Redundancy Transmission System Based on Multidisciplinary Object Compatibility Design Optimization

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***Abstract*—In studying multidisciplinary design optimization method for non-hierarchic system, Multidisciplinary Object Compatibility Design Optimization method based on simulated annealing algorithm is presented. In order to coordinate the independent optimization of subsystems, the compatibility constraint in system level and compatibility objective in subsystem work together. As optimization process continued, the coupling relationship between system level and different subsystems is gradually improved by state accepting function which is embedded in compatibility constraint. In this way, abnormal program termination and premature convergence will be avoided and ideal global optimal solution will be achieved effectually. Then the method is used in the optimization design of TR-B triple-redundancy transmission system. The multidisciplinary object compatibility design optimization model is established and the comprehensive optimal solution is obtained which meets the matching relationship of gear teeth, strength requirement and dynamic requirement, etc.**

***Index Terms*—Multidisciplinary Design Optimization (MDO); Simulated Annealing (SA); Redundancy Transmission**

I. Introduction

Multidisciplinary Design Optimization (MDO) [1-4] is a kind of design method for complex systems, its basic idea is to take effective design optimization strategies to solve complex systems, and fully considerate synergy effects produced by the interaction between disciplines. The optimization cycle can be reduced through parallel design and we can also obtain the overall optimal solution at the same time. MDO approach can solve complex engineering system more effectively by decomposing a large complex system into several smaller, more manageable subsystems. In this way, the scale of optimization design can be reduced and a potential parallel processing environment can be made available. In recent years, MDO have been successfully applied to handle a number of engineering problem [5-9]. In general, complex multidisciplinary system can be deco posited into many subsystems belonging to different disciplines. Typically, according to the relationship between subsystems, there are three types of complex system [2] [10]: hierarchic system, non-hierarchic system and hybrid-hierarchic system. Most of the complex systems including subsystems are non-hierarchic systems in nature where the intercoupling subsystems are at the same level. Multidisciplinary Object Compatibility Design Optimization includes system level and subsystem level optimization, it is an effective method to deal with the design optimization of non-hierarchic system which have system level optimization objective. The goal of the optimization in subsystem is to reduce the discrepancy with optimal solution in system level while satisfying the constraints of individual discipline. At the system level optimization, the goal is to minimize the system objective function while satisfying the constraints. Compatibility constraint in system level optimization is constructed to coordinate the coupling relationship of the subsystems and reduce the discrepancy between subsystems. In order to improve the quality and stability of the solution, Simulated Annealing algorithm is introduced in the system level optimization and state accepting function is embedded in compatibility constraint to control the optimization process. In this way, the problem that coupling relationship in subsystems satisfied early and get stuck at local optimal solution will be avoided and ideal global optimal solution will be achieved effectually. And the method is proved by used in the optimization design of TR-B triple-redundancy transmission system. TR-B Triple-redundancy transmission system use compound gear train structure consisted of two stage differential gear train and fixed axis gear train. The system is decomposed into two subsystems and the multidisciplinary design optimization model is established which contains two subsystems and the optimal scheme is obtained which meets the matching relationship of gear teeth, strength requirement and dynamic requirement, etc.

II. Proposed Scheme

A. Basic Idea

To solve the multidisciplinary design optimization problem which includes a system level and subsystem level optimization, Multidisciplinary Object Compatibility Design Optimization method based on simulated annealing algorithm for non-hierarchy complex system is presented. The basic idea is: First, the complex system is decomposed into several independent

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subsystems which corresponding to different disciplines, then the optimization can be carried out in their respective subspace. To the coupling relationship of design variables between subsystems, an approach of variable substitution is introduced, so each subsystem can be optimized independently. By variable substitution, the substitute variables take place of design variables in subsystem optimization. After optimization of subsystem, the optimum results of the substitute variables are transferred to system level. The system level optimization target decided by global design requirements. Compatibility constraint in system level is constructed corresponding to the compatibility objective in subsystems. As the optimization process continued, discrepancy between the optimum of substitute variables from each subsystem is reduced by the combination of compatibility constraints in system level optimization and compatibility objective in subsystem level optimization. Finally, the ideal global optimal solution will be achieved while the individual discipline's constraints are satisfied.

B. Definition of Variables

In the process of modeling, the method of variable substitution is introduced to deal with coupled interaction amongst design variables in multidisciplinary design optimization. All kinds of design variables and their substitution variables of the Multidisciplinary Object Compatibility Design Optimization method are defined as follows.

