Depth of penetration of bubbles entrained by a plunging water jet

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A model is proposed to predict the depth of penetration of the air bubbles entrained by a round water jet impacting into a flat, liquid pool. This depth is shown to be determined only by the initial jet momentum and by the non-monotonic nature of the bubble terminal velocities as a function of their size. The model is shown to be in excellent agreement with measurements of the depth and width of penetration of the bubbles performed over a wide range of jet diameters, velocities, and plunging angles. [S1070-6631(97)00907-0]

Many industrial and environmental processes involve the aeration of a liquid by the entrainment of air bubbles produced when another liquid of the same or different properties impacts on its surface; e.g., a jet plunging into a pool, a breaking wave plunging in the ocean, a droplet impinging on a liquid surface, etc. As a consequence, *plunging jets* have been thoroughly investigated over the years, and an extensive bibliography exists on this issue. For a summary of the most recent studies, see the comprehensive review by Bín.[1] Despite this large body of work, several of the mechanisms involved in this complex two-phase flow are still poorly understood,[2] and the prediction of important parameters such as the volume of entrained air, the bubble size distribution function, or the depth of penetration of the bubbles still relies on semi-empirical correlations valid only over a restrictive range of parameters.

In this Letter, we shall report on some experiments conducted with round plunging jets that reveal interesting new results concerning the depth of penetration of the bubble cloud under a wide range of jet diameters, velocities and plunging angles. We will also propose new scaling laws derived from a simplified model, and we will show that they are in excellent agreement with our experimental observations as well as with previous results. Other issues such as entrainment threshold values, flow rate of air entrained, and bubble size distributions throughout the submerged jet will be presented elsewhere.[3]

The problem concerns the air entrained by a round water jet ejecting from a nozzle of diameter, D, with a mean exit velocity, V_0, plunging at an angle, θ, into a pool located at a distance, h, from the nozzle, as shown in Fig. 1. The inertia, viscous, surface tension, and gravity effects are described by means of the nondimensional Reynolds, Weber, and Froude numbers defined by $\mathrm{Re}=V_0D/\nu$, $\mathrm{We}=\rho V_0^2D/\sigma$, $\mathrm{Fr}=V_0^2/gh$, respectively, where g is the acceleration due to gravity, ρ is the liquid density and σ is the surface tension of water ($\rho_{\mathrm{water}}=1000\ \mathrm{kg/m^3}$ and $\sigma_{\mathrm{water}}=0.073\ \mathrm{kg/s^2}$).

The experiments reported here were carried out in a tank 50 cm tall, 50 cm wide, and 200 cm long. The ratio h/D was always kept constant and equal to 20, and the ratio of the maximum penetration depth to the total depth never exceeded 0.35 in order to minimize the effect of the finite size of the pool. De-ionized water was supplied to the nozzle at a constant pressure through a regulator and a flowmeter. All nozzles consisted of stainless steel needles with length to diameter ratios, L_0/D, larger than 50. The mean flow rate of the jets was measured with high accuracy flow meters, while the exit velocity profile was estimated assuming a fully developed pipe flow. The range of variation of the control parameters are reported in Table I, where subindex 1 corresponds to the continuous entrainment threshold, and 2 to the maximum values tested.

The bubble penetration depth was measured experimentally applying an *edge finding* filter to the average image resulting from 125 video frames acquired over a 13 s span. The video images were recorded with a Sony xc-77RR camera (768 pixels×493 pixels), and processed with the *NIH image 1.60* image processing package.

Let us consider first the case of a jet plunging normal to a flat liquid surface ($\theta=0$). For a given nozzle diameter, when the velocity of the jet, at the point of impact, is above the critical value V_1, the suction generated by the submerged jet is strong enough to overcome the capillary forces, and a shroud of air is entrained into the receiving liquid (Fig. 1). The resulting air bubbles are then transported in both the axial and radial directions by the large scale eddies dominating the evolution of the submerged jet.[4] As the bubbles are further convected by the flow they grow in size, according to the equilibrium established between the break up and coalescence processes,[5] and reach a maximum depth where the buoyancy force surpasses the viscous drag exerted by the downward moving jet, and the bubbles begin to rise to the surface. Experimentally, it is observed that this flow reversal occurs sharply at a location we shall define as the maximum penetration depth, H (see Fig.1). Although at this point a large fraction of the entrained bubbles is seen rising to the surface, we observe that a certain portion of the entrained air, composed of smaller bubbles, is transported to deeper distances into the pool (see Fig. 1).

