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Alex Cukierman
University of Tel-Aviv

Allan H. Meltzer
Carnegie Mellon University, am05@andrew.cmu.edu

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A THEORY OF AMBIGUITY, CREDIBILITY, AND
INFLATION UNDER DISCRETION
AND ASYMMETRIC INFORMATION

BY ALEX CUKIERMAN AND ALLAN H. MELTZER¹

This paper develops a positive theory of credibility, ambiguity, and inflation under discretion and asymmetric information. The monetary policymaker maximizes his own (politically motivated) objective function that is positively related to economic stimulation through monetary surprises and negatively related to monetary growth. The relative importance he assigns to each target shifts stochastically through time. His current preference trade-off is known to him but not to the public. When choosing the (state contingent) path of money growth for the present and the future, the policymaker compares the benefits from current stimulation with the costs associated with higher future inflation expectations. Current monetary growth conveys information to the public about future money growth because there is persistence in the policymaker's objectives. Although expectations are rational, information is imperfect because monetary control procedures are imprecise. As a result the public cannot correctly distinguish persistent changes of emphasis on different policy objectives from transitory monetary control errors. The public becomes aware of changes gradually by observing past monetary growth. Credibility is defined in terms of the speed with which the public recognizes changes in the objectives of the policymaker. Credibility is lower the noisier monetary control and the more stable the objectives of the policymaker. Looser monetary control and a higher degree of time preference on the part of the policymaker induce him to produce higher and more variable monetary growth.

When the policymaker is free to determine the accuracy of monetary control he does not always choose the most effective control available in spite of the fact that monetary surprises always have an expected value of zero. The reason is that ambiguous control procedures enable the policymaker to generate positive surprises when he cares more than on average about economic stimulation. He leaves the inevitable negative surprises for periods in which he cares more about inflation prevention. This result provides an explanation for the Fed's preference for ambiguity, recently documented by Goodfriend (1986). The policymaker is more likely to pick more ambiguous control procedures the more uncertain his objectives and the higher his time preference.

The paper also provides a theoretical underpinning for the well documented cross-country positive correlation between the level and the variability of inflation.

KEYWORDS: Ambiguity, credibility, dynamic policy games, asymmetric information, inflation.

INTRODUCTION

CENTRAL BANKS usually do not follow well specified policy rules. Instead they move policy variables to reflect their changing emphasis on objectives like high employment and low inflation. As emphasized by Weintraub (1978), Woolley (1984), and Hetzel (1984), the Federal Reserve is influenced by pressures from

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Congress, interest groups, and the electorate. Arthur Burns (1979) appears to share this view. He believes that the Federal Reserve can work to achieve price stability only if the policy does not create too large an adverse effect on production and employment and does not irritate the Congress, the body to which the Federal Reserve is formally responsible. In Burns' words the role of the Fed is to continue "probing the limits of its freedom to undernourish . . . inflation" (Burns, 1979, p. 16). Evaluation of those limits involves a judgmental process which uses as inputs both economic and political factors that shift in an unpredictable manner over time and about which the public has less timely information than the monetary authority.

This paper explores the implications of this informational advantage for inflation and the credibility of the policymaker when the latter maximizes his own objective function. A central objective of the paper is to establish conditions under which ambiguity and imperfect credibility are preferable from the point of view of the policymaker to explicit formulation of objectives and perfect credibility.² The strong penchant of the Federal Reserve for secrecy has recently been revealed in the legal record of a case in which the Federal Open Market Committee (FOMC) was sued under the Freedom of Information Act of 1966 to make public immediately after each FOMC meeting the policy directives and minutes for that meeting (Goodfriend, 1986). The Federal Reserve resisted and argued the case for secrecy on a number of different grounds. The present paper provides a theoretical explanation for the Federal Reserve's desire for secrecy and ambiguity. We believe the analysis is applicable also to other central banks.

The paper also analyzes the quality of monetary control. Imperfect control of money increases opportunities for ambiguity. The reason is that the public is less able to detect shifts in policy when they occur. A policymaker who desires less than perfect credibility can reduce the quality of monetary control by avoiding efficient control procedures. Our paper relates the quality of monetary control and the credibility of the policymaker. We show that policymakers do not necessarily choose the most efficient control procedure that is technologically available. A certain degree of ambiguity enables policymakers to stimulate economic activity when they care most about such stimulation. Ambiguity enables policymakers to create monetary surprises when stimulation via surprises is most advantageous to the policymaker. As a result the optimal level of political ambiguity in the conduct of monetary policy may be larger than the minimum that is technologically attainable.

The paper also investigates the relationship between the structure of the policymaker's objectives and his credibility. Credibility is characterized in terms of the speed with which the public is convinced that new policies have been adopted when this is the case.

Recent literature on credibility and reputation has focused on finding conditions under which a shift to a policy rule as opposed to discretion is sustainable and

² This view of choice of policies by the monetary authority is consistent with the observation by students of the Fed like Brunner and Meltzer (1964), Lombra and Moran (1980) and Kane (1982) that monetary policy is not formulated within a precise analytical framework.

therefore credible (Taylor, 1982; Barro and Gordon, 1983a). The motivation of this literature is normative. The present paper focuses on positive rather than normative issues. The maintained assumption is that the policymaker is free to follow discretionary policies. This seems to be roughly consistent with the actual state of affairs in the U.S. and other countries. The Federal Reserve and other central banks are not bound by any particular monetary rule.