Sharing variable: The set of sharing variables contains design variables that are needed by each subsystem and system level model. The sharing variable vector can be expressed as $\mathbf{X}_{\mathrm{SH}}=[x_{\mathrm{SH1}},x_{\mathrm{SH2}},\cdots]^T$. A sharing variable will get different optimum results from system level optimization and subsystem optimization. Then the coupling problem of sharing variable emerged due to discrepancy between the different optimum results. To solve this kind of coupling problem, an approach of variable substitution is introduced. Sharing substitute variable is proposed in each subspace corresponding to sharing variable. The sharing substitute variable vector in subsystem i can be expressed as $\mathbf{X}_{\mathrm{SH}i}=[x_{\mathrm{SH}i1},x_{\mathrm{SH}i2},\cdots]^T$ and the sharing substitute variable vector in system level is $\mathbf{Z}_{\mathrm{SH}i}=[z_{\mathrm{SH}i1},z_{\mathrm{SH}i2},\cdots]^T$. In this way, subsystem optimal solution would not be influenced each other because the sharing substitute variables in each subsystem are different variables.

State variable: State variable is determined by optimum solutions of a particular subsystem and act as design variable in other subsystem. In other words, it is the output of a subsystem and also used as input in another subsystem. The set of state variable vector in subsystem j can be expressed as $\mathbf{Y}_i=[y_{i1},y_{i2},\cdots]^T$. The coupling problem of state variable emerged due to discrepancy between optimum solution of state variable while acting as an output from subsystem and as a design variable in other subsystem. To solve this kind of coupling problem, an approach of variable substitution is introduced. Sharing substitute variable is proposed in each subspace corresponding to sharing variable. In subsystem i the design variables that are represented as substitute variable of the state variable from subsystem j are called state substitute variable vector, and expressed as $\mathbf{X}_{\mathrm{Y}ij}=[x_{\mathrm{Y}ij1},x_{\mathrm{Y}ij2},\cdots]^T$ (where $i,j=1,2,\cdots N_{\mathrm{CA}}, j\neq i$, N_{CA} is the total number of subsystem); State substitute variables in system level which corresponding to state variables of subsystem i are expressed as $\mathbf{Z}_{\mathrm{Y}i}=[z_{\mathrm{Y}i1},z_{\mathrm{Y}i2},\cdots]^T$.

Local variable: The variables only have effect on one subsystem are called local variables. Local variables in subsystem i expressed as $\mathbf{X}_{\mathrm{D}i}=[x_{\mathrm{D}i1},x_{\mathrm{D}i2},\cdots]^T$.

Thus, the system-level design variable vector can be expressed as: $\mathbf{Z}=[\mathbf{Z}_{\mathrm{SH}i},\mathbf{Z}_{\mathrm{Y}i}]^T$; the design variable vector of subsystem i can be expressed as: $\mathbf{X}_i=[\mathbf{X}_{\mathrm{SH}i},\mathbf{X}_{\mathrm{D}i},\mathbf{X}_{\mathrm{Y}ij}]^T$ ($j=1,2,\cdots N_{\mathrm{CA}}, j\neq i$).

C. Approach of Object Compatibility Based on Simulated Annealing Algorithm

In MDO, the coupling problem between system level and each subsystem is a key problem during optimization design. On one hand, the issue is to reduce the discrepancy between coupling variables in different subspaces while satisfying discipline's constraints. On the other hand, optimization process should be controlled by an efficient way to avoid abnormal program termination or premature convergence caused by getting stuck at local optimal solution. Thus, more flexible and slack constraint or control strategy is required.

Simulated annealing algorithm (SA) [11-14] is developed from physical process of annealing, belongs to heuristic algorithm. In recent years, SA have been successfully applied to handle a number of optimization problem [15-18]. This algorithm explores the analogy between the search for a minimum in an optimization process and the gradual cooling of a metal into a minimum energy crystalline structure. And SA is an approach to search the global optimal solution that accept most worsening moves at the start of SA to avoid entrapment in poor local optimal solution. At each stage, the new solution taken from the feasible region of the optimization problem is accepted as the new current solution if it has a lower or equal objective function value; if it has a higher value it is accepted with a probability that decreases as the difference in the objective function values increases and as the temperature of the method decreases. Thus at the start of SA, most uphill moves are accepted, but at the end only improving ones have an opportunity to be accepted. Since the new solution generated randomly, SA can avoid getting stuck at local optimal solution effectively.

In consideration of the characteristic of SA mentioned above, SA is introduced and Multidisciplinary Object Compatibility Design Optimization based on simulated annealing algorithm is proposed.

