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## Soliton-like Pulses along Electrical Nonlinear Transmission Line

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### Introduction

Nonlinearity is a fascinating feature of nature whose importance has been thought of for many years when considering large-amplitude wave motions observed in various fields ranging from fluids and plasmas to solidstate, biological and chemical systems. In that respect solitons represent one of the most striking aspects of nonlinear phenomena.

Solitons are a special class of pulse-shaped waves that propagate without changing their shape in nonlinear dispersive media. A balancing mechanism between nonlinearity and dispersion is responsible for the appearance of soliton phenomena [1]. Nature offers a variety of soliton examples. The first-reported soliton was a mono-pulse water wave in a narrow canal where the shallow water possessed both nonlinearity and dispersion [2]. The propagation of modulated waves, such as bright (envelope) solitons or dark (hole) solitons, has been the subject of considerable interest for many years. The optical fiber is yet another example of a nonlinear dispersive medium where optical solitons are observed. Optical solitons with higher order dispersion are subject of wide mathematical interest and development of new methods for their analytical and numerical solutions [3].

In electronics, nonlinear transmission lines (NLTLs) serve as nonlinear dispersive media where electrical solitons can propagate in the form of voltage waves. The NLTL is constructed by periodically loading a normal transmission line with varactors (e.g., reverse-biased p-n junction diodes or MOS capacitors), Fig. 1 (a), or alternatively, by arranging inductors and varactors in a 1-D lattice, Fig. 1 (b). The nonlinearity of the NLTL originates from the varactors whose capacitance changes with applied voltage, while its dispersion arises from its structural periodicity. Fig. 1 (c) shows the general soliton waveform on an NLTL in the absence of loss, which is a periodic train of voltage solitons. This waveform is called the cnoidal wave, and is a solution to what is known as the KdV equation [4]. For a given NLTL, an infinite number of cnoidal waves are possible via different combinations of the amplitude A, pulse spacing A, and pulse width W. Initial or boundary conditions will determine a specific cnoidal wave that can propagate. Fig. 2 (d) shows the special mono-pulse electrical soliton, whose cousin in shallow water was observed by Russell [5]. In addition to maintaining their shape, the solitary waves are typically bell shaped pulses which can survive collisions with other solitary waves [2]. When loss is present solitons cannot maintain their shape, but they still maintain spatial localization of energy in a pulse shape through a unique damping process [5].



**Fig. 1.** (a) - NLTL consisting of a linear transmission line and varactors; (b) - alternative form of the NLTL; (c) - general soliton wave formed on an infinitely long NLTL; (d) - the monopulse case [2]

These lines are of interest because of their applications in several fields. For example, in the linear regime, the NLTLs can be used as phase shifters in phased antenna arrays, where time delay can be controlled by means of a DC bias applied to Schottky diodes acting as variable reactance. Under large signal conditions, NLTLs can serve as impulse compressors or frequency multipliers [6]. Very recently, attempts have been made to study analytically and numerically nonlinear excitations using NLTLs, and it has been shown experimentally that solitons can exist in such systems [7]. Recently, these lines have proven to be of great practical use in extremely wideband (frequencies from dc to 100 GHz) focusing and shaping of signals [8] which is usually a hard problem.

In this paper, we will present a method to study solitary waves in NLTLs by focusing our attention on lossy NLTL. A lossy NLTL can be viewed as a multiple combination of small circuit segments shown in Fig. 2. The series inductance is due to magnetic field effects, and the capacitance is due to electric field coupling between the lines. The losses in the transmission media are depicted by the series and the shunt resistors. These resistors represent the finite conductivity of the conductors and the dielectric insulator between the conductors, respectively. The constants  $r_1$  and  $r_2$ , accounting for the transmission line losses, the linear inductance l, and the voltage-dependent capacitance c are the circuit parameters, and the resulting circuit is referred to as a distributed model of a lossy NLTL (a distributed model of the NLTL has been proposed to deal with wide-bandwidth signals) [6]. To study soliton propagation in lossy NLTL, a KdV approach cannot generally be applied, even with similar c(V)nonlinearity. Therefore, a procedure to obtain soliton solutions in lossy nonlinear transmission will be presented. Although device and transmission line losses can degrade the performance of NLTLs and cannot be discarded in realistic circuit simulations, this work is just focused on the propagation of pulse-like solitons in the case of lossy NLTL shown in Fig. 2.