To estimate the mean velocity of the submerged jet as a function of the downstream location, we model the jet as an equivalent water jet with an initial momentum flux given by $\pi\rho V_{j0}^2D_{j0}^2/4$, where V_{j0} and D_{j0} are the diameter and velocity of the jet at the point of impact with the flat interface. Since most of the reported data correspond to turbulent jets ($\mathrm{Re}>2300$), the assumption $V_{j0}=V_0$, and $D_{j0}=D$ holds. Applying conservation of linear momentum between the impact section and the section of the submerged jet located at

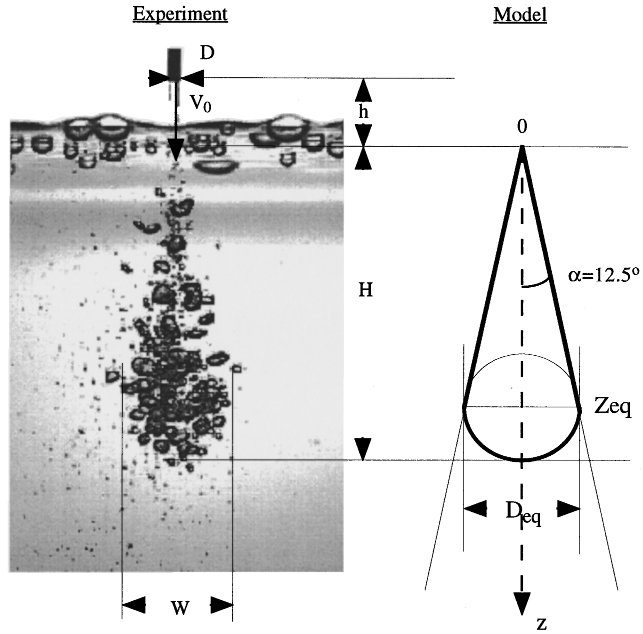

FIG. 1. Plunging jet ($V_0 = 2.4$ m/s, $D = 2.159$ mm, $h/D = 7$ and $\theta = 0°$).

$z = H$, we estimate the mean velocity of the jet $\overline{U_j(z)}$ as:

$$\overline{U_j(z)} = V_0 \frac{D}{D_j(z)}. \qquad (1)$$

Consistent with our experimental observation, and previously reported measurements for high Reynolds numbers,[1,6] we assume that the submerged biphasic jet spreads linearly with a constant half angle, $\alpha \approx 12.5°$, independent of Re (a result well known in turbulent single phase jets[7,8]). The penetration depth H, can then be estimated as the distance downstream where the mean downward velocity of the jet equals the terminal velocity of the bubbles ($\overline{U_j(H)} = U_T$).[6] Thus,