In the process of the system optimization, the goals of the compatibility constraints of system-level and compatibility objective of subsystems are to coordinate

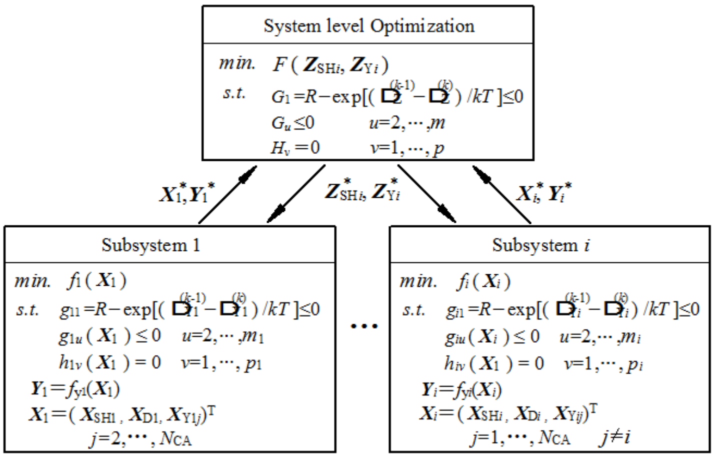

Figure 1. Model of multidisciplinary object compatibility design optimization method based on SA

the coupling relationship of the sub-disciplines and reduce the discrepancy between subsystems. The compatibility constraint of system level which is constructed by state accepting function is

$$G = R - \exp[(\Delta_Z^{(k-1)} - \Delta_Z^{(k)})/kT] \le 0 \quad (1)$$

where R is random number between(0,1); k is the number of iteration; T is the simulated annealing temperature; $\Delta_Z^{(k)}$ is the energy function of SA in system level optimization, can be expressed as

$$\Delta_Z^{(k)} = \sum_{i=1}^{N_{CA}} \left\| \mathbf{X}_{SHi}^* - \mathbf{Z}_{SHi}^{(k)} \right\| + \sum_{i=1}^{N_{CA}} \sum_{\substack{j=1 \\ j \ne i}}^{N_{CA}} \left\| \mathbf{X}_{Yij}^* - \mathbf{Z}_{Yi}^{(k)} \right\| + \sum_{i=1}^{N_{CA}} \left\| \mathbf{Y}_i^* - \mathbf{Z}_{Yi}^{(k)} \right\| \quad (2)$$

where, $\mathbf{X}_{SHi}^*, \mathbf{X}_{Yij}^*, \mathbf{Y}_i^*$ are current solution of sharing substitute variable, state substitute variable and state variable in subsystem i, respectively; N_{CA} is the total number of subsystem; k is the number of iteration. The state accepting function can adjust compatibility constraint to avoid too rigidly to be satisfied during initial stage of system level optimization. The coupling problem between design variable in subsystem and corresponding design variable in system is improved by subsystem compatibility objective function, which is expressed as

$$f_i = \left\| \mathbf{X}_{SHi} - \mathbf{Z}_{SH}^* \right\| + \sum_{\substack{j=1 \\ j \ne i}}^{N_{CA}} \left\| \mathbf{X}_{Yij} - \mathbf{Z}_{Yi}^* \right\| \quad (3)$$

where, $\mathbf{Z}_{SH}^*, \mathbf{Z}_{Yi}^*$ are current solution of sharing substitute variable and state substitute variable in system level optimization, respectively; N_{CA} is the total number of subsystem. The coupling problem between state function in subsystem and corresponding state variable in system is improved by compatibility constraints, which is expressed as

$$g_{i1} = R - (\Delta_{Yi}^{(k-1)} - \Delta_{Yi}^{(k)})/kT \le 0 \quad (4)$$

where, $\Delta_{Yi}^{(k)}$ is the energy function of SA in subsystem optimization, can be expressed as

$$\Delta_{Yi}^{(k)} = \left\| \mathbf{Y}_i^{(k)} - \mathbf{Z}_{Yi}^* \right\| \quad (5)$$

In this way, at the system level optimization, the goal is to minimize the system objective function while satisfying the compatibility constraints. At the subsystem level, the goal of the optimization is to minimize the subsystem compatibility objective function while satisfying the compatibility constraints and other constraints of that subsystem, then improve the coupling behavior between system and subsystem. By compatibility constraints of system level and compatibility objective function of subsystem level, the coupling relationship is satisfied, and finally the global optimum is found while satisfying all constraints of each subsystem.

Based on the above analysis, the system optimization model can be expressed as in Fig.1 and the flowchart of the Multidisciplinary Object Compatibility Design Optimization method based on simulated annealing algorithm is shown in Fig. 2.