Although the public forms expectations rationally it cannot, in the presence of noisy monetary control, distinguish perfectly between persistent shifts in the setting of policy instruments and transitory control deviations. A consequence of the inability to separate these persistent and transitory shifts is that low credibility can persist for a time particularly when objectives change substantially. The model also permits us to parametrize credibility as a continuous variable whose time path and laws of motion depend on the underlying characteristics of the policymaker's objective function and on the quality of monetary control.

Casual empiricism suggests that individuals use resources to monitor Federal Reserve actions and use current reports on the money supply as an indicator of future policies (Bull, 1982). This behavior is consistent with the thesis of this paper—that expectations of future monetary growth react to actual monetary trends. A consequence of the public's behavior is that the policymaker takes into consideration the effect of current policy actions on future expectations even under discretion. However, since the policymaker has positive time preference, he partly discounts the effects of current policies on future expectations.

The model is consistent with observations showing that planned rates of monetary growth vary substantially both over time³ and across countries. Other implications of the theory are:

a. Policy has an inflationary bias in the presence of control errors. Looser control of money raises the average rate of monetary growth above the rate required for price stability and decreases the credibility of shifts to new policies.

b. Monetary variability is not minimized. The variance of monetary growth is larger the higher the degree of time preference of the policymaker and the looser his control of money.

c. For a given quality of monetary control, the level of monetary uncertainty inflicted on the public increases with the time preference of the policymaker.

d. When the policymaker is free to determine the accuracy of monetary control he does not always choose the most effective control available. A deliberate choice of ambiguous control procedures is more likely the larger is the uncertain component of the policymaker's objectives in comparison to the certain component. We refer to countries with relatively large uncertainty in objectives as politically unstable.

e. The larger political instability the larger both the mean and the variance of monetary growth. This result provides a theoretical underpinning for the well documented cross country positive correlation between the level and the variability of inflation (Okun, 1971; Logue and Willet, 1976).

³ Large, frequent changes in planned monetary growth may be required by actual policy procedures. See Brunner and Meltzer (1964).

f. A policymaker is more likely to make a deliberate choice of ambiguous control procedures the higher his rate of time preference.

Taylor (1982) and Barro and Gordon (1983a) have argued that in the absence of constitutional enforcement, policy rules, as opposed to discretion, will not be credible because they are not dynamically consistent. There is no problem of dynamic inconsistency⁴ here, and the absence of a binding constitutional rule is not the reason for imperfect credibility. Policy is discretionary, and credibility is imperfect because the policymaker has imperfect control of money, changing objectives, and an incentive to maintain some degree of ambiguity.

Section 1 presents the model and the policymaker's objective function. Section 2 shows the process by which the public forms its perceptions about the setting of the government's policy instruments and discusses the definition and the determinant of credibility under discretion. Section 3 derives the optimal strategy of the policymaker and demonstrates the rationality of expectations. The implications for the distribution of money growth appear in Section 4. In Section 5, we compare our model and results to other literature. Section 6 derives the politically optimal level of accuracy in monetary control when this level can be altered by appropriate choice of institutions. The main result of this section is that the policymaker does not necessarily choose maximum accuracy. This is followed by concluding remarks.

1. THE MODEL—GENERAL STRUCTURE

This section develops a framework for analyzing the choice of monetary growth by a policymaker whose objectives are described by a multi-period, state dependent objective function. Each period the policymaker chooses the rate of money growth so as to maximize the value of this function. When making his choices the policymaker uses information that the public does not have; he knows the state in which his own objective function is.

The public knows the structure of government's decision-making process, but it does not know with certainty the state in which the government's objective function lies. The public forms a rational expectation about the current rate of money growth using information about past money growth.

The policymaker's objective function summarizes the government's preferences as a political entity. The government is concerned about public support and

⁴Section 5 discusses this issue. We define dynamic inconsistency as the circumstance in which a dynamic optimizer, based on his objective function, (i) sets both policy for period t (today) and also sets contingent policies for future periods, but (ii) actual policy in period $t+k$ differs from planned policy in $t+k$ set at time t . This definition is taken from the important paper by Kydland and Prescott (1977, 475-476). Kydland and Prescott also introduce an example (1977, 477-480) using a model that is not explicitly dynamic and in which consistent and optimal policies differ. Some readers of our paper have interpreted "dynamic inconsistency" in terms of this example. Our paper corresponds conceptually to Kydland and Prescott's consistent solution but in an explicitly stochastic and dynamic framework. Credibility in our framework refers to changes in the policymaker's objectives within a discretionary and dynamically consistent framework.

desires to stay in power.⁵ We hypothesize that the government perceives that its chance of staying in power depends on the level of economic activity and on the rate of inflation. This view is consistent with public opinion polls showing that public support for the government rises with economic stimulation and decreases with the rate of inflation (Goodhart and Bhansali, 1970; Frey and Schneider, 1978; and Fischer and Huizinga, 1982).