$$\frac{H}{D} = \frac{1}{2 \tan \alpha} \frac{V_0}{U_T}. \qquad (2)$$

The remaining problem is to calculate the terminal velocity U_T of the bubbles that reach the maximum penetration depth, H. First, recall that beyond the point defined as the maximum penetration, we observed a sharp drop in the bubble size whereby bubbles much smaller than those rising were seen being transported deeper into the pool by the submerged jet. The explanation for this sharp transition in bubble size observed at $z = H$ rests on the dependency of the bubble terminal velocity on its size. If this velocity increased monotonically with the size of the bubbles, the bubbles would have been sorted out by the pull of gravity rising to the surface from depths which would increase as their size is decreased. The actual behavior of the bubbles in the submerged jet is drastically different since the terminal velocity follows a non-monotonic law and undergoes a marked change of behavior between the gravity and viscosity dominated regime (Stokes regime), and the purely buoyancy dominated regime (Taylor regime) as shown in Fig. 2 (see Mendelson,[9] Clift *et al.*,[10] and Maxworthy *et al.*[11]). In our experiments, $\mathrm{Mo} \approx 2.5 \times 10^{-11}$. Thus, as the mean jet velocity decreases, it eventually reaches the velocity corresponding to the minimum in Fig. 2 (note that the terminal velocity to this minimum is $U_T \approx 22$ cm/s independent of the Morton number Mo), at which point all bubbles with sizes greater than 1 mm will suddenly rise to the surface (see Fig. 2), thereby, producing a sharp drop in the bubble size from 6 mm to 1 mm. Our preliminary measurements of the bubble size at the

TABLE I. Range of variation of the different relevant parameters.

D(mm)	V_1 (m/s)	V_2(m/s)	Re_1	Re_2	We_1	We_2	Fr_1	Fr_2
2.16	1.9	6.3	4102	13601	106	1173	8	93
1.60	2.1	7.3	3360	11680	96	1168	14	169
1.19	2.5	12	2978	14292	101	2349	26	616
0.838	2.8	15	2346	12570	89	2582	47	1368
0.584	3.9	13	2278	7591	121	1351	132	1474
0.394	5.2	13	2049	5122	145	912	349	2186
0.241	8.0	18	1928	4338	211	1069	1353	6852

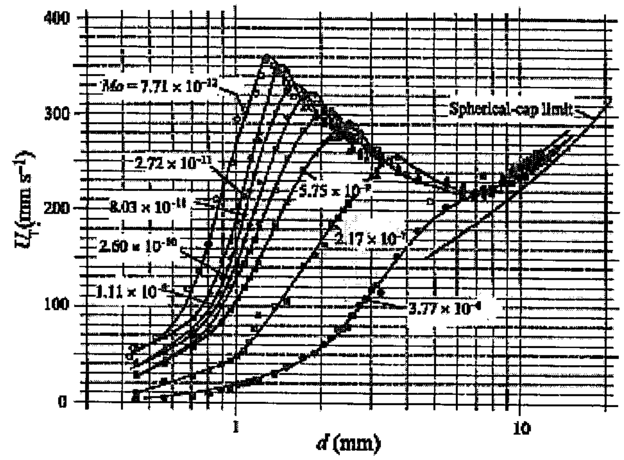

FIG. 2. Raw data curves of the terminal velocity of U_T vs the diameter d of the bubble for selected values of the Morton number $\mathrm{Mo} = g\nu^4\rho^3/\sigma^3$ (Fig. 4 from reference 11). Note that the terminal velocity corresponding to the minimum is $U_T \approx 22$ cm/s independent of the Morton number Mo).

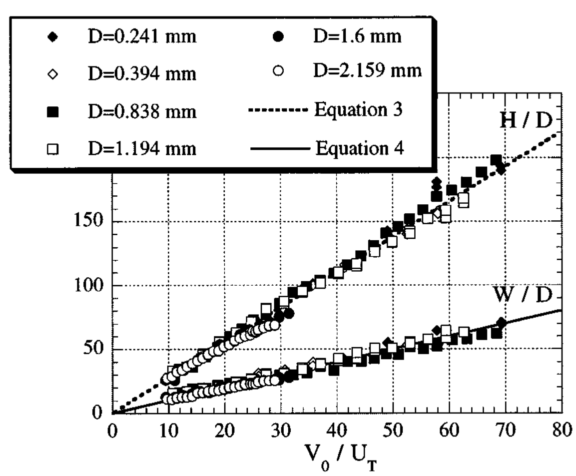

FIG. 3. Penetration depth H/D and width W/D as a function of V_0/U_T for different diameters.