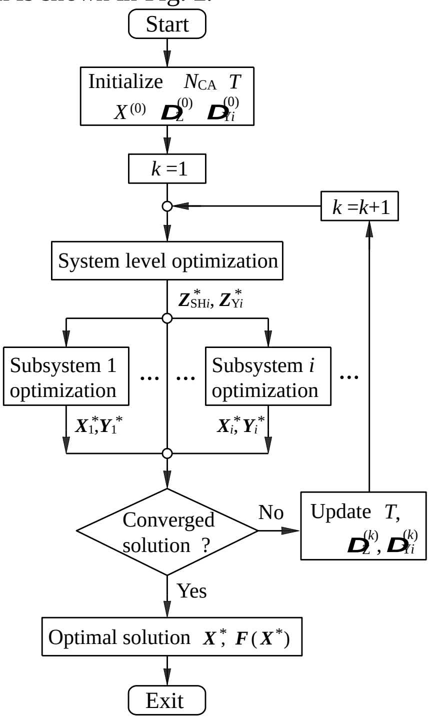

Figure 2. Flowchart of Multidisciplinary Object Compatibility Design Optimization method based on SA

III. System Design

Redundancy transmission system [19] [20] is a technical measure using multiple redundant backup power input to ensure important system equipment not due to failure to stop running, it has the advantages of safety and reliable, less affected by environmental factors, precise movements, fast response and high strength, etc, and have a lot of potential in fields such as aerospace and other high reliability requirements. The TR-B triple-redundancy transmission system in Fig. 3 using three sets of parallel input motors, each serves as back up to the other, when one or two motor failure caused part of the input shaft locked, others can still working properly to maintain a stable output and ensure system security and stability under unexpected failures. Triple-redundancy transmission system use compound gear train structure consisted of two stage differential gear train and fixed axis gear train. Multidisciplinary Object Compatibility Design Optimization method based on simulated annealing algorithm is introduced to the optimization of the TR-B triple-redundancy transmission system. The system is decomposed into two subsystems: differential gear train subsystem and fixed axis gear train subsystem. System level and subsystem-level optimization models are established.

A. Optimization Model of System Level

1) Design Variable

The design variable of system level include sharing substitute variables: modulus of differential gear train m_0; the number of teeth Z_6 , Z_7 ; modulus of fixed axis gear train m_1 , m_2 , m_3 ; the number of teeth Z_1 , Z_2 , Z_3 , Z_4 , Z_5 , Z_{11} . state substitute variables: the diameter of addendum of gear 8 d_{f8} ; the transmission ratio i_{14} .

2) Objective Function

The objective function for system-level optimization is to minimize the volume of the transmission system in order to obtain an effective solution with compact size and light weight. It can be expressed as follows:

$$F(\mathbf{Z}) = \sum_{i=1}^{12} V_i = m_1^3 Z_1 (Z_1^2 + Z_4^2) + (m_2 Z_2)^3 + (m_3 Z_3)^3 + [(m_2 Z_5)^2 - d_{f8}^2] m_2 Z_2 + [(m_3 Z_{11})^2 - d_{f8}^2] m_3 Z_3 + 2 m_0^3 Z_{\min} \left[3Z_7^2 + Z_6^2 + 4(Z_6 + 2Z_7 + 1) \right] \quad (6)$$

where, V_i is the volume of gear i ; $Z_{\min} = \min(Z_6, Z_7)$.

3) Constraint Function

Constraint functions of system level are as follows:

Compatibility constraints

$$G_1 = R - \exp[(\Delta_Z^{(k-1)} - \Delta_Z^{(k)}) / kT] \le 0, \quad (7)$$

where,

$$\Delta_Z^{(k)} = \sum_{i=1}^{2} \left\| \mathbf{X}_{\mathrm{SH}i}^* - \mathbf{Z}_{\mathrm{SH}i}^{(k)} \right\| + \sum_{i=1}^{2} \sum_{\substack{j=1 \\ j \ne i}}^{2} \left\| \mathbf{X}_{\mathrm{Y}ij}^* - \mathbf{Z}_{\mathrm{Y}i}^{(k)} \right\| + \sum_{i=1}^{2} \left\| \mathbf{Y}_i^* - \mathbf{Z}_{\mathrm{Y}i}^{(k)} \right\| .$$

Teeth matching condition: Keep the three transmission equivalent, that is to get matching relationship of gears for same transmission ratio of each input when working individually. The relationship between input and output is formulated as in:

$$n_{\mathrm{X}} = \frac{1}{Z_{12} + Z_9} \begin{bmatrix} \dfrac{Z_1 Z_6 Z_9}{Z_4 (Z_6 + Z_8)} & 0 & 0 \\ 0 & \dfrac{Z_2 Z_8 Z_9}{Z_5 (Z_6 + Z_8)} & 0 \\ 0 & 0 & \dfrac{Z_{12} Z_3}{Z_{11}} \end{bmatrix} \begin{bmatrix} n_{\mathrm{I}} \\ n_{\mathrm{II}} \\ n_{\mathrm{III}} \end{bmatrix} \quad (8)$$

where, n_{X} is output revs; n_{I} , n_{II} , n_{III} are input revs of the three input shaft I, II, III, respectively.

