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INDUCTION METHODS PARADOXICALITY

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1. INTRODUCTION

The problem of induction (machine learning in expert systems, recognition, prediction, hypotheses formation, etc.) is a real and also complicated problem of artificial intelligence. The following definition of an inductive method is usually considered to be the most general one: the inductive method is the means of transition from facts to general statements about the world.

Socrates was one of the first to ponder over the means of general concepts formation. Aristotle described the process later called induction through simple enumeration which was further developed into the scientific induction method by F. Bekon and J. Mill. A large group of investigators have considered the possibility of constructing induction methods in formal languages[1-6].

In Refs. 1-6, induction methods are constructed as formalization of certain heuristical methods of hypotheses reinforcement. An approach to the induction problem which is different in essence is proposed by K. F. Samochwalov[7,8,9], who has investigated the possibility of constructing induction methods not on the basis of heuristics but by means of revealing the following necessary requirements: non-triviality and non-contradictoriness of a hypothesis to initial data and linguistic invariance. The induction method satisfying these demands is called regular[8]. However, even these necessary requirements formalizing the main notions about induction methods, when in total, result is the absence of nongenerate regular induction methods. In other words, these notions about induction methods are paradoxical. Thus, it would be natural to investigate the reason for the appearance of this paradoxicality. With this purpose the authors have made use of the following methods: for each requirement (or the whole complex of requirements) the possibility of including it (them) in a formal definition of the induction method is considered. If it is managed to substantiate that such an inclusion does not diminish the generality of consideration, then this requirement can be included in the definition and, thus, excluded from consideration when the reasons for paradoxicality are investigated.

For this paper, we have taken a formal definition of induction methods, using more usual and visual language in which the observation results are presented by points in the space of features R^n. It turned out that in this case the requirements of non-triviality and non-contradictoriness of a hypothesis to initial data can be included in formalization without the loss of generality. Moreover, it becomes possible to carry out the formalization in such a way that it excludes the appearance of degenerate induction methods.

As for the linguistic invariance requirement, the authors managed to prove that there were no induction methods satisfying it (see Theorem I). Therefore, this demand cannot be in principal included in a formal definition of induction methods.

[a] It is natural that the efficiency condition requires that T_0, pr_0 should be defined effectively as well.

In this paper as the formalization has been carried out on a language different from the one used in Refs. 7 and 8, the problem of the substantiation of this requirement appears. Carrying out reasonings analogous to those in Ref. 8, we managed to prove that non-compliance with the linguistic invariance requirement results in paradoxical situations, when the induction method application to two different languages chosen arbitrarily gives incompatible empirical hypotheses. In this sense the requirement of linguistic invariance is equivalent to the requirement of non-paradoxicality. Such a substantiation of the necessity of the linguistic invariance requirement is stronger than that of its "reasonable character in a strong sense" given in Ref. 8. It follows from these results that if the induction methods exist, then paradoxical situations, which are generalizations of Goodman's paradox are to apply for them.

Thus, the induction methods satisfying the formal definition introduced by us, proceeding from the same empirical data recorded in two different languages chosen arbitrarily, give incompatible results. In this sense induction methods are paradoxical. We also hope that the determination of the paradoxicality source and the use of more usual and visual language will allow revision in future of basic notions about the induction methods. The first author is sure that in this case induction will turn into deduction from data and *a priori* knowledge of a special type.

2. THE EMPIRICAL HYPOTHESIS DEFENITION

The empirical hypothesis in physics, chemistry or any other branch of science can be presented in the form of a statement: if we carry out observations by means of a certain measuring procedure, we shall fail to obtain the results of observations of a definite type. Let us specify some concepts.

A measuring procedure *Obs* is a certain strictly fixed sequence of actions over a set of objects with the aim of obtaining some observation result.

It is required from a measuring procedure that, firstly, one can always determine, whether it is possible to carry out an observation over a given set of objects *A*; secondly, one could always say, whether the observation was really carried out; thirdly, if it was, then the observation result is a certain formal record called a protocol. The application of a measuring procedure *Obs* to a certain set of objects *A* with the receipt of a protocol as a result, is called *an observation*.