The paper is organized as follows: In Section 2, we write down the circuit equations governing smallamplitude pulses on the lossy NLTL shown in Fig. 2. We undertake the study of the propagation of solitons on the line in Section 3. Exact soliton-like solutions of nonlinear partial differential equation are obtained by symbolic computation using improved tanh-function (ITF) method. Section 4 provides conclusions stemming from the results of this paper.

#### Model description and circuit equation

Interestingly, in constructing proposed lossy nonlinear network we were motivated with a very interesting example of nanobioelectronics problem. Namely, our group was recently modeled ionic currents along actin filaments in living cells in order to explain experimental results obtained by Lin and Cantiello [9]. Experiment revealed that pulses of ionic charges propagate along surface of helicoidally wound threads of actin filaments preserving stable localized shape. Our model includes nonlinear capacitive, inductive and resistive components. The results of this modeling provided the reasonable explanations of mentioned experimental assay.

Now, we consider a nonlinear electrical lossy network with N cells, as illustrated in Fig. 2. Each cell consists of a linear inductive element of inductance l and a

linear resistor with resistance  $r_1$  in the series branch, which constitute the linear dispersive element, and a nonlinear capacitor of capacitance c and a linear resistor with resistance  $r_2$  in the shunt branch.



Fig. 2. One cell of the lossy NLTL

The capacitance *c* is voltage-dependent and is biased by constant voltage  $V_0$  [6]

$$c(V_0 + V_n) = \frac{dQ_n}{dV_n} = c_0 \left( 1 + 2\alpha V_n + 3\beta V_n^2 + ... \right), \quad (1)$$

where  $\alpha$  and  $\beta$  are nonlinear coefficients that determine electric charge  $Q_n$  stored in the n-th capacitor in the line, and  $V_n$  is the voltage across the n-th capacitor. In this paper, we only consider the case when the perturbation voltage V is small in comparison with the equilibrium voltage  $V_0$ , therefore, we neglect higher-order terms in Eq. (1), keeping only the first two terms of the expansion.

Eq. (1) can be the second order curve fitting for the diode characteristics or the MOS varactor characteristics according to the sign of  $\alpha$  and  $\beta$ . For the diode the sign of  $\alpha$  is negative and the sign of  $\beta$  is positive and vice versa for the MOS [10]. The MOS varactor characteristic is shown in Fig. 3.



Fig. 3. Characteristic of MOS varactor [8]

By applying the Kirchhoff current law at node n whose voltage with respect to the ground is  $V_n$ , and applying the Kirchhoff voltage law across the two inductors connected to this node, we obtain:

$$I_{n-1} - I_n = \frac{dQ_n}{dt},\tag{2}$$

$$V_n - V_{n+1} = l \frac{dI_n}{dt} + I_n r_1.$$
 (3)

Similarly, if the voltage across the capacitor is  $V_n + V_0$ , where  $V_0$  is the bias voltage of the capacitor, we have

$$V_n = r_2(I_{n-1} - I_n) + V_0 + V_n.$$
(4)

From Eq. (3) we have:

$$\begin{cases} l \frac{dI_{n-1}}{dt} = V_{n-1} - V_n - I_{n-1}r_1, \\ l \frac{dI_n}{dt} = V_n - V_{n+1} - I_n r_1. \end{cases}$$
(5)

From Eq. (2) we have

$$l\frac{d^2Q_n}{dt^2} = l\frac{dI_{n-1}}{dt} - l\frac{dI_n}{dt}$$
(6)

and including Eq. (5) in Eq. (6), we get

$$l\frac{d^2Q_n}{dt^2} = V_{n+1} + V_{n-1} - 2V_n + r_1(I_n - I_{n-1}).$$
 (7)

Replacing  $V_{n+1}$ ,  $V_n$  and  $V_{n-1}$  from Eq. (4) to Eq. (7), we obtain that voltages of the adjacent nodes on this lossy NLTL are related via partial differential equation as follows

$$l\frac{d^{2}}{dt^{2}}\left[c_{0}\left(V_{n}+\alpha V_{n}^{2}\right)\right]+r_{1}c_{0}\frac{d}{dt}\left(V_{n}+\alpha V_{n}^{2}\right)+r_{2}c_{0}+r_{2}c_{0}\left\{-\frac{d}{dt}\left(V_{n-1}+\alpha V_{n-1}^{2}\right)+2\frac{d}{dt}\left(V_{n}+\alpha V_{n}^{2}\right)-\frac{d}{dt}\left(V_{n+1}+\alpha V_{n+1}^{2}\right)\right\}=V_{n+1}-2V_{n}+V_{n-1}.$$
(8)

The right-hand side of Eq. (8) can be approximated with partial derivatives with respect to distance x, from the beginning of the line, assuming that the spacing between two adjacent sections is  $\delta$  (i.e.,  $x_n=n\delta$ ). Let V(x, t) be a continuous function of the variables x and t so that V(n, t) = $V_n(t)$  and  $V_{n\pm l}(t) = V(x \pm \delta, t)$ . An approximate continuous partial differential equation can be obtained by using the Taylor expansions

$$V_{n\pm 1} = V(x\pm\delta,t) = V\pm\delta\frac{\partial V}{\partial x} + \frac{\delta^2}{2!}\frac{\partial^2 V}{\partial x^2} \pm \frac{\delta^3}{3!}\frac{\partial^3 V}{\partial x^3} + \dots, (9)$$

to evaluate the right-hand side of Eq. (8), i.e.

$$V_{n+1} - 2V_n + V_{n-1} = \delta^2 \frac{\partial^2 V}{\partial x^2} + \frac{\delta^4}{12} \frac{\partial^4 V}{\partial x^4}.$$
 (10)

By eliminating terms of higher order than  $\delta^4$  as negligible small and considering that the time variations of local voltage V are small compared to the constant background voltage  $V_0$ , we could safely assume that time derivative is of the order of small parameter  $\varepsilon$  as well as the nonlinear voltage terms  $\alpha V^2$  are of the order of  $\varepsilon^2$ . Thus, we obtain the following nonlinear PDE for the perturbed voltage V

$$R_{2}C_{0}\delta^{2}\frac{\partial}{\partial t}\left(\frac{\partial^{2}V}{\partial x^{2}}\right) + \delta^{2}\frac{\partial^{2}V}{\partial x^{2}} - LC_{0}\frac{\partial^{2}V}{\partial t^{2}} + R_{1}C_{0}2\alpha V\frac{\partial V}{\partial t} - R_{1}C_{0}\frac{\partial V}{\partial t} = 0, \qquad (11)$$

where  $L = l/\delta$ ,  $R_1 = r_1/\delta$ ,  $R_2 = r_2/\delta$  and  $C(V) = c(V)/\delta$ ( $C_0=c_0$ ) are defined as the inductance, resistance and capacitance per unit length, respectively.

#### Soliton-like pulses

In this paper, the improved tanh method (ITM) is applied in order to find the analytical solution of the lossy NLTL [10]. The first step in this method is to introduce the voltage in the form of the traveling wave

$$V(x,t) = V(\xi); \quad \xi = x - v_0 t,$$
 (12)

where  $v_0$  is undetermined parameter that represents the velocity of propagation. A propagating mode solution of Eq. (11) can be obtained by substituting Eq. (12) into the partial differential equation (11). Then Eq. (11) becomes the ordinary differential equation of the third order

$$\frac{R_2}{R_1} \delta^2 \mathbf{v}_0 \frac{d^3 V}{d\xi^3} + \frac{L}{R_1} \left( \mathbf{v}_0^2 - \delta^2 \omega_0^2 \right) \frac{d^2 V}{d\xi^2} + 2\alpha \mathbf{v}_0 V \frac{dV}{d\xi} - \mathbf{v}_0 \frac{dV}{d\xi} = 0.$$
(13)

Here the parameter  $\omega_0^2$ , meaning the cutoff angular frequency, is defined by  $\omega_0^2 = 1/(LC_0)$  and the maximum velocity of the propagation of waves is  $v_{0max} = \delta \omega_0$ . After first integration of Eq. (13), we obtain

$$A\frac{d^{2}V}{d\xi^{2}} + B\frac{dV}{d\xi} - v_{0}V + DV^{2} = c_{1},$$
 (14)

where  $c_1$  is an arbitrary constant, whose value depends on the initial conditions. Requiring that all functions  $V(\xi)$ ,  $dV(\xi)/d\xi$  and  $d^2V(\xi)/d\xi^2$  tend to zero as  $\xi \to \infty$ , the integration constant  $c_1$  is set to be zero. The parameters of Eq. (14) read as follows  $A = \frac{R_2}{R_1} \delta^2 v_0$ ,  $B = \frac{L}{R_1} \left( v_0^2 - \delta^2 \omega_0^2 \right)$ ,  $D = 2\alpha v_0$ .

It is observed from Eq. (13) that the sign of the dispersion coefficient (*A*) is always positive, but the sign of the nonlinearity coefficient (*D*) depends on the nonlinear characteristics of the capacitance. Therefore, the sign of the product *AD* can be positive or negative, which leads to appearance of dark soliton (AD < 0) or bright soliton (AD > 0) [11]. The parameter *B* is always positive and involves the effect of dissipation.

We now introduce the new independent variable  $Y = tanh\xi$  and  $W(Y) = V(\xi)$ , as a polynomial serie, due to which Eq. (14) transforms to the shape

$$A(1-Y^{2})^{2} \frac{d^{2}W}{dY^{2}} + \left\{B(1-Y^{2}) - 2AY(1-Y^{2})\right\}\frac{dW}{dY} - v_{0}W + DW^{2} = 0.$$
(15)

We compare the derivative term of highest-order with the highest order nonlinear term. So, balancing the order of W'' with the order of  $W^2$  in Eq. (15), we obtain m + 2 = 2m $\rightarrow m = 2$ . So the solution take the form

$$W(Y) = \sum_{i=0}^{m=2} a_i Y^i = a_0 + a_1 Y + a_2 Y^2, \qquad (16)$$

where  $a_0$ ,  $a_1$ ,  $a_2$ , are to be determined. Inserting Eq. (16) into Eq. (15), we get a system of algebraic equations:

$$Y^{0}: 2a_{2}A + a_{1}B + a_{0}^{2}D - v_{0}a_{0} = 0,$$
  

$$Y^{1}: -2a_{1}A + 2a_{2}B + 2a_{0}a_{1}D - v_{0}a_{1} = 0,$$
  

$$Y^{2}: -8a_{2}A - a_{1}B + a_{1}^{2}D + 2a_{0}a_{2}D - v_{0}a_{2} = 0, (17)$$
  

$$Y^{3}: 2a_{1}A - 2a_{2}B + 2a_{1}a_{2}D = 0,$$
  

$$Y^{4}: 6a_{2}A + a_{2}^{2}D = 0.$$

Solving the above set of equations (17) with the aid of Matlab, we can distinguish two different cases, as follows:

*Case I.* Using the determined parameters as  $a_0 = \frac{1}{D} \left( 6A + \frac{1}{2} v_0 \right); \quad a_1 = \frac{6}{5} \frac{B}{D}; \quad a_2 = -6 \frac{A}{D}, \text{ together}$  with a relation between *B* and *A* as  $B^2 = 100A^2$ , finally, the solution of the voltage equation (11) reads explicitly

$$V(x,t) = \frac{1}{D} \left( 6A + \frac{1}{2} \mathbf{v}_0 \right) + \frac{6}{5} \frac{B}{D} \tanh(x - \mathbf{v}_0 t) - \frac{6}{D} \tanh^2(x - \mathbf{v}_0 t).$$
(18)

It represents a particular combination of a bell-shaped solitary wave, third term of Eq. (18), with a kink-like shock-wave, second term. Here, the solitary wave velocity  $v_0$  is given by the following implicite expression

$$\mathbf{v}_0^2 = 4 \left( 24A^2 + \frac{6}{5}B^2 \right).$$
 (19)

Case II. We have the set of calculated parameters:

 $a_0 = \frac{1}{D} \left( A + \frac{1}{2} \mathbf{v}_0 \right); \quad a_1 = \frac{B}{D}; \quad a_2 = 0, \text{ so that the}$ 

second solution of the voltage equation (11) reads explicitely

$$V(x,t) = \frac{1}{D} \left( A + \frac{1}{2} v_0 \right) + \frac{B}{D} \tanh(x - v_0 t).$$
(20)

It represents the kink-like form of solitary wave with velocity  $v_0$  that is given by the following equation

$$v_0^2 = 4\left(A^2 + B^2\right).$$
 (21)

This solution agree with the result of papaer [9].

In order to verify our analytical prediction based on Eq. (18), we have also analyzed the electric circuit of Fig. 3 using well-known methods of circuit analysis. The number of cells between the source and the load are assumed to be 120. The circuit is analyzed using a timedomain technique for nonlinear circuits. The method uses finite-difference applied to the differential equations governing the voltages and current of the circuit. A circuit simulator is implemented in Matlab software package.

Firstly, we consider the lossy NLTL with a linear inductance  $L = 10 \mu$ H, the linear resistances  $R_1 = 5 \Omega$  and  $R_2 = 10 \Omega$ , and a varactor characterized by  $C(V) = 10 pF(1+2\alpha V)$ , with the nonlinearity factor  $\alpha = -0.25 V^{-1}$ . In this case, the sign of the product *AD* is negative. Therefore, if this circuit is excited by a dark initial signal [11], a dark soliton will be generated. The envelope waveform obtained from the finite-difference method at different positions *x* depicted in Fig. 4. As observed in Fig. 5, b, good agreement between the analytical and simulated results. According to the analytical calculations, this circuit because of its negative nonlinearity cannot support bright solutions.



**Fig. 4.** The 3-D plot of dark soliton at different positions x obtained from simulation of the NLTL



**Fig. 5.** The 2-D plots of dark soliton at different positions x (a); envelope waveforms obtained after of propagation of  $t = 5 \mu s$  (b)

Now, we assume a nonlinear characteristic for the varactor as  $C(V) = 10 pF(1+2\alpha V)$ , with the nonlinearity factor  $\alpha = 0.25 V^{-1}$ . In this case, the NLTL is excited by a secant hyperbolic (i.e., a bright soliton). Because of the positive sign of the product AD, as predicted by the analytical solution, a bright soliton will be formed along the transmission line. The envelope waveforms obtained from simulation at different positions x are shown in Fig.

6, where l is length of the lossy NLTL. From the results shown in Fig. 7, b, it is observed the analytical solution is in good agreement with the simulation result. According to the analytical calculations, this circuit because of its positive nonlinearity cannot support dark solutions.



Fig. 6. The 3-D plot of bright soliton at different positions x obtained from simulation of the NLTL



**Fig. 7.** The 2-D plots of dark soliton at different positions *x*, (a); envelope waveforms obtained after of propagation of  $t = 5 \mu s$  (b)

From Fig. 4–Fig. 7, we observe that:

- The velocity of the solitary wave increases with its amplitude;
- 2. The soliton-like pulse width decreases with increasing the pulse velocity;
- 3. The width shrinks for higher amplitude;
- The sign of the solution depends on the sign of the nonlinearity factor α. For negative α we will get a dark soliton-like pulse and for positive α we will get a bright soliton-like pulse on NLTL;
- 5. To maintain the amplitude while narrowing the pulse the inductance and the capacitance of the NLTL must be as small as possible, and the nonlinearity factor,  $\alpha$  should be large enough to compensate for the dispersion of the line [10].

#### Conclusions

In this paper, we studied the propagation of solitonlike signal in a nonlinear lossy RLC transmission line. We first used Kirchhoff's laws to show that the voltages along the network are governed by a third-order differential equation. In the small amplitude limit, we have used the reductive perturbation method and the continuum limit approximation to transform the nonlinear partial differential equation into ordinary differential equations which govern the propagation of solitary pulses in the NLTLs. The parameters of the obtained equation show that the dispersion coefficient (A) is always positive, but the sign of the nonlinearity coefficient (D) is determined by the characteristics of the varactor. Depending on the sign of the product AD, the proposed lossy NLTL can sustain either bright or dark solitons. The parameter B is always positive and reflects the effect of dissipation. Also, we found the soliton-like solution of equation using symbolic computation and improved tanh-function (ITF) method. From the analytical solution we can choose the suitable parameters for the nonlinear elements to satisfy the required application or to compare the analytical results with the experimental results.

The circumstance that solutions, Eq. (18) and Eq. (20), do not suffer attenuation nor degradation could be conceived as a consequence of the fact that thermal losses were balanced by the energy supply provided by biasing voltage  $V_0$ . We here mention the interesting paper by H. Ghafouri-Shiraz and P. Shum [12] which analyzed the propagation of a soliton in a nonlinear biased lossy LC ladder network using the Fourier series analysis technique and testing experimentally the same network. The input rectangular voltage pulse evolved in the bell-shaped soliton of the same form as Eq. (18) of present paper. The increased bias voltage increased the peak voltage of pertaining soliton and narrowed its pulse width.

What is very interesting to mention, we were motivated for this approach by an interesting natural phenomenon. Namely, along cellular cylindrical biopolymers called microtubules, currents of positive ions propagate in the form of stable localized pulses. These pulses can explain some properties of microtubules resembling to unipolar transistors enabling the rectification and amplification of ionic currents. Very recently, we elaborated the model of these currents relying on the concept of nonlinear transmission lines [13, 14].

We are planning to perform an experimental assay in order to test our circuit in the context of the model proposed here.

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# D. L. Sekulic, M. V. Sataric, M. B. Zivanov, J. S. Bajic. Soliton-like Pulses along Electrical Nonlinear Transmission Line // Electronics and Electrical Engineering. – Kaunas: Technologija, 2012. – No. 5(121). – P. 53–58.

The nonlinear transmission line is a structure where short-duration pulses called electrical solitons can be created and propagated. In this paper, we analytically and numerically investigated the propagation of soliton-like pulses along a lossy nonlinear transmission RLC line. Based on Kirchhoff's law taken in the continuum limit, a nonlinear partial differential equation is derived. Exact soliton solutions of voltage equation are obtained by symbolic computation using an improved tanh-function (ITF) method. It is found that the parameters of the transmission line play an important role in controlling the shape of the soliton. We discussed the fundamental properties of each soliton-like pulse, including the width and velocity. Ill. 7, bibl. 14 (in English; abstracts in English and Lithuanian).

# D. L. Sekulic, M. V. Sataric, M. B. Zivanov, J. S. Bajic. Solitoniniai impulsai elektrinėje netiesinėje perdavimo linijoje // Elektronika ir elektrotechnika. – Kaunas: Technologija, 2012. – Nr. 5(121). – P. 53–58.

Netiesinė perdavimo linija yra struktūra, kurioje gali būti sukurti ir sklisti trumpi impulsai, vadinami solitonais. Analitiškai ir skaitmeniškai ištirtas solitonų sklidimas netiesine RCL perdavimo linija. Remiantis Kirchofo dėsniu tęstinėje riboje, išvesta netiesinė diferencinė lygtis. Tiksli solitonų įtampos lygties išraiška gauta naudojant pagerintą tangento funkcijos metodą. Nustatyta, kad perdavimo funkcijos parametrai yra svarbūs solitono formai valdyti. Aptartos pagrindinės kiekvieno solitono savybės, įskaitant plotį ir greitį. Il. 7, bibl. 14 (anglų kalba; santraukos anglų ir lietuvių k.).