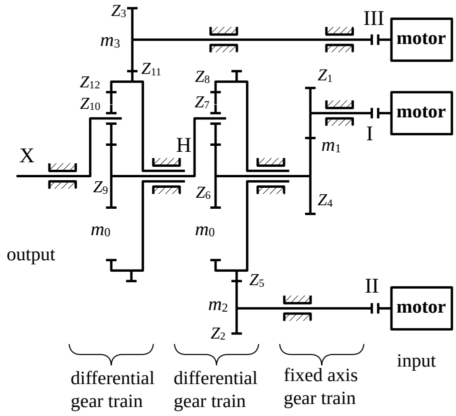

Figure 3. The diagram of TR-B triple-redundancy transmission system

Due to structure constraints, we can give the relationship between gears: $Z_6 = Z_9$, $Z_7 = Z_{10}$, $Z_8 = Z_{12}$. Based on concentric conditions $Z_8 = Z_6 + 2Z_7$, establish variable association, exchanging Z_8 in the model. So the constraint function of teeth matching condition can be expressed as follows:

$$G_2 = Z_6 Z_1 / Z_4 - (Z_6 + 2Z_7) Z_2 / Z_5 = 0 \quad (9)$$

$$G_3 = 2 Z_3 Z_5 (Z_6 + Z_7) - Z_2 Z_6 Z_{11} = 0 \quad (10)$$

B. Optimization Model of the Differential Gear Train Subsystem

1) Design Variable

The design variables of this subsystem include sharing substitute variable: modulus of differential gear train m_0 ; the number of teeth Z_6 , Z_7 ; state substitute variables: the transmission ratio i_{14} .

2) Objective Function

The optimization objective of the differential gear train subsystem is compatibility objective, the aim of which is to satisfy coupling relationship between system level and this subsystem. It can be expressed as follows:

$$f_1 = \left\| \mathbf{X}_{\mathrm{SH1}} - \mathbf{Z}_{\mathrm{SH}}^* \right\| + \left\| \mathbf{X}_{\mathrm{Y12}} - \mathbf{Z}_{\mathrm{Y1}}^* \right\| \quad (11)$$

3) Constraint Function

Constraint functions of the differential gear train subsystem are as follows:

Compatibility constraints

$$g_{11} = R-(\Delta_{Y1}^{(k-1)} - \Delta_{Y1}^{(k)})/kT \le 0 \qquad (12)$$

where, $\Delta_{Y1}^{(k)} = \left\|\mathbf{Y}_1^{(k)} - \mathbf{Z}_{Y1}^*\right\|$.

Assembly conditions of differential gear train

$$g_{12} = \mathrm{mod}[2(Z_6+Z_7), C_s] - 0.5 \le 0 \qquad (13)$$

where, C_s is number of planetary gear, $C_s = 3$.

Adjacency conditions of differential gear train.

$$g_{13} = Z_7 + 2 - (Z_6 + 3Z_7)\cdot \sin(\pi/3) \le 0 \qquad (14)$$

Gear-tooth bending fatigue strength condition

$$g_{14} = \frac{2K_A K_\alpha K_\beta K_v T_{ca} Y_{FS}}{\varphi_d m_0^3 Z_{min}^2} - [\sigma_F] \le 0 \qquad (15)$$

where, K_A, K_α, K_β and K_V are application factor, load distribution factor of gear teeth, longitudinal load distribution factor of gear teeth and dynamic load factor, respectively; Y_{FS} is tooth form factor; φ_d is coefficient of tooth width; $Z_{min} = \min(Z_6, Z_7)$; $[\sigma_F]$ is allowable bending fatigue stress; T_{ca} is calculation torque which can expressed as follows:

$$T_{ca} = \frac{k_C}{C_S}\frac{Z_7}{Z_6} i_{14}\eta T_1 \qquad (16)$$

where, k_C is uneven load factor load sharing coefficient; η is mechanical efficiency; C_s is number of planetary gear; T_1 is input torque of the input shaft I.

Gear-tooth contact fatigue strength condition

$$g_{15} = Z_H Z_E Z_\varepsilon \sqrt{\frac{2K_A K_\alpha K_\beta K_v T_{ca}(u+1)}{\varphi_d m_0^3 Z_{min}^3 u}} - [\sigma_H] \le 0 \qquad (17)$$

where, Z_H is pitch region coefficient; Z_E is elasticity coefficient; Z_ε is coincidence coefficient; u is tooth number ratio; $[\sigma_H]$ is allowable contact fatigue stress.