Let us suppose that the empirical hypotheses under consideration are used in situations completely determined by the definition of individual numerical characteristics of each object taken separately. In this case the measuring procedure *Obs* includes the measurements of all the object characteristics under consideration. The protocol of the observation over the set *A* is a set of *m* vectors in euclidean space R^n, where *n* is the number of the characteristics under consideration and *m* is the number of objects. Thus, a partial mapping determined on the sets of objects *A* and assuming values in a set of protocols, is called *a measuring procedure Obs*. Any finite subset R^n is called *a protocol pr*

$$pr = Obs(A) = \{\bar{x}_1,...,\bar{x}_m\},\ \bar{x}_i \in R^n,\ i = 1,\dots, m,$$
$$pr = Obs(a_i) = \bar{x}_i \in R^n,\ a_i \in A,\ \ i = 1,\dots, m.$$

A permissible set of the results of the observation *T* (or simply permissible set) is an open connected subset R^n. It determines the set of vectors which the accepted empirical hypothesis permits as possible results of the observation carried out over an object *a* taken separately. The protocol *pr* is said to confirm the empirical hypothesis (permissible) if it lies in *T*, $pr \subset T$ and falsify the empirical hypothesis (non-permissible) if it does not lie in *T*.

Let us substantiate the supposition of openness and connectedness of the permissible set *T*. In the process of measuring, certain errors always appear, so it is reasonable to suppose that if some vector in R^n is permissible, the observation results lying in its particular neighborhood are to be permissible as well. The supposition of connectedness of the set *T* is made to simplify the proof. It can be weakened (see Consequence 2).

[a] It is natural that the efficiency condition requires that T_0, pr_0 should be defined effectively as well.

An empirical hypothesis h is a pair $\langle Obs, T \rangle$, where *Obs* is a measuring procedure and T is a permissible set of observation results. The empirical hypothesis consists in the following: for any set of objects A the protocol $pr = Obs(A)$ is to lie in T, $pr \subset T$.

3. THE INDUCTION METHOD DEFINITION

On the basis of the empirical hypothesis $h_0 = \langle Obs, T_0 \rangle$ and the protocol of observation $pr_0 = Obs(A)$ over a certain set of objects A, the induction method f is to give a new empirical hypothesis $h_1 = \langle Obs, T_1 \rangle$ which is to be stronger than the initial one. The hypotheses h_0, h_1 and protocol pr_0 have the same measuring procedure *Obs*. Thus, we are fixing the following notion about induction methods: the hypotheses h_0, h_1 and protocol pr_0 are to carry information about the same phenomena, properties, magnitudes, etc. Otherwise, it is not sensible to combine them in a statement, that is, the hypothesis h_1 is a result of the hypothesis h_0 reinforced by means of protocol pr_0.

Thus, to define the induction method $f: \langle h_0, pr_0 \rangle \to h_1$ it is enough to define the method of constructing the permissible set T_1. As our aim is to investigate the possibility of constructing formal and effective induction methods, we are going to consider the induction methods f only where the permissible set T_1 is obtained as a result of a certain formal induction method application.

Definition 1. A formal induction method is the following effective mapping

ind_f : $T_1 = ind_f(T_0, pr_0)$[a], where pr_0 is a protocol, $pr_0 \neq \varnothing$; T_0, T_1 are the permissible sets satisfying the condition $pr_0 \subset T_1 \subset T_0$.

Definition 2. The induction method f if the following mapping $f: \langle h_0, pr_0 \rangle \to h_1$, determined on any pair $\langle h_0, pr_0 \rangle$, where $h_0 = \langle Obs, T_0 \rangle$, $pr_0 = Obs(A)$, A is a set of objects; $h_1 = \langle Obs, T_1 \rangle$,

$T_1 = ind_f(T_0, pr_0)$, ind_f is a certain formal induction method, which satisfies the following condition: a pair $\langle h_0, pr_0 \rangle$ exists, such that $f(\langle h_0, pr_0 \rangle) = h_1$ and $T_1 \neq T_0$.