The objective function is state dependent. The relative importance assigned to inflation and stimulation shifts in unpredictable ways as individuals within the decision-making body of government change their positions, alliances, and views. The changing weights may also reflect annual changes in the composition of a committee like the Federal Open Market Committee.⁶ We assume, as suggested by Burns (1979), that the policymaker has more information about the timing of shifts than the public.

The model is rational in the sense that the actual behavior that emerges is the same as the behavior on which the public relies to form expectations about future money growth. The government knows how the public forecasts monetary growth and inflation, so it can calculate, up to a random shock, the effect of a given choice of monetary growth on surprise creation. The government chooses the rate of money growth by comparing the benefits (to the government) from surprise creation against the costs of higher inflation.

Each period the policymaker plans to achieve a particular rate of monetary growth, m_i^p . Actual money growth, m_i , may differ from the planned rate because control is imperfect. More specifically

$$(1) \quad m_i = m_i^p + \psi_i$$

where ψ_i is period i 's realization of a stochastic serially uncorrelated normal variate with zero mean and variance σ_ψ^2 . This variance reflects the extent to which the operating procedures and the institutional environment prevent perfect control of money growth. At this stage σ_ψ^2 is taken as a technological parameter.

The policymaker's decision-making strategy is:

$$(2) \quad \max_{\{m_i^p, i=0,1,\dots\}} E_{G0} \sum_{i=0}^{\infty} \beta^i \left(e_i x_i - \frac{(m_i^p)^2}{2} \right),$$

$$(3) \quad e_i \equiv m_i - E[m_i | I_i],$$

$$(4) \quad x_i = A + p_i, \quad A > 0,$$

$$(5) \quad p_i = \rho p_{i-1} + v_i, \quad 0 < \rho < 1,$$

⁵ Note that the objective function is *not* a social welfare function of the type used by economists (or planners) to reach normative judgments about optimal policy. We use the terms government and policymaker interchangeably, and do not relate the objective function to the behavior of voters or the policymaker's perception of the shifting weights voters place on inflation and unemployment.

⁶ The fact that committee membership does not change completely at any one time would then be consistent with the persistence of objectives introduced below. Changes in the weights that the Federal Open Market Committee assigned to inflation or real income and to achieving monetary targets can be inferred from discussions in consecutive issues of the *Federal Reserve Bulletin*.

where v is a serially uncorrelated normal variate with zero mean, variance σ_v^2 , and is distributed independently of the control error ψ . Here e_i is the unanticipated rate of money growth in period i , I_i the information available to the public at the beginning of period i , and $E[m_i|I_i]$ the public's forecast of m_i given the information set I_i . The information set I_i includes all past values of m_i up to and including period $i-1$. β is the government's subjective discount factor, and E_{G0} is a conditional expected value operator that is conditioned on the information available to government in period 0 including a direct observation on x_0 . *Ceteris paribus* the policymaker prefers lower to higher inflation.

x_i is a random shift parameter which determines the shifts in the policymaker's objectives between economic stimulation achieved through surprise creation and inflation. The higher x_i , the more willing is the policymaker to trade higher inflation for more stimulation. Equations (4) and (5) specify the stochastic behavior of the shift parameter x_i and indicate that governmental objectives exhibit a certain degree of persistence which depends on the size of A and ρ .

The probability that x_i will have a positive realization is larger than the probability that it will have a negative realization.⁷ This reflects the hypothesis that unanticipated monetary growth stimulates employment and output⁸ and that, *ceteris paribus*, the policymaker is more likely to prefer more or less stimulation. Since changes in the level of economic activity and in the rate of inflation have redistributive consequences,⁹ shifts in x_i may originate from changes in the relative importance that the policymaker attaches to the welfare of various groups.

The public does not observe x_i directly but can draw inferences about the policymaker's objectives from observations of past money growth. However, inferences are not perfect because actual money growth reflects both the (persistent) plans of the policymaker and transitory control deviations.

Our formulation of the objective function reflects the basic uncertainty confronting the public. Society (including the policymaker) frequently experiences unanticipated events. Policymakers respond to these events by changing the relative weights that they place on the two objectives—stimulation and inflation. The policymaker does not reveal his current objectives or the tradeoffs confronting him, so the public has less information than the policymaker about the relative weight currently assigned to surprise creation.¹⁰

⁷ The distributional assumptions on v imply that $x \sim N(A, \sigma_p^2)$ where $\sigma_p^2 = \sigma_v^2 / (1 - \rho^2)$. The statement in the text follows from the positivity of A and the fact that $\rho < 1$. As A increases in relation to σ_p^2 the probability of negative x_i 's becomes smaller. For example when $A = \sigma_p$ this probability is 0.16 and when $A = 2\sigma_p$ it is less than 0.03.

⁸ This effect can operate through any of the following: (a) the mechanism described in Lucas (1973); (b) via nominal contracts of the type analyzed by Fischer (1977) and Taylor (1980a); (c) through a temporary decrease in the real rate (Brunner, Cukierman, and Meltzer, 1983); (d) through the price level (Bomhoff, 1982).

⁹ For example more stimulation accompanied by higher inflation increases welfare of the currently unemployed and decreases the welfare of retired individuals who hold fixed interest financial assets. See also Hetzel (1984).