4) State Variable

State variable of the differential gear train subsystem is the diameter of addendum of gear 8:

$$d_{f8} = m_0(Z_6 + 2Z_7) + 2.5m_0 \qquad (18)$$

B. Optimization Model of the Fixed Axis Gear Train Subsystem

1) Design Variable

The design variables of this subsystem include sharing substitute variable: modulus of the fixed axis gear train m_1, m_2, m_3, the number of teeth Z_1, Z_2, Z_3, Z_4, Z_5, Z_{11}; state substitute variables: the diameter of addendum of gear 8 d_{f8}.

2) Objective Function

The optimization objective is to satisfy coupling relationship between system level and this subsystem. It can be expressed as follows:

$$f_2 = \left\|\mathbf{X}_{SH2} - \mathbf{Z}_{SH}^*\right\| + \left\|\mathbf{X}_{Y21} - \mathbf{Z}_{Y2}^*\right\| \qquad (19)$$

3) Constraint Function

Constraints to be met are as follows:

Compatibility constraint

$$g_{21} = R-(\Delta_{Y2}^{(k-1)} - \Delta_{Y2}^{(k)})/kT \le 0 \qquad (20)$$

where, $\Delta_{Y2}^{(k)} = \left\|\mathbf{Y}_2^{(k)} - \mathbf{Z}_{Y2}^*\right\|$.

Geometric constraints

$$g_{22} = 1.2d_{f8} - m_2 Z_5 \le 0 \qquad (21)$$

$$g_{23} = 1.2d_{f8} - m_3 Z_{11} \le 0 \qquad (22)$$

Gear-tooth bending fatigue strength condition

$$g_{24} = \frac{2K_A K_\alpha K_\beta K_v T_j Y_{FS}}{\varphi_d m_j^3 Z_j^2} - [\sigma_F] \le 0 \quad (j=1,2,3) \qquad (23)$$

where, T_1, T_2, T_3 are input torque of the input shaft I, II, III, respectively.

Gear-tooth contact fatigue strength condition

$$g_{25} = Z_H Z_E Z_\varepsilon \sqrt{\frac{2K_A K_\alpha K_\beta K_v T_j (u_j+1)}{\varphi_d m_j^3 Z_j^3 u_j}} - [\sigma_H] \le 0$$

$$(j = 1,2,3) \qquad (24)$$

where, $u_1 = Z_1/Z_4$; $u_2 = Z_5/Z_2$; $u_3 = Z_{11}/Z_3$

Dynamic characteristic constraints: Rotational speed of high-speed gear should be much smaller than the critical speed of resonance.

$$g_{26} = n_j - n_{cr} = n_j - \frac{1.53\times 10^2 \cos\alpha}{m_j Z_k^2 Z_j}\sqrt{\pi E(Z_j^2 + Z_k^2)} \le 0$$

$$(j = 1,2,3 \quad k = 4,5,11) \qquad (25)$$

where, n_1, n_2, n_3 are rotational speed of input shaft I, II, III, respectively; n_{cr} is critical speed of resonance; E is young modulus; α is pressure angle.

4) State Variable

State variable of the fixed axis gear train subsystem is the transmission ratio $i_{14}\, i_{14}$

$$i_{14} = Z_4 / Z_1 \qquad (26)$$

To sum up, system level and subsystem level optimization models are expressed as in Fig. 4.

The Multidisciplinary Object Compatibility Design Optimization method based on simulated annealing algorithm is used to the optimization of this model. Most design variables of this model such as the number of teeth and modulus of gears are discrete variables, so discrete variable optimization algorithm is adopted for optimization in system level and subsystem. In this way, the deviation between discrete solution and quasi discrete solution rounded by optimal solution of continuous variable optimization could be avoided. The result of optimization is stated as in Tab. I:

	System-level optimization model
min .	$F(\mathbf{Z})=\sum_{i=1}^{12}V_i=m_1^3Z_1(Z_1^2+Z_4^2)+(m_2Z_2)^3+(m_3Z_3)^3+[(m_2Z_5)^2-d_{f8}^2]m_2Z_2$ $+[(m_3Z_{11})^2-d_{f8}^2]m_3Z_3+2m_0^3Z_{\min}\left[3Z_7^2+Z_6^2+4(Z_6+2Z_7+1)\right]$ $Z_{\min}=\min(Z_6,Z_7)$
s.t.	$G_1=R-\exp[(\Delta_Z^{(k-1)}-\Delta_Z^{(k)})/kT]\le 0 \quad \Delta_Z^{(k)}=\sum_{i=1}^{N_{CA}}\lVert\mathbf{X}_{SHi}^*-\mathbf{Z}_{SHi}^{(k)}\rVert+\sum_{i=1}^{N_{CA}}\sum_{j=1, j\ne i}^{N_{CA}}\lVert\mathbf{X}_{Yij}^*-\mathbf{Z}_{Yi}^{(k)}\rVert+\sum_{i=1}^{N_{CA}}\lVert\mathbf{Y}_i^*-\mathbf{Z}_{Yi}^{(k)}\rVert$ $G_2=Z_6Z_1/Z_4-(Z_6+2Z_7)Z_2/Z_5=0$ $G_3=2Z_3Z_5(Z_6+Z_7)-Z_2Z_6Z_{11}=0$
Z	$\mathbf{Z}=[m_0,Z_6,Z_7,m_1,m_2,m_3,Z_1,Z_2,Z_3,Z_4,Z_5,Z_{11},d_{f8},i_{14}]^T=[\mathbf{Z}_{SH1},\mathbf{Z}_{SH2},\mathbf{Z}_{Y1},\mathbf{Z}_{Y2}]^T$ $=[Z_{SH11},Z_{SH12},Z_{SH13},Z_{SH21},Z_{SH22},Z_{SH23},Z_{SH24},Z_{SH25},Z_{SH26},Z_{SH27},Z_{SH28},Z_{SH29},Z_{Y11},Z_{Y21}]^T$