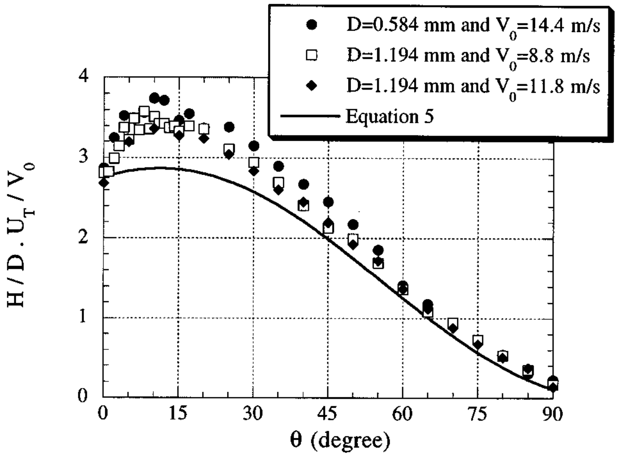

FIG. 4. Non dimensional penetration depth H/D. U_T/V_0 as a function of the penetration angle θ, for different diameters and velocities.

maximum penetration depth ($z=H$) agree qualitatively with the above values. In our experiments, we measure a transition in the bubble size from approximately 4 mm to 0.5 mm. The reasons for the observed differences could be due to two main effects: an error in our estimation of σ due to surfactant contamination, and bubble-bubble interaction or clustering effects that are not considered here. The data in Fig. 2 correspond to isolated single bubble experiments, however, one could modify the expression for the rise velocity of the bubble in a swarm as $U_T=U_{T_{\text{bubble}}}\sqrt{1-\alpha_G}$, where α_G is the gas volume fraction. Measurements of the void fraction and its influence on the prediction of the maximum penetration depth will be presented elsewhere.[3]

The above discussion implies that the terminal velocity of the bubbles located at the maximum depth is determined exclusively by the value of the minimum in Fig. 2. Therefore, it depends only on the density and surface tension of the liquid and is independent of the jet's parameters. Taking $U_T=0.22$ m/s in Eq. (2), leads to $H/D\approx 10.25V_0$ (which is very close to the well known result of Suciu and Smigelschi[6] $H/D\approx 10V_0$). The penetration reported by these authors and the one evaluated by the above method corresponds to the distance Z_{eq} in Fig. 1. However, the way we measured the *maximum* penetration H also involves the turbulent fluctuations at the centerline of the jet. The largest scales of these fluctuations at the location $z=Z_{\text{eq}}$, are of the order of magnitude of $D_{\text{eq}}=2\tan\alpha Z_{\text{eq}}$ (see Fig. 1). Equation (2) can then be further corrected to give:

$$\frac{H}{D}=\frac{1+\tan\alpha}{2\tan\alpha}\frac{V_0}{U_T}. \tag{3}$$

The maximum width of the bubble cloud, W, can then be evaluated as the width of the jet at the Z_{eq} location (Fig. 1):

$$\frac{W}{D}=\frac{V_0}{U_T}. \tag{4}$$

In Fig. 3, we present the measurements of the penetration depth, H/D, and of the width W/D for the whole range of diameters and velocities (Table I) and compare them with the predictions given by Eqs. (3) and (4). Noteworthy in these results is the excellent agreement of all the data with the predictions, where the bubble terminal velocity has been assumed to be in all cases that of the minimum in Fig. 2 ($U_T=22$ cm/s), and the spreading angle of the submerged jet $\alpha=12.5°$.

Finally, the above argument can be extended to jets plunging into the free liquid surface at an arbitrary angle, θ. The maximum penetration of the cloud is then calculated from simple geometrical relations as:

$$\frac{H}{D}=\frac{(1+\tan\alpha)\cos\theta+\tan\alpha\sin\theta}{2\tan\alpha}\frac{\cos(\theta-\alpha)}{\cos\alpha}\frac{V_0}{U_T}. \tag{5}$$

The function $(H/D)(U_T/V_0)$, that only depends on θ, is presented in Fig. 4. The comparison of the predicted values [Eq. (5)] with the measurements reveals an agreement within 15% for $\theta>30°$, while the ''overshoot'' observed for $0<\theta<30$ is underestimated by our model.

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