¹⁰ Our model does not give the government superior knowledge about the position of the economy. We allow the government, but not the public, to know the effect of current events on current policy objectives. The weights on the government's objectives may reflect the desires of the voters, but the public does not know the extent to which the government responds to voters in the current period.

The policymaker knows the process by which the public forms its perception, $E[m_i|I_i]$, of the current rate of monetary expansion. This process must be consistent with the actual policy strategy followed by government. The government's strategy is derived, in turn, by solving the maximization problem in equation (2) taking the process for the formation of $E[m_i|I_i]$ as given. In other words, $E[m_i|I_i]$ is a rational expectation of m_i formed by using the public's knowledge about the policymaker's strategy in conjunction with all the relevant information available. It is convenient to proceed in two steps. First, we postulate the public's beliefs about the strategy that the government uses to set m_i^p . Then, we show that when government optimizes (2), given the public's beliefs, the strategy that emerges is identical to the strategy which the public expects. The public's beliefs and the consequent form of the optimal predictor $E[m_i|I_i]$ are discussed in Section 2. The solution of government's optimization in (2) and a proof of the rationality of the model are in Section 3.

2. THE PUBLIC'S BELIEFS AND THE EXPECTED RATE OF MONETARY EXPANSION

The public believes that the rate of monetary expansion planned by government is the following linear function of A and p_i :

$$(6) \quad m_i^p = B_0A + Bp_i \quad \text{for all } i,$$

where B_0 and B are known constants which ultimately depend on the underlying parameters of government's objective function. The public does not observe m_i^p directly. It observes

$$(7) \quad m_j = B_0A + Bp_j + \psi_j, \quad j \leq i-1,$$

which is a noisy indicator of m_j^p because of the existence of a control error. Since p_j displays a certain degree of persistence (as measured by ρ) past values of m are relevant for predicting the current rate of monetary growth. The information set of the public also contains the constants A , B_0 , B , ρ and the variances σ_v^2 and σ_ψ^2 . As a consequence, from each past observation on m the public can, using (7), infer

$$(8) \quad y_j \equiv Bp_j + \psi_j.$$

In Section 3, we show that equation (7) is implied by the policymaker's actions given the public's belief, so this inference is correct for equilibrium positions.

It follows from this remark and equation (6) that

$$(9) \quad E[m_i|I_i] = B_0A + BE[p_i|y_{i-1}, y_{i-2}, \dots].$$

It is shown in Part 1 of the Appendix that the conditional expected value on the right-hand side of (9) is

$$(10a) \quad E[p_i|y_{i-1}, y_{i-2}, \dots] = \frac{(\rho - \lambda)}{B} \sum_{j=0}^{\infty} \lambda^j y_{i-1-j},$$

$$(10b) \quad \lambda \equiv \frac{1}{2} \left[\frac{1+r}{\rho} + \rho \right] - \sqrt{\frac{1}{4} \left(\frac{1+r}{\rho} + \rho \right)^2 - 1},$$

$$(10c) \quad r \equiv B^2 \frac{\sigma_v^2}{\sigma_\psi^2}.$$

Substituting (10a) into (9), using (7) to express y_j in terms of m_j , and rearranging the resulting expression, we obtain

$$(11a) \quad E[m_i | I_i] = \sum_{j=0}^{\infty} \lambda^j [(1-\rho)\bar{m}^p + (\rho-\lambda)m_{i-1-j}],$$

$$(11b) \quad \bar{m}^p \equiv B_0 A.$$

Equations (6) and (11b) show that \bar{m}^p is the mean and median (since m_i^p is normally distributed) value of planned monetary growth. The coefficient λ is bounded between zero and one. The optimal predictor of monetary growth is, therefore, a geometrically distributed lag, with decreasing weights, of weighted averages of the unconditional mean money growth and actually observed past rates of money growth. In general, individuals give some weight to mean governmental planned money growth and assign the rest of the weights to observations on actual past money growth.¹¹ It is easily checked that the sum of the weights on the mean and past rates of money growth is one.

The relative weight accorded to \bar{m}^p is a measure of how strongly the public sticks to preconceptions rather than relying on actual developments. In the limit, as ρ tends to 1 the public abandons preconceptions entirely.¹² In this case, governmental preferences tend towards nonstationarity, so the information on the fixed mean \bar{m}^p becomes of lesser significance. At the other extreme, when ρ tends to zero there is hardly any persistence in the stochastic component of governmental preferences, so the information on actual past rates of growth becomes less relevant for predicting the future. Hence individuals stick to preconceptions and give negligible weight to actual developments.¹³

3. DERIVATION OF GOVERNMENT'S DECISION RULE AND PROOF OF THE RATIONALITY OF EXPECTATIONS

The policymaker chooses the current planned rate of money growth using his objective function and his knowledge of the current value of the random shock to this function. The past history of policy constrains the effect of his choice on his objectives. In this section we first derive the policy strategy. Then, we show that the policymaker chooses the strategy that the public expects.

¹¹ Note that λ is always smaller than or equal to ρ so that the weight $\rho - \lambda$ is always nonnegative. This can be seen by noting from (10b) that the condition $\rho - \lambda \geq 0$ is equivalent to the condition $n \geq 0$ which is always the case.

¹² Formally when $\rho = 1$ the predictor reduces to Muth's (1960) predictor.

¹³ In the limit when $\rho \rightarrow 0$, $\lambda \rightarrow 0$ and the predictor tends to \bar{m}^p . However, we have not been able to show that the weight given to \bar{m}^p is a decreasing function of ρ in all the range between zero and one although such a result seems plausible.

Substituting (11b) into (11a), substituting the resulting expression into (3), substituting this result into equation (2), using (1), and rearranging, the policymaker's problem can be rewritten as

$$(12) \quad \max_{\{m_i^p, i=0,1,\dots\}} E_{G_0} \sum_{i=0}^{\infty} \beta^i \left[\left(m_i^p + \psi_i - \frac{1-\rho}{1-\lambda} B_0 A (\rho - \lambda) \sum_{j=0}^{\infty} \lambda^j (m_{i-1-j}^p + \psi_{i-1-j}) \right) x_i - \frac{(m_i^p)^2}{2} \right].$$

The policymaker chooses the actual value of m_0^p and a contingency plan for $m_i^p, i \geq 1$. Recognizing that in each period in the future the policymaker faces a problem that has the same structure as period zero's problem, the stochastic Euler equations necessary for an internal maximum of this problem are, following Sargent (1979, Chapter XIV),

$$(13) \quad x_i - (\rho - \lambda) \beta E_{G_i} (x_{i+1} + \beta \lambda x_{i+2} + (\beta \lambda)^2 x_{i+3} + \dots) - m_i^p = 0 \quad (i = 0, 1, \dots).$$

Equations (13) yield the actual choice of m_0^p and the contingency plan for all future rates of money growth for $i \geq 1$.¹⁴

Although the policymaker knows x_i in period i (and the public does not), he is uncertain about values of x beyond period i . Based on the information available to him in period i , he computes a conditional expected value for $x_{i+j}, j \geq 1$. In view of (4) and (5) this expected value is

$$(14) \quad E_{G_i} x_{i+j} = A + E_{G_i} p_{i+j} = A + \rho^j p_i = \rho^j x_i + (1 - \rho^j) A, \quad j \geq 0.$$

Substituting (14) into (13) using (4) and the formulas for infinite geometric progressions, and rearranging,

$$(15) \quad m_i^p = \frac{1 - \beta \rho}{1 - \beta \lambda} A + \frac{1 - \beta \rho^2}{1 - \beta \rho \lambda} p_i.$$

Rationality of expectations implies that the coefficients of A and of p_i should be the same across equations (15) and (6) respectively, so

$$(16a) \quad B_0 = \frac{1 - \beta \rho}{1 - \beta \lambda(B)},$$

$$(16b) \quad B = \frac{1 - \beta \rho^2}{1 - \beta \rho \lambda(B)}.$$

¹⁴ Note that the transversality condition

$$\lim \beta^i E_{G_0} \left[x_i - (\rho - \lambda) \beta E_{G_i} \sum_{j=0}^{\infty} (\beta \lambda)^j x_{i+j+1} - m_i^p \right] = 0$$

is satisfied for any $\beta < 1$ since the term inside the brackets following E_{G_0} is finite. This condition is sufficient for an internal maximum.

The dependence of λ on B through equation (10) is stressed by writing λ as a function of B . Equation (16b) determines B uniquely as an implicit function of β , ρ , σ_v^2 , and σ_ψ^2 . Uniqueness is demonstrated by noting that the right-hand side of (16b) is increasing in λ and that λ is decreasing in r .¹⁵ The definition of r in terms of B from equation (10c) implies therefore that both λ and the right-hand side of (16b) are monotonically decreasing in B . Hence this equation yields a unique solution for B . Given this solution equation (16a) determines B_0 .

As long as the optimal predictor of money growth is linear in the information set of the public, this solution is also unique. This can be shown by writing the optimal predictor as a general linear function of all past rates of money growth and by allowing m_i^p to be a general function

$$m_i^p = F(p_i, p_{i-1}, p_{i-2}, \dots)$$

of the entire history of governmental objectives. Note that this formulation is similar to that of Green and Porter (1984) in which the current action of a representative firm depends on the entire history of the industry price.

The proof of uniqueness proceeds by substituting the general linear predictor into government's objective function in (2), deriving the Euler equations and showing that they imply the function F above to be a linear function of p_i only, as postulated in equation (6). A proof is available upon request.

Note that even if the costs of inflation to the policymaker had been expressed in terms of actual money growth rather than in terms of planned money growth, the resulting equilibrium would be the same. The reason is that for any $i \geq 0$

$$(m_i)^2 = (m_i^p)^2 + \psi_i^2 + 2m_i^p\psi_i.$$

The second term on the right-hand side of this equation does not depend on m_i^p and the third term has an expected value of zero, given the policymaker's information in period 0. Hence the Euler equations for this reformulation are still given by (13) leading to the equilibrium solution in (15).

4. CREDIBILITY AND THE DETERMINANTS OF THE DISTRIBUTION OF MONETARY GROWTH

Credibility is defined as the absolute value of the difference between the policymaker's plans and the public's beliefs about those plans. The smaller this difference, the higher the credibility of planned monetary policy. Credibility is relatively low when governmental objectives undergo large changes. In addition it is lower on average the longer it takes the public to recognize a change in governmental objectives. The weight λ in equation (10b) measures the degree of sluggishness in expectations. The higher λ , the longer is the "memory" of the public and the less important are recent developments for the formation of current expectations. With a low λ past policies are quickly forgotten. It can be shown

¹⁵ Let $b = \rho + (1+r)/\rho$. Then from (10b) $\partial\lambda/\partial b = (1/2)\sqrt{b^2/4 - 1}(\sqrt{b^2/4 - 1} - b)$ which is negative since $(3/4)b^2 + 1 > 0$. Since b is increasing in r , λ is decreasing in r .

that λ is a decreasing function of σ_v^2 and an increasing function of σ_ψ^2 ;¹⁶ the effects of past choices of monetary growth on current expectations are smaller in comparison to more recent choices the larger σ_v^2 and the lower σ_ψ^2 . People give less weight to the more distant past the larger the variance of the innovation to governmental objectives and the lower the variance of the control error. The worse the control of the money stock (high σ_ψ^2), the longer will past policies affect future expectations.

Since λ is related to the speed with which the public recognizes shifts in governmental objectives it is a prime determinant of the credibility accorded to new objectives of the government. Suppose that after remaining above its mean value m_t^p decreases below the mean. This more conservative attitude towards inflation will take longer to be recognized by the public the larger is λ . Therefore, the worse the control of the money stock the lower the credibility of shifts to rates of monetary growth that differ from those previously experienced. λ can therefore be taken as a measure of credibility. The higher λ , the longer it takes the public to recognize a change in governmental objectives and the lower, therefore, the government's credibility.

We turn now to the investigation of the distribution of money growth. Substituting (15) into (7), actual money growth can be rewritten

$$m_t = \frac{1-\beta\rho}{1-\beta\lambda}A + \frac{1-\beta\rho^2}{1-\beta\rho\lambda}p_t + \psi_t$$

with unconditional mean and variance that are given respectively by

$$(17) \quad Em_t = \frac{1-\beta\rho}{1-\beta\lambda}A,$$

$$(18) \quad V(m_t) = \left[\frac{1-\beta\rho^2}{1-\beta\rho\lambda} \right]^2 \frac{\sigma_v^2}{1-\rho^2} + \sigma_\psi^2 \equiv B^2 \frac{\sigma_v^2}{1-\rho^2} + \sigma_\psi^2.$$

Since $A > 0$, $0 \leq \lambda$, $\rho \leq 1$, the average rate of monetary expansion is positive provided policymakers have some degree of time preference ($\beta < 1$). Equation (17) suggests that mean monetary expansion is systematically related to the underlying parameters of the model. The following proposition summarizes the effects of some of those parameters on average monetary growth.

PROPOSITION 1: For any $\beta < 1$ average monetary growth is larger (a) the higher A , and (b) the higher σ_ψ^2 .

¹⁶ Let $a = \sigma_v^2/\sigma_\psi^2$. The total effect of a change in a on λ , taking the dependence of B on a through (16b) into consideration, is

$$d\lambda/da = (\partial\lambda/\partial b)[B^2(1-\beta\rho\lambda)/\rho(1-\beta\rho\lambda - 2a\beta\rho B^2\partial\lambda/\partial r)]$$

which is negative since $\partial\lambda/\partial b$ and $\partial\lambda/\partial r$ are both negative as implied by footnote 15. The result in the text follows by noting that a is positively related to σ_v^2 and negatively related to σ_ψ^2 .

Part (a) follows immediately from (17) by noting that $\beta < 1$, $0 \leq \rho$, $\lambda \leq 1$. Part (b) follows by noting that Em_i is an increasing function of λ which is in turn an increasing function of σ_ψ^2 (footnote 16).

Part (a) of Proposition 1 says that average monetary growth is higher when the policymaker is more biased towards economic stimulation than to preventing inflation (high A). Part (b) implies that average monetary growth is also higher the less effective is control of monetary growth as measured by a relatively high σ_ψ^2 . The reason is that at higher σ_ψ^2 it takes longer for the public to recognize a shift to a more expansionary policy. The negative effects of increased monetary expansion on government's objectives are delayed, so the government gains more from trading future inflation for current economic stimulation.

The following proposition summarizes the effects of the underlying parameters of the model on the variability of monetary growth.

PROPOSITION 2: (a) For any $\beta < 1$ the variance of monetary growth is larger the larger σ_ψ^2 . (b) For a given finite value of the variance of the monetary control error, σ_ψ^2 , the variance of monetary growth is larger the lower the discount factor β .

Part (a) follows from (18) by noting that $V(m_i)$ is an increasing function of σ_ψ^2 both directly and through its dependence on λ . Part (b) is proved by noting that $V(m_i)$ is increasing in B and that B is decreasing in β .¹⁷

Variability and uncertainty are not identical (Cukierman, 1984, Chapter 4, Section 4). A natural measure of monetary uncertainty is the variance of the money growth forecast error

$$V(e) = E[m_i - E[m_i | I_i]]^2.$$

PROPOSITION 3: For a given value of the variance of the monetary control error, σ_ψ^2 , monetary uncertainty as measured by $V(e)$ is larger the lower the discount factor β . The proof is developed in Part 2 of the Appendix.

Part (a) of Proposition 2 states that the variance of monetary growth is larger the larger the variance of the control error in money growth. There are two reasons. First is the direct effect. For any level of planned growth actual monetary growth is more variable. Second is the effect on λ . The public is slower to detect shifts in governmental objectives, so it pays government to induce a higher degree of stimulation by creating more uncertainty. For a similar reason when the discount factor is low, government discounts the costs associated with expectations of future inflation more heavily, and chooses more current stimulus. As suggested

¹⁷ The last relation follows by differentiation of (16b) with respect to β . This yields $\partial B / \partial \beta = 2B\rho(\lambda - \rho) / (1 - \beta\rho\lambda)^2$. This expression is nonpositive since $\lambda \leq \rho$ and strictly negative for a finite σ_ψ^2 .

by Proposition 3 this is done by creating more uncertainty, part of which takes the form of higher variability.

5. COMPARISON TO PREVIOUS AND CURRENT LITERATURE

The deterministic component of the policymaker's objective function in (2) is similar to that used by Barro and Gordon (1983b). As a result, our solution exhibits a similar inflationary bias which increases with A , the average relative preference of the policymaker for economic stimulation (Proposition 1a). A novel element of our framework is that the public's information about the shifting objectives of the policymaker changes over time.¹⁸ Shifting objectives and noisy control permit people to supplement the information on average objectives, given by A , by observing past money growth. Our solution specializes to Barro and Gordon's discretionary solution when asymmetric information is removed from the model. Formally, asymmetric information is eliminated when for a given (nonzero) σ_v^2 the variability of governmental objectives, σ_v^2 , is zero. In this case the actual relative preference of the policymaker for stimulation is common knowledge and is given by A . When $\sigma_v^2 = 0$, $r = 0$ which implies through (10b) that $\lambda = \rho$ ¹⁹ and through (4) and (5) that $p_i = 0$ for all i . In this case the solution in (15) specializes to

$$m_i^p = A \quad \text{for all } i \geq 0,$$

which is the solution obtained by Barro and Gordon under discretion. In addition it can be seen from (9) that in this case

$$E[m_i | I_i] = A.$$

That is, as in Barro and Gordon (1983b, p. 595), individuals do not pay attention to actual rates of money growth in forming expectations. Since they know with certainty the structure of governmental objectives they do not need to use information about observed rates of money growth to forecast future growth. Whatever the realization of money growth, individuals interpret its deviation from A as a transitory control deviation and stick to the preconceived notion that future monetary growth will be A . In this particular case the only way to change expectations is by convincing the public that discretion has been abandoned in favor of a different regime.

In the more general case considered here, expectations depend both on the preconception, \bar{m}^p in (11a), and on the past history of money growth. This last dependence is induced by asymmetric information between the policymaker and the public. Correspondingly, as can be seen from (15), planned monetary growth is composed of two components. The first, which depends on A , is common knowledge. The second, which depends on p_i , is known with certainty only to

¹⁸ Barro and Gordon (1983a and 1983b) limit their analysis of discretionary policy to the case in which the public does not need to learn about changes in the policymaker's objectives because those objectives are fixed.

¹⁹ For $r = 0$, (10b) implies that $\lambda = (1/2)(1/\rho + \rho) - \sqrt{(1/4)(1/\rho + \rho)^2 - 1}$ which is equal to ρ .

the policymaker. It is this second component that makes expectations dependent on past rates of money growth and makes it possible to change expectations without necessarily changing the discretionary nature of the policy regime. Note that the relative weights given to the preconception \bar{m}^p and the past history of monetary growth depend on the degree of persistence, ρ , in that part of governmental objectives about which the policymaker possesses an information advantage. When ρ tends to one the weight given to the preconception \bar{m}^p becomes negligible (see (11a)).

In Barro and Gordon (1983b) and in the example introduced by Kydland and Prescott (1977, 477-480) a rule which binds the policymaker to set $m_t^p = 0$ in all periods is believed, so the expectation of money growth is zero. The rule achieves a better value for the policymaker's objective function than discretion. However, none of these frameworks incorporates asymmetric information. In the presence of asymmetric information a zero rate of money growth rule does not always achieve a better value for the policymaker's objective function. The reason is that the public is slow to recognize shifts in governmental objectives. As a result, for sufficiently unstable objectives and slow adjustment of expectations, the positive contribution of a current positive e_t to the policymaker's objective function in (2) can dominate the negative effects of higher inflation and future negative $e_t - s$ by enough to make discretion preferable to a binding zero rate of money growth rule.²⁰ In such cases there is no difference between the discretionary-consistent solution and the optimal solution.

In their reputational paper Barro and Gordon (1983a) postulate an exogenously given "punishment" period.²¹ They also briefly consider an extension of their basic model in which government has an information advantage. However, even in the extension, the expected rate of monetary growth is fixed and independent of actual changes in monetary growth. This expectation is rational, given the stochastic structure postulated by Barro and Gordon. But their structure lacks descriptive realism, since forecasts of inflation are usually influenced by actual inflation and monetary growth. A dramatic example is the decrease in expected inflation between the end of the seventies and the present. Further, recent empirical work leaves no doubt that expectations regarding future monetary growth and inflation change within a discretionary framework. See *inter alia*, Hardouvelis (1984) and the many references there. Explicit modeling of the way expectations change is essential for discussing changes in credibility in the absence of a constitutional rule.

The present framework links expectations to observed money growth by introducing persistence in the policymaker's objectives and imperfect monetary control. This permits us to discuss different degrees of credibility within a discretionary regime. The evidence presented in Hardouvelis (1984) suggests that in spite of the fact that U.S. monetary policy has remained discretionary, the

²⁰ An example for which this is the case is discussed at the end of Section 6.

²¹ This term is borrowed from the repeated games literature to describe the relationship between past choices of monetary growth and current expectations. In this literature, the public chooses its expectation so as to induce socially desirable behavior by government in a supergame.

1979 change in the Fed's operating procedures changed the public's perception of the objectives of the Fed. The model of Barro and Gordon does not handle a change in perceptions of this type.

An additional advantage of our formulation is that it relates the speed with which expectations adjust to the quality of monetary control and other parameters of the environment. The determinants of the size and timing of "punishment" are identified rather than postulated exogeneously as in Barro and Gordon. Further, as stressed by Backus and Driffill (1985) a weakness of the Barro and Gordon analysis is that their equilibrium solution critically depends on the form this punishment strategy takes. As a result their model has multiple equilibria with no mechanism for choosing among them. We have shown that at least within the class of linear minimum square error predictors of money growth our equilibrium is unique. Given linearity, the attempt by each individual to minimize his error of forecast determines a unique pattern of learning that acts as a deterrent to the policymaker. This differs conceptually from Barro and Gordon's (1983a) punishment mechanism. In Barro and Gordon, the public is viewed as a single player who picks his expectation strategically in order to induce "better" behavior on the part of the policymaker. Here the structure of deterrence is induced, as a by-product, of each individual's attempt to minimize his forecast error.

As stressed by Bull (1982) casual empiricism suggests that a positive amount of resources is devoted to monitoring the central bank. Our framework is consistent with this observation; it is rational to study central bank behavior. Further, because expectations are influenced by past monetary growth, our concept of discretionary policy permits the policymaker to consider the effects of current policy on future expectations. In contrast, Barro and Gordon (1983a, p. 106) restrict discretion to mean that "...the policymaker treats the current inflationary expectation, and all future expectations, as given when choosing the current inflation rate." Actual policymaking in the absence of a constitutional rule recognizes the effect of current policy actions on future expectations, perhaps with some discounting. Barro and Gordon's definition of discretion seems overly restrictive, since it applies only to a world in which current policy actions do not affect future expectations.²²

The wider notion of discretion we use applies to any arrangement in which there is no constitutional rule to restrict the range of possible actions of the policymaker. The policymaker may follow a decision rule, in the sense of the optimal control literature, that takes account of the effect of present policy actions on future expectations. The use of a decision rule permits the policymaker to maximize his objective function. The policymaker, in our analysis, is free to

²² Our framework differs from that of BG in several other respects. Our objective function represents the attempt of the policymaker to elicit support for his policies while BG interpret the objective function of the policymaker as a social welfare function. Imperfect credibility in BG concerns the socially optimal constitutional rule and occurs because the policy rule is not dynamically consistent. In our framework credibility is imperfect even with a dynamically consistent discretionary policy because of noisy control and shifting objectives. Taylor (1983) raises doubts about the relevance of the Barro-Gordon model as a positive theory of inflation. Canzoneri (1985) tries to resolve those doubts by appealing to asymmetric information.

choose the weights he places on the arguments of the decision rule. The contingent decision rule that we derive corresponds to discretionary policy, since no a priori restrictions are imposed on the feasible set of policy actions. In contrast, a Friedman-type rule in which policy actions are subject to a priori, binding constraints is a constitutional rule.

Recently Backus and Driffill (1985) have formulated credibility as the outcome of a sequential equilibrium in a repeated game. Our model shares with theirs the asymmetric information about government objectives and the notion that discretionary policy is dynamically consistent. It differs in that government objectives are allowed to change continuously through time and to assume an infinite number of values. In Backus and Driffill there are only two possible types of government, and these types never change. Consequently, inflationary expectations can assume only two possible values. Moreover, once a government inflates it is revealed to be the weak type and asymmetric information is eliminated forever. Backus and Driffill restrict government's discount factor to 1 and do not analyze the effect of the precision of monetary control on the choice of policies. They define credibility in terms of the probability that government is "hard nosed" (has no incentive to inflate) whereas we conceive of credibility as the speed with which the public detects changes in the policymaker's objectives.

As in Backus and Driffill imperfect credibility arises here without any dynamic inconsistency of the type defined by Kydland and Prescott (1977, p. 475). The reason dynamic inconsistency does not arise in our framework is that the "action" taken by the public—evaluating $E[m_i | I_i]$ —does not depend on the future settings, m_{i+j}^p ($j \geq 1$), of government's choice of instruments. See (11). In cases of this kind, Kydland and Prescott (1977, p. 476) point out that the time consistent solution is also optimal. When period $i+j$ ($j \geq 1$) arrives, the government follows the contingency plan made in