Subsystem 1	differential gear train	Subsystem2	fixed axis gear train
min.	$f_1(\mathbf{X}_1)=\lVert\mathbf{X}_{SH1}-\mathbf{Z}_{SH1}\rVert+\lVert\mathbf{X}_{Y12}-\mathbf{Z}_{Y2}\rVert$	*min.*	$f_2(\mathbf{X}_2)=\lVert\mathbf{X}_{SH2}-\mathbf{Z}_{SH2}\rVert+\lVert\mathbf{X}_{Y21}-\mathbf{Z}_{Y1}\rVert$
s.t.	$g_{11}=R-(\Delta_{Y1}^{(k-1)}-\Delta_{Y1}^{(k)})/kT\le 0$ $g_{12}=\mathrm{mod}[2(Z_6+Z_7),C_s]-0.5\le 0$ $g_{13}=Z_7+2-(Z_6+3Z_7)\cdot\sin(\pi/3)\le 0$ $g_{14}=\frac{2K_AK_\alpha K_\beta K_vT_{ca}Y_{FS}}{\varphi_d m_0^3Z_{\min}^2}-[\sigma_F]\le 0$ $g_{15}=Z_HZ_EZ_\varepsilon\sqrt{\frac{2K_AK_\alpha K_\beta K_vT_{ca}(u+1)}{\varphi_d m_0^3Z_{\min}^3u}}-[\sigma_H]\le 0$ $T_{ca}=\frac{k_C}{C_S}\frac{Z_7}{Z_6}i_{14}\eta T_I$	*s.t.*	$g_{21}=R-(\Delta_{Y2}^{(k-1)}-\Delta_{Y2}^{(k)})/kT\le 0$ $g_{22}=1.2d_{f8}-m_2Z_5\le 0$ $g_{23}=1.2d_{f8}-m_3Z_{11}\le 0$ $g_{24}=\frac{2K_AK_\alpha K_\beta K_vT_jY_{FS}}{\varphi_d m_j^3Z_j^2}-[\sigma_F]\le 0 \quad j=1,2,3$ $g_{25}=Z_HZ_EZ_\varepsilon\sqrt{\frac{2K_AK_\alpha K_\beta K_vT_j(u_j+1)}{\varphi_d m_j^3Z_j^3u_j}}-[\sigma_H]\le 0$ $j=1,2,3$ $g_{26}=n_j-n_{cr}=n_j-\frac{1.53\times 10^2\cos\alpha}{m_jZ_k^2Z_j}\sqrt{\pi E(Z_j^2+Z_k^2)}\le 0$ $j=1,2,3 \quad k=4,5,11$
y_{11}	$y_{11}=d_{f8}=m_0(Z_6+2Z_7)+2.5m_0$	y_{21}	$y_{21}=i_{14}=Z_4/Z_1$
X_1	$\mathbf{X}_1=[m_0,Z_6,Z_7,i_{14}]^T=[\mathbf{X}_{SH1},\mathbf{X}_{Y12}]^T$ $=[x_{SH11},x_{SH12},x_{SH13},x_{Y121}]^T$	X_2	$\mathbf{X}_2=[m_1,m_2,m_3,Z_1,Z_2,Z_3,Z_4,Z_5,Z_{11},d_{f8}]^T$ $=[\mathbf{X}_{SH2},\mathbf{X}_{Y21}]^T=[x_{SH21},x_{SH22},x_{SH23},x_{SH24},$ $x_{SH25},x_{SH26},x_{SH27},x_{SH28},x_{SH29},x_{Y211}]^T$

