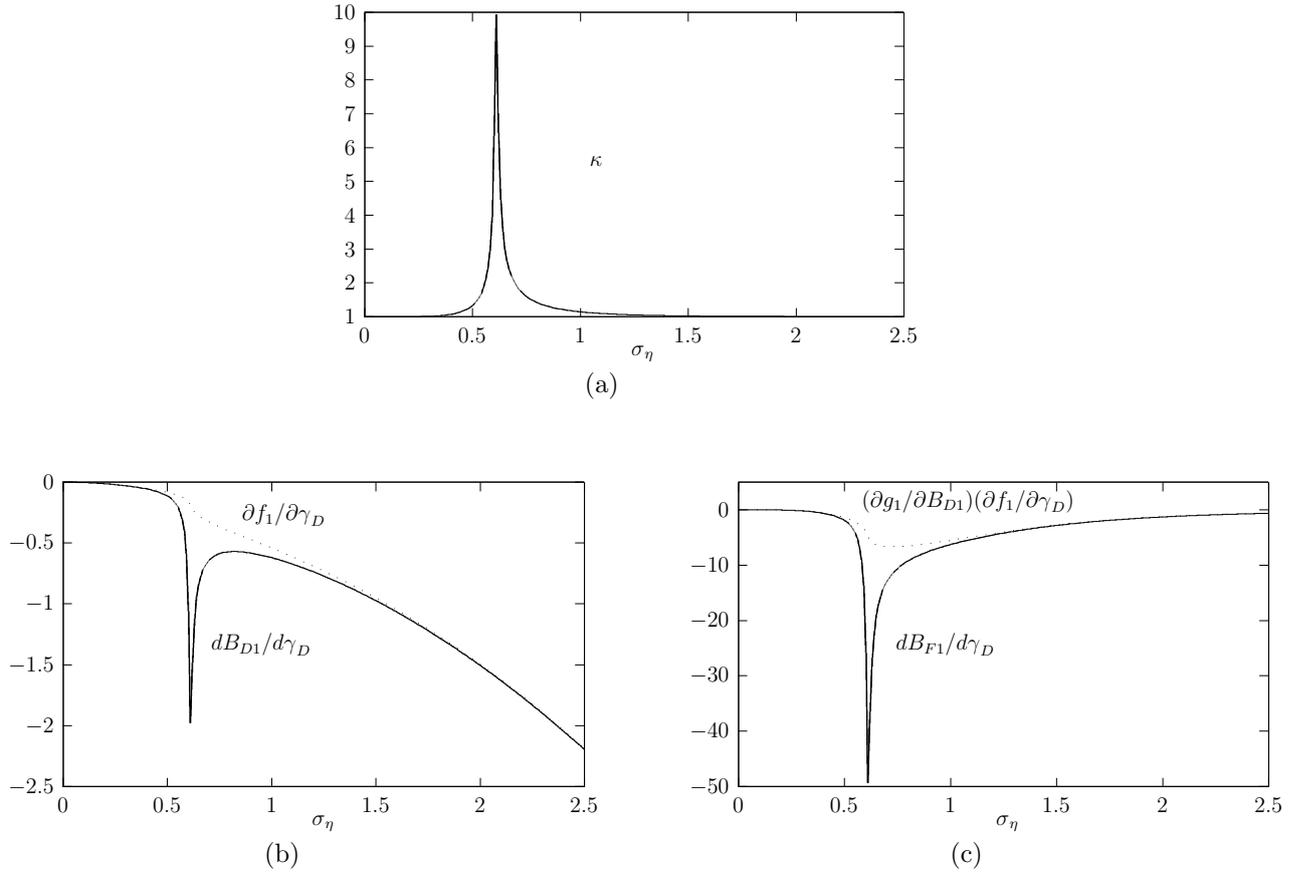


Figure 4: Illiquidity multiplier. In panel (a) we plot κ as a function of σ_η . Panels (b) and (c) show the direct effect (dotted line) and total effect (plain line) of a change in the risk tolerance of the dealers in security D on the illiquidity of securities D and F , respectively as a function of σ_η . Other parameter values are $d = 1$, $\sigma_{u_F} = .1$, $\sigma_{u_D} = 1.6$, $\gamma_D = 1.8$, and $\gamma_F = 1/24$.

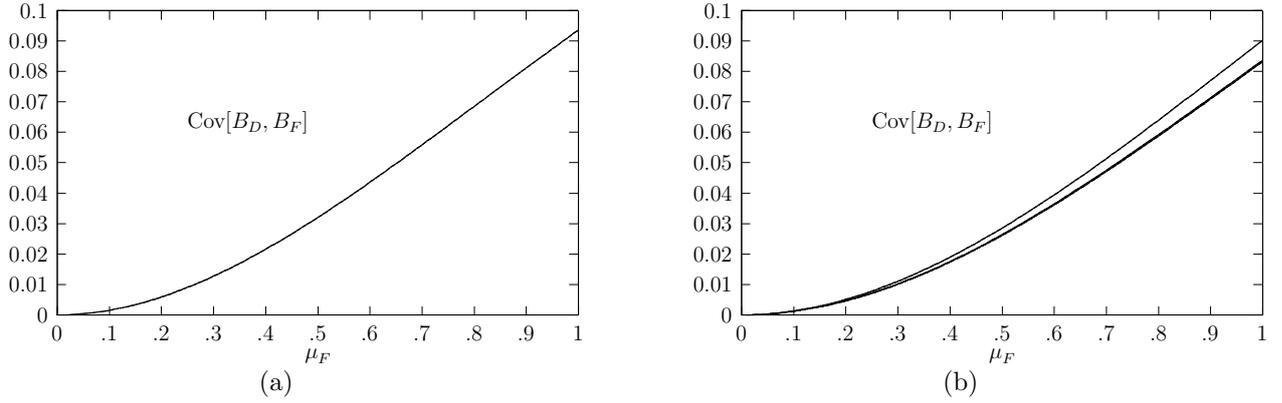


Figure 5: Comovement in illiquidity. The figure displays the covariance between the illiquidity of security F and the illiquidity of security D as a function of μ_F when $d = 0$ (panel (a)) and $d = 0.9$ (panel (b)). In panel (b) the covariance between the illiquidity of the two securities is higher when $\mu_D = 0.9$ (light curve) than when $\mu_D = 0.1$ (bold curve), for all values of $\mu_F > 0$. Other parameter values are $\sigma_{u_F} = \sigma_{u_D} = 1/2$, $\sigma_\eta = 2$, $\gamma_F = 1/2$, and $\mu_D \in \{0.1, 0.9\}$.

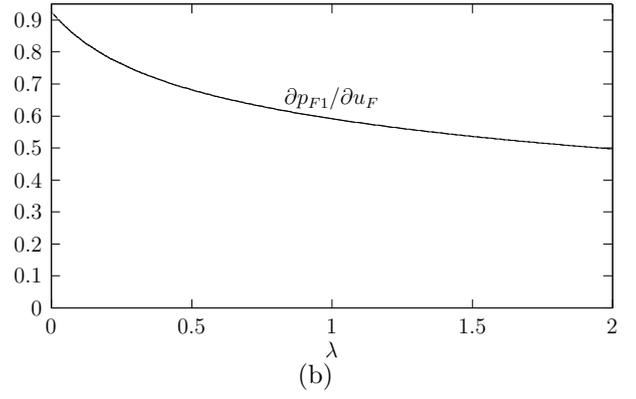
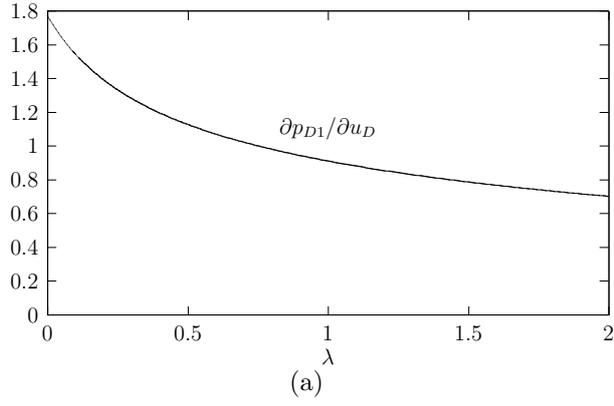


Figure 6: Illiquidity in market D (panel (a)) and F (panel (b)) for $\lambda \in \{0, 0.01, \dots, 2\}$. Other parameter values: $\sigma_{u_j} = 2$, $\sigma_\eta = 1$, $\gamma_j = \gamma^H = 1$, $d = 1$.

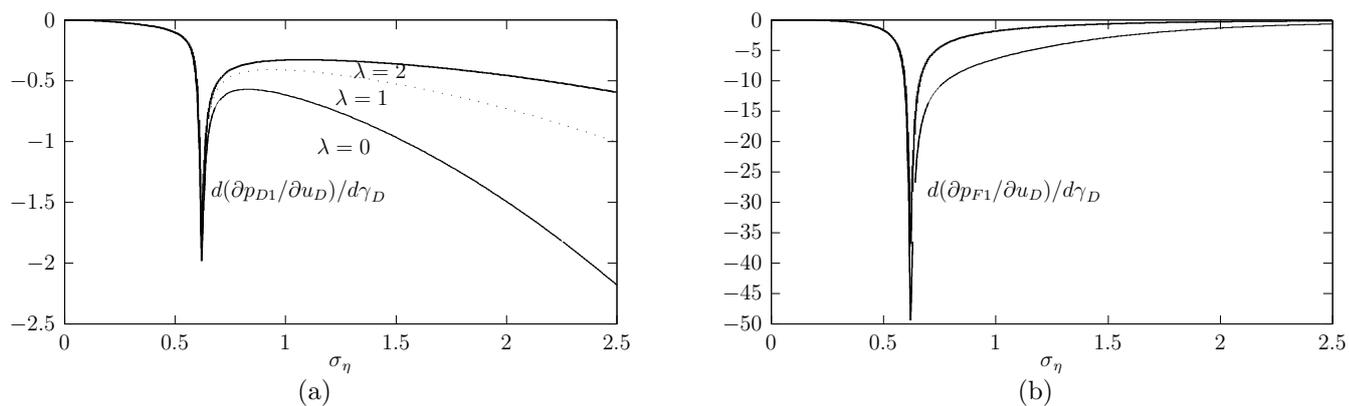


Figure 7: Total effect of an increase in γ_D on the illiquidity in market D (panel (a)) and F (panel (b)) for $\lambda = 0$ (low continuous curve), $\lambda = 1$ (dotted curve), and $\lambda = 2$ (high continuous curve), as a function of σ_η . Other parameter values are $d = 1$, $\sigma_{u_F} = .1$, $\sigma_{u_D} = 1.6$, $\gamma_D = 1.8$, and $\gamma_F = 1/24$

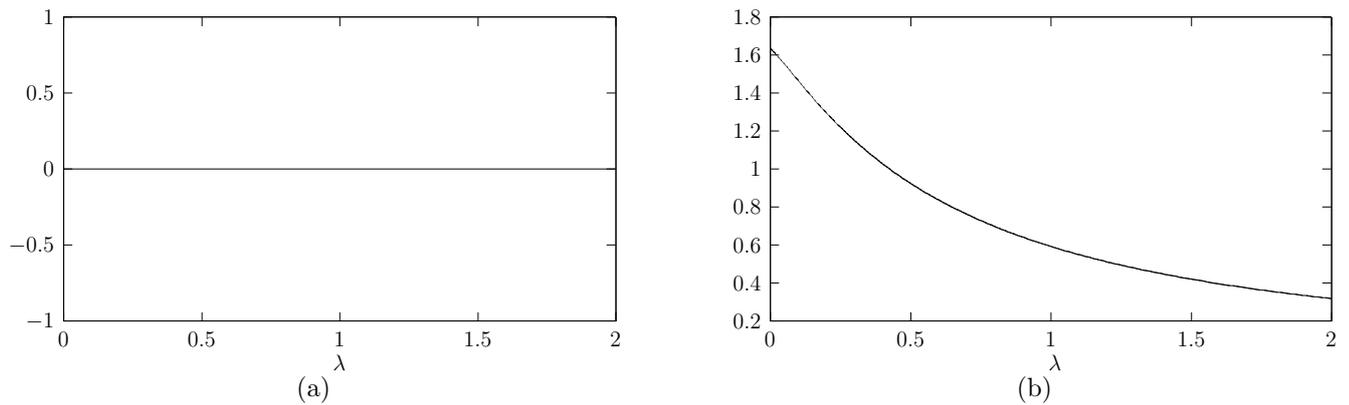


Figure 8: Comovement in illiquidity. The figure displays the covariance between the illiquidity of security F and the illiquidity of security D as a function of λ in the market with only arbitrageurs (panel (a)), and in the market with arbitrageurs and pricewatchers (panel (b)). Parameters' values are as follows: $\gamma_D = \gamma_F = \gamma_H = 1/2$, $\sigma_{u_j} = 1$, $d = .9$, $\sigma_\eta \in \{0.01, 0.02, \dots, 2.5\}$, and $\lambda \in \{0.01, 0.02, \dots, 2\}$.

γ_D	κ	B_{D1}	B_{F1}	Elasticities			
				$\mathcal{E}_{B_{D1},\gamma_D}$	$\hat{\mathcal{E}}_{B_{D1},\gamma_D}$	$\mathcal{E}_{B_{F1},\gamma_D}$	$\hat{\mathcal{E}}_{B_{F1},\gamma_D}$
2.2	1.54	0.19	2.11	-1.54	-1.00	-2.80	-1.81
2	2.16	0.23	2.87	-2.16	-1.00	-3.80	-1.76
1.8	9.94	0.36	5.94	-9.49	-1.00	-14.95	-1.50
1.62	2.35	0.57	11.01	-2.35	-1.00	-2.54	-1.08
1.46	1.65	0.70	13.41	-1.65	-1.00	-1.45	-0.88
1.31	1.39	0.82	15.29	-1.39	-1.00	-1.00	-0.72

Table 1: The table shows the impact of the illiquidity multiplier for different shocks to the risk aversion of dealers in market D . Other parameter values are $d = 1$, $\sigma_\eta = .62$, $\sigma_{u_F} = .1$, $\sigma_{u_D} = 1.6$, $\gamma_D = 1.8$, and $\gamma_F = 1/24$.

$\gamma_j = \gamma^H = 1, \sigma_{w_j} = 2, d = 1, \lambda = 1/2$						
σ_η	$R_D B_D$	$R_F B_F$	$(R_D B_D)^H / (R_D B_D)^L$	$(R_F B_F)^H / (R_F B_F)^L$	$\text{Var}[p_D - p_F]$	$\text{Var}[p_D - p_F]^H / \text{Var}[p_D - p_F]^L$
0.1	{0.395, 0.009}	{0.39, 0.0003}	39.52	982.81	{0.0002, 7.79 × 10 ⁻⁹ }	34, 273
0.2	{0.436, 0.040}	{0.427, 0.006}	10.88	66.62	{0.004, 2.03 × 10 ⁻⁶ }	2, 069
0.25	{0.459, 0.063}	{0.445, 0.015}	7.27	28.04	{0.009, 1.32 × 10 ⁻⁵ }	750
$\gamma_j = \gamma^H = 1, \sigma_{w_j} = 2, d = 1, \lambda = 1$						
σ_η	$R_D B_D$	$R_F B_F$	$(R_D B_D)^H / (R_D B_D)^L$	$(R_F B_F)^H / (R_F B_F)^L$	$\text{Var}[p_D - p_F]$	
0.1	{0.379, 0.009}	{0.377, 0.0003}	37.93	943.63	{4 × 10 ⁻⁵ , 5.01 × 10 ⁻¹⁴ }	8.7 × 10 ⁸
0.2	{0.411, 0.040}	{0.404, 0.006}	10.28	63.03	{7 × 10 ⁻⁴ , 3.1 × 10 ⁻⁹ }	226, 705
0.25	{0.429, 0.063}	{0.417, 0.015}	6.81	26.31	{0.001, 1.1 × 10 ⁻⁷ }	15, 280

Table 2: Liquidity crashes with closely substitutable securities.