Let us substantiate these definitions. The initial data for the induction method is a pair $\langle h_0, pr_0 \rangle$, $h_0 = \langle Obs, T_0 \rangle$. In a general case only a pair $\langle T_0, pr_0 \rangle$ can be a formal part of these data. A measuring procedure *Obs* is a sequence of actions which is not formal. Even if it is possible in principal to describe this sequence formally, we are not going to consider such a possibility in the present paper. Thus, a formal induction method depends on the pair $\langle T_0, pr_0 \rangle$ only.

The induction method should reinforce the initial hypothesis by the use of some new results of the observation pr_0, so $pr_0 \neq \varnothing$. These observation results should not contradict the hypothesis h_0, so $pr_0 \subset T_0$. If this condition is not satisfied, the hypothesis h_0 is falsified by the protocol pr_0 and should be revised. It is not sensible to reinforce a hypothesis on the basis of the observations falsifying it.

The hypothesis h_1 obtained as a result of the induction method application should not be falsified by the protocol pr_0 as well, so $pr_0 \subset T_1$. The reinforced hypothesis h_1 should not contradict the initial hypothesis h_0, i.e. if $pr \subset T_1$, then $pr \subset T_1$, so $T_1 \subset T_0$.

The induction method f is determined on any pair $\langle h_0, pr_0 \rangle$, but we do not know whether, at least, one of such a pair exists or not. It is supposed that it exists for the domain of the method f not to be empty. Besides, the induction method f should not be trivial on the existing pairs, i.e. proceeding from the initial hypothesis h_0 the induction method should not always give the hypothesis h_0 again. So a pair $\langle h_0, pr_0 \rangle$ is to exist for which $T_1 \neq T_0$.

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4. THE LINGUISTIC INVARIANCE REQUIREMENT

Let F be a certain homomorphism of the space R^n on itself. Let us define the concept of F-conversion of the empirical hypothesis. F- conversion of a measuring procedure *Obs* is a measuring procedure F *Obs* placing each set of the objects A in correspondence with protocol $pr' = FObs(A) = F(pr)$, $pr = Obs(A)$. Homomorphism F in a measuring procedure F *Obs* can be realized, for instance, with the help of a certain analog transformation. It is clear that an analog or electronic scheme realizing homomorphism does not physically exist for any one. We consider only such classes of homomorphisms in which each homomorphism F and the inverse one F^{-1} can be realized physically. Let Φ be such class of homomorphisms.

F- conversion of an observation protocol pr is a protocol $F(pr)$; F- conversion of a permissible set of observation results is a permissible set $F(T)$ (homomorphism preserves the properties of openness and connectedness of the set T), F- conversion of the empirical hypothesis $h=\langle Obs, T \rangle$ is a hypothesis $F * h = \langle FObs, F(T)\rangle$. The two hypotheses are called empirically equivalent, if it can be proved that the observations over the same set of objects always simultaneously confirm or falsify these hypotheses.

Statement 1. For any $F \in \Phi$ the hypotheses h and $F * h$ are empirically equivalent.

Proof. Let an arbitrary set of objects A be given. After carrying out an experiment by a measuring procedure *Obs* we obtain protocol $pr = Obs(A)$. Having applied the transformation F to it we obtain protocol $pr' = FObs(A) = F(Obs(A)) = F(pr)$ as well, which is a result of the observation over the set A according to procedure F *Obs*. As the permissible sets T and $F(T)$ of the hypotheses h and $F * h$ are also connected by the transformation F, so $pr \subset T \Rightarrow F(pr) \subset F(T)$, which proves the statement. ■

This statement shows that the hypotheses h and $F * h$ cannot be distinguished by carrying out experiments. They are distinguished by means of recording only. If we want the induction method not to depend on this means of recording but on the empirical contents of the hypotheses we should accept the following requirement to induction methods. Supposing we have the initial data $\langle h_0, pr_0 \rangle$ for the induction method f. Then we have the initial data $\langle F * h_0, F(pr_0)\rangle, F \in \Phi$, as well. We can either first do F- conversion of the data $\langle h_0, pr_0 \rangle$ and then use the induction method f or, vice versa, first use the induction method and then do F- conversion. The result is the same.

Definition 3. The induction method f is said to satisfy the requirement of linguistic invariance (or is invariable) with respect to the class of homomorphisms Φ and initial data $\langle h_0, pr_0 \rangle$, if for any homomorphism $F \in \Phi$ the following condition is satisfied:

$$F(ind_f(T_0, pr_0)) = ind_f(F(T_0), F(pr_0)). \tag{1}$$

Definition 4. The induction method f is said to satisfy the requirement of linguistic invariance (or is invariable) with respect to the class of homomorphisms Φ, if it is invariable with respect to Φ for any initial data.

Let us show that the requirement of invariance with respect to Φ is too strong.

Theorem 1. For any space R^n, $n \geq 2$, one can find such a class of homomophisms Φ in R^n that there are no induction methods satisfying the requirement of linguistic invariance with respect to Φ.

Let us prove the following lemma.

Lemma. For any open connected subsystem A of the space R^n, $n \leq 2$, containing an open subset D, $D \neq A$, there are two points $a \in D$ and $b \in A, b \notin D$ and homomorphism F of the space R^n, such that $F(a) = b$, $F(b) = a$, $F(A) = A$ and F is identical out of A.

Proof of Lemma. Let the set A and its subset D be given. Let us construct the required homomorphism. Let take such a point $u \in A \setminus D$ that in its neighborhood $O \subseteq A$, the points a and b can be found, such that $b \in D, a \in A \setminus D, b \neq u, a \neq u$. Let us prove that such a point u exists.

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According to the definition, space E is connected, if it cannot be presented in the form of the unification of the two non-empty non-crossing sets, open in E. A set in a topological space is connected, if it is connected as a subspace[10]. As A is connected and D is open, so $A \setminus D$ is not open. Therefore, a point of the set $A \setminus D$ exists which is not internal. Let us denote this point as u (Fig. 1).

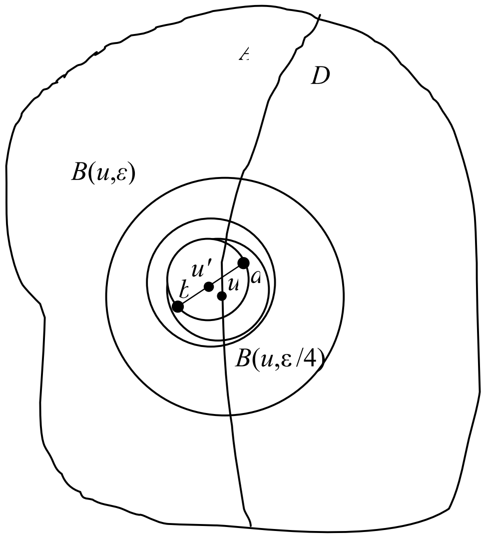

Fig. 1.

As $u \in A$ and A is open, the neighborhood $O \subseteq A$ of point u, lying completely in A, exists. As point u is not internal for the set $A \setminus D$, so for any neighborhood O of point u there exists a point b, $b \neq u$, lying in the set D.

If point u is such that it has the neighborhood O completely (except for point u itself) lying in D, then denoting the set of such isolated points as S one can obtain the following partition of the set A: $A = ((A \setminus D) \setminus S) \bigcup (D \bigcup S))$, i.e. we have unified the sets D and S. The set $D \bigcup S$ is open, as D is open and each point of the set S has the neighborhood completely lying in D. As A is connected and $D \bigcup S$ is open, so $(A \setminus D) \setminus S$ is not open; therefore, the point $u \in (A \setminus D) \setminus S$ exists, such that in its neighborhood $O \subseteq A$, the points a and b can be found and $b \in D, a \in A \setminus D, b \neq u, a \neq u$. *Q.E.D.*

As the subsystem of open spheres is the basis of topology[10,p.72] in the space R^n, so an open sphere with the center at the point u and with radius ε lying completely in A can be taken as the neighborhood $O \subseteq A$. Let us denote this sphere as $B(u,\varepsilon)$. In this sphere we take a sphere $B(u,\varepsilon/4)$ with the center at point u and with radius $\varepsilon/4$. In the sphere $B(u,\varepsilon/4)$ the points $a \in A \setminus D$ and $b \in D$, $a \neq u, b \neq u$, can be found. By this points a and b we construct a new sphere so that a segment $[a, b]$ is its diameter. Let the new sphere be $B(u', \frac{[a,b]}{2})$. Let us envelope the sphere $B(u', \frac{[a,b]}{2})$ in a sphere $B(u',[a,b])$ with the center at the point u' as well and radius $p[a, b]$. Taking into consideration the following inequality:

$$p(u,x) \leq p(u',u) + p(u',x) \leq \frac{\varepsilon}{4} + p(a,b) \leq \frac{\varepsilon}{4}\varepsilon,$$

where x is an arbitrary point of the sphere $B(u',[a,b])$ and p is euclidean metrics, we can make a conclusion that sphere $B(u',[a,b])$ lies completely in sphere $B(u,\varepsilon)$. Let us take a homomorphic transformation G of the space R^n consisting of the point u' with transference to the beginning of the coordinates and then turning of the space R^n so as to transfer the points a and b to the points

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$G(a) = (0,0,...,r_1,r_2)$ and $G(b) = (0,0,...,-r_1,-r_2)$. Such a transformation can be obviously written out if the coordinates of the points a and b are known. It is evident, that $p(a,b) = 2\sqrt{r_1^2 + r_2^2}$.

Let us make one more homomorphic transformation F' of the space R^n.

$$F'(x) = \begin{pmatrix} 1 & 0 & 0 & . & . & 0 \\ 0 & 1 & 0 & . & . & 0 \\ . & . & . & . & . & . \\ 0 & 0 & . & . & . & \cos\varphi(x) \;\; -\sin\varphi(x) \\ 0 & 0 & . & . & . & \sin\varphi(x) \;\; \cos\varphi(x) \end{pmatrix},$$

where

$$\pi, \text{ if } p(x,0) \le p(a,b)/2,$$
$$\pi - \frac{\pi(p(x,0)-r)}{r}, \text{ if } \frac{p(a,b)}{2} \le p(x,0) \le p(a,b),$$
$$0, \text{ if } p(x,0) > p(a,b), \text{ where } r = \frac{p(a,b)}{2}.$$

Let us explain how this transformation acts for the case of three-dimensional space. In Fig. 2 a section of the sphere is obtained by a plane passing through the beginning of the coordinates and the points $G(a)$ and $G(b)$ so that it is perpendicular to that axis of the coordinates, the value of which is equal to zero at the points $G(a)$ and $G(b)$. Out of sphere $G(B(u',[a,b])) = B(0, p(a,b))$ transformation F' is identical, and sphere $B(0, \frac{p(a,b)}{2})$ rotates (it is shaded in Fig.2) as a solid body around the axis perpendicular to the plane of the figure and rearrange the points $G(a)$ and $G(b)$. The actions of F' on the rest of the field are shown in the same figure on the example of segments l and l'. It is easy to understand from the above how transformation F' acts on other sections of the sphere by the planes parallel to a given one.

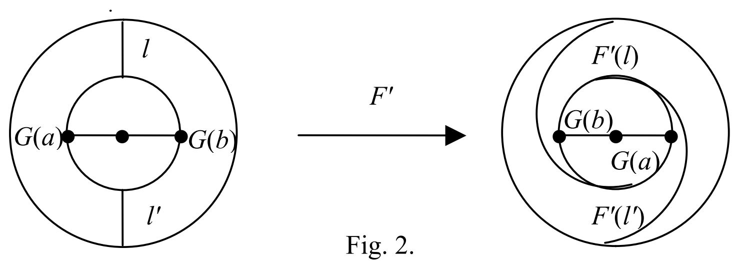

Fig. 2.

Now we can write out the unknown homomorphism F: $F = GF^1G^{-1}$. This homomorphism rearranges the points a and b and satisfies the condition $F(A) = A$.

Proof of Theorem 1. Let us show the process of the proof in Fig. 3.

Let us define the class of homomorphisms Φ. Note that homomotphisms F constructed in the lemma, can firstly be realized physically, secondly, they are completely determined by the choice of the two points a and b from R^n, $n \ge 2$. Let Φ be the class of such homomorphisms determined by various pairs of points a and b from R^n.

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The theorem is proved from the converse. Suppose the induction method f exists satisfying the requirement of linguistic invariance with respect to the class Φ. Then, according to the induction method definition, a pair $\langle h_0, pr_0 \rangle$ exists, such that. $f(\langle h_0, pr_0 \rangle) = h_1, T_1 \neq T_0, T_1 \subset T_0, T_1 = ind_f(T_0, pr_0)$. Let us consider the set $A=T_0 \backslash pr_0$ and its subset $D = T_1 \backslash pr_0$. As the set pr_0 is finite, so the sets A and D are open and connected. By lemma there are two one-point protocols pr_0 and pr_1, and homomorphism $F \in \Phi$, such that $pr_1 \not\subset pr_0, pr_2 \not\subset pr_0, pr_1 \subset T_0 \setminus T_1, pr_2 \subset T_1$, F is identical out of T_0 and

$$F(pr_1) = pr_2, F(pr_2) = pr_1, F(T_1) \neq T_1,$$
$$F(pr_0) = pr_0, F(T_0) \neq T_0.$$

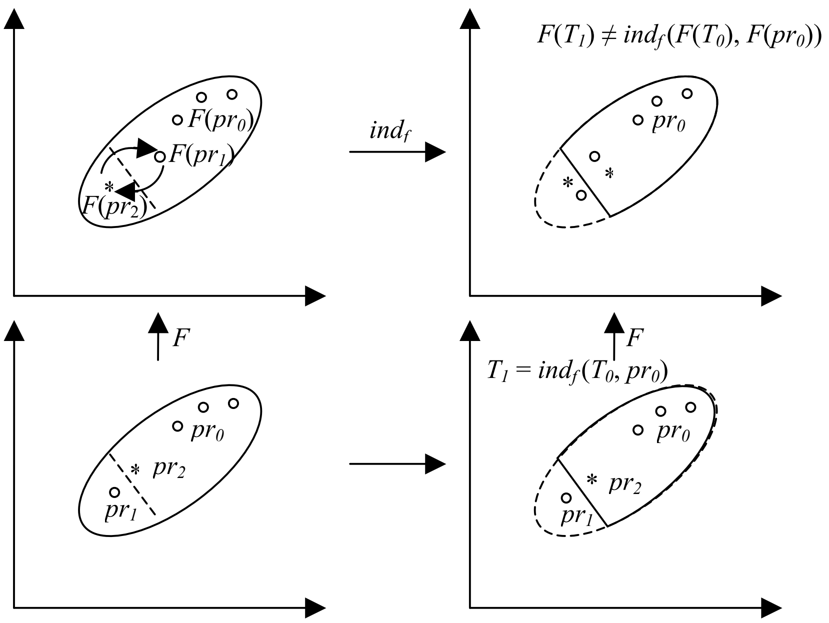

Fig. 3.

Let us show that for homomorphism F the requirement of linguistic invariance is violated. Apart from the pair $\langle h_0, pr_0 \rangle$ we consider the pair $\langle F * h_0, F(pr_0) \rangle$. The induction method f should be applicable to a pair f: $\langle F * h_0, F(pr_0) \rangle = h_1'$. Let us consider the permissible sets $T_1 = ind_f(T_0, pr_0)$ and $T_1' = ind_f(F(T_0), F(pr_0))$. As $F(T_0) = T_0, F(pr_0) = pr_0$, the arguments of the mapping ind_f will be the same in both cases. Thus, $T_1 \neq T_1'$. At the same time the equality $F(T_1) \neq T_1'$ is to be fulfilled in correspondence with the requirement of linguistic invariance. The equality $F(T_1) = T_1$ that cannot be fulfilled by the construction F follows from these two equalities. The theorem is proved.■

The condition $n \geq 2$ for the space R^n is not essential. One can prove the theorem for $n = 1$ as well.

Consequence 1. If the hypothesis $h_0 = \langle Obs, R^n \rangle$, $n \geq 2$, i.e. the hypothesis "everything is possible" is taken as the initial empirical hypothesis, it is till not possible to begin the reinforcement of a hypothesis on a certain protocol pr_0, as R^n is an open connected set; in correspondence with Theorem 1 there are no induction methods invariable with respect to Φ.

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Consequence 2. It is easy to show that Theorem 1 is also true for such empirical hypotheses in which a permissible set is not connected. But in this case each component of connectedness is contain, at least, one point in protocol pr_0.

Thus, it is not possible to accept the requirement of linguistic invariance as there are no induction methods satisfying it. Let us show that it is impossible to accept it as well, when it results in paradoxical situations defined below.

It follows from the theorem that for any induction method *f* the initial data $\langle h_0, pr_0\rangle$ exist for which the requirement of linguistic invariance with respect to Φ is broken. Let us determine for which initial data this requirement is violated. For the space R^n, $n \ge 2$, we are going to make use of the class of homomorphisms Φ only, defined in the proof of Theorem 1.

Theorem 2. Any induction method *f* does not satisfy the requirement of linguistic invariance with respect to Φ and any initial data $\langle h_0, pr_0\rangle$, such that $f(\langle h_0, pr_0\rangle) = h_1$, $T_1 \ne T_0$.

Proof. The proof is analogous to that of Theorem 1.

Thus, the requirement of linguistic invariance is violated for that initial data upon which the induction method produces a non-trivial reinforcement. Let us show that in each case the violation of the linguistic invariance requirement results in paradoxical situations consisting in the following: the induction method *f* proceeding from empirically equivalent initial data gives different reinforcements resulting in a contradiction, i.e. one reinforcement states that a certain protocol is permissible and another – that it is not permissible.

Definition 5. A paradoxical situation is said to appear for the induction method *f*, if for the initial data $\langle h_0, pr_0\rangle$, $h_0 = \langle Obs, T_0\rangle$, ($pr_0 = Obs(A)$, A is a certain set of objects), protocol *pr* and homomorphism $F \in \Phi$ the following correspondences take place

$$h_1 = f(\langle h_0, pr_0\rangle) = \langle Obs, T_1\rangle, T_1 = ind_f(T_0, pr_0),$$

$$h_1^F = f(\langle F * h_0, F(pr_0)\rangle) = \langle FObs, T_1^F\rangle, T_1^F = ind_f(F(T_0), F(pr_0)),$$

and either $pr \subset T_1, F(pr) \not\subset T_1^F$, or $pr \not\subset T_1, F(pr) \subset T_1^F$.

Statement 2. If the requirement of linguistic invariance with respect to the class of homomorphisms Φ and initial data $\langle h_0, pr_0\rangle$ is not satisfied for the induction method *f*, then homomorphism $F \in \Phi$ and protocol *pr* exist for which a paradoxical situation takes place.

The proof follows directly from the definition of a paradoxical situation and linguistic invariance. Really, the violation of the linguistic invariance requirement means that homomorphism $F \in \Phi$ exists for which the equality (1) is violated. But it immediately follows from it that protocol *pr* exists, satisfying the conditions of a paradoxical situation appearance.

Let us show the essence of the paradoxicality of the situation under consideration and the induction methods. Supposing we have a certain induction method *f*. We want to apply it to the initial data $\langle h_0, pr_0\rangle$ so as to obtain a stronger hypothesis $h_1, T \ne T_0$. This would allow one to speak about probable observation results more definitely. For instance, a certain set of objects A is given, then before carrying out observations we can say that an observation protocol is to lie in T_0 only, but even in $T_1, T_1 \subset T_0, T_1 \ne T_0$.

Let $f(\langle h_0, pr_0\rangle) = h_1$, $h_1, T \ne T_0$ (we are not interested in the case $T = T_0$). Then the requirement of linguistic invariance with respect to the class of homomorphisms Φ is violated for the induction method in correspondence with Theorem 2, and a paradoxical situation appears for a certain $F \in \Phi$ and protocol *pr* in correspondence with Statement 2. Its paradoxicality is in the following: for the initial data $\langle h_0, pr_0\rangle$ we can immediately find the empirically equivalent initial data $\langle F * h_0, F(pr_0)\rangle$ which are simply a different record of the same initial data. The application of the induction method *f* to these data gives the hypothesis h_1^F different from the hypothesis $F * h_1$. These two hypotheses can be distinguished on protocol *pr*. One of the hypotheses, for instance h_1, states that protocol *pr* is

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permissible, the other h_1^F – that *F*(*pr*) is not permissible. The paradoxicality of the present situation consists in this informal contradiction.

We cannot prefer the statement made by one hypothesis to the other one as the hypotheses h_1, h_1^F are obtained under absolutely symmetrical conditions. Since homomorphisms *F* and F^{-1} can be realized physically, therefore, as soon as the initial data $\langle h_0, pr_0 \rangle$ are given, we have the initial data, $\langle F * h_0, F(pr_0) \rangle$ as well. From the data $\langle F * h_0, F(pr_0) \rangle$ one can pass on to the data $\langle h_0, pr_0 \rangle$ again, so it makes no difference whether these data are of primary or secondary nature. The induction method *f* should be applied to all data. But the reinforcements obtained differ, in fact, they depend on our choice of homomorphism $F \in \Phi$.

The hypotheses h_1^F, $F \in \Phi$ should contain more definite statements about observation results than the corresponding initial hypotheses $F * h_0$, $F \in \Phi$. But a more definite character of the statements depends, in fact, on our subjective arbitrary choice of *F*, not on objective observations pr_0 giving initial data for the method *f* together with the hypothesis h_0.

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REFERENCES

1. G. Kieberg, *Probability and Inductive Logic.* Progress, Moscow, 1978, 375 pp.
2. I. Hintikka, "A two-dimensional continuum of inductive methods", *Aspects of Inductive Logic*, eds. I. Hintikka and Suppes, North-Holland, 1956, pp. 113-132.
3. R. Carnap, " The aim of inductive logic", *Logic Methodology and Philosophy of Science*, eds. H. H. Nagel, Suppes and Tarski, Stanford University Press, 1963, pp. 303-318.
4. R. Carnap, *The Continuum of Inductive Methods,* Chicago Press, 1952.
5. R. Carnap, *The Logical Foundation of Probability,* University of Chicago Press, 1963, 613 pp.
6. *Machine Learning – An Artificial Intelligence Approach,* eds. R.S. Michalski, J. G. Carbonnel and T. M. Mitchell, Springer-Verlag, 1984, 572 pp.
7. K. F. Samochwalov, " On the theory of empirical predictions", *Computer Systems* **N55** (1973) 3-35.
8. N. G. Zagoruik, K. F. Samochwalov and D. I. Sviridenko, *The Logic of Empirical Research,* Novosibirsk, 1978, 65 pp.
9. K. F. Samochwalov, " The Impossibility Theorem for Universal Theory of Predictions." Reports of Formal Methodology of Empirical Science. (March 1974 - May 1974), 19 pp. Inst. Philos. and Sociology, Polish Acad. Sci. Wroclaw, 1974.
10. J. L. Kelly, *General Topology,* Nauka, 1968, 380 pp.

[a] It is natural that the efficiency condition requires that T_0, pr_0 should be defined effectively as well.