Figure 4. MDO model of triple-redundancy transmission system

TABLE I. MDO RESULTS OF TRIPLE-REDUNDANCY TRANSMISSION SYSTEM

	system-level	
objective funciton	F= 5.244 ×10^5	
design variable	$z_{SH11}=m_0$= 0.9 mm $z_{SH12}=Z_6$= 22 $z_{SH13}=Z_7$= 17 $z_{SH21}=m_1$= 0.8 mm $z_{SH22}=m_2$= 1.25 mm $z_{SH23}=m_3$= 1 mm $z_{SH24}=Z_1$=39 $z_{SH25}=Z_2$=39 $z_{SH26}=Z_3$=22 $z_{SH27}=Z_4$=22 $z_{SH28}=Z_5$=56 $z_{SH29}=Z_{11}$=112 $z_{Y11}=d_{f8}$ = 52.65 $z_{Y21}=i_{14}$ =0.5640	
variable/function	Subsystem 1	Subsystem 2
Sharing substitute variable	$x_{SH11}=m_0$= 0.9 mm $x_{SH12}=Z_6$= 22 $x_{SH13}=Z_7$= 17	$x_{SH21}=m_1$= 0.8 mm $x_{SH22}=m_2$= 1.25 mm $x_{SH23}=m_3$= 1 mm $x_{SH24}=Z_1$=39 $x_{SH25}=Z_2$=39 $x_{SH26}=Z_3$=22 $x_{SH27}=Z_4$=22 $x_{SH28}=Z_5$=56 $x_{SH29}=Z_{11}$=112
State substitute variable	$x_{Y121}=i_{14}$ = 0.5638	$x_{Y211}=d_{f8}$= 52.648
State variable	$y_{11}=d_{f8}$ = 52.65	$y_{21}=i_{14}$ =0.5641

TABLE II. COMPARISON BETWEEN OPTIMAL RESULT AND INITIAL DATA OF TRIPLE-REDUNDANCY TRANSMISSION SYSTEM

	m_0 (mm)	m_1 (mm)	m_2 (mm)	m_3 (mm)	Z_1	Z_2	Z_2	Z_4	Z_5	Z_6	Z_7	Z_{11}	F (mm^3)
initial scheme	0.9	0.9	1.25	1.25	36	36	19	19	53	19	17	106	6.267×10^5
Optimal scheme	0.9	0.8	1.25	1	39	39	22	22	56	22	17	112	5.244×10^5

According to the design results in Tab. I, between sharing substitute variables of system-level and sharing substitute variables of two corresponding subsystems, the state substitute variables of system-level and state variable of two corresponding subsystems, the optimal solution are the basically same. Hence, the coupling relationship between the system and subsystem are satisfied. The relationship between objective function and the total number of iterations is shown in Fig.5. As the optimization process continued, the objective function of system level decreases and the compatibility objective function of subsystem decrease and close to zero, it means that the coupling problem is solved. The result indicated that the Multidisciplinary Object Compatibility Design Optimization method based on simulated annealing algorithm is effective and feasible. Compared with the initial design scheme (Tab. II), the value of objective function was reduced by 16.32%. The optimization effect is remarkable.

IV. Conclusion

To solve the multidisciplinary design optimization problem of non-hierarchic system, the Multidisciplinary Object Compatibility Design Optimization method based on simulated annealing algorithm is proposed. As optimization process continued, the coupling relationship between system level and different subsystems is improved by state accepting function which is embedded in compatibility constraint. In this way, abnormal program termination and premature convergence could be avoided and ideal global optimal solution could be achieved effectually. By taking the TR-B triple-redundancy transmission system as an research object, MDO model contain system level optimization and two subsystem optimization is established. The Multidisciplinary Object Compatibility Design Optimization method based on Simulated Annealing algorithm is applied to the optimization of TR-B triple-redundancy transmission system. The optimal results meet all requirement of the system and the method is proved to be practicality and feasibility in dealing with the optimization problem of complex system, and it is also effective for optimization of similar design problem.

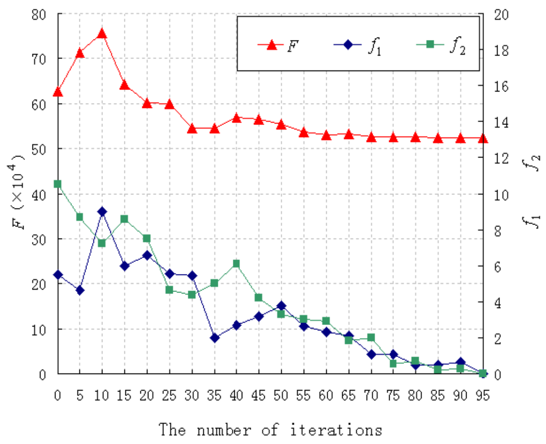

Figure 5. Graph of objective function(F, f1 and f2) vs the total number of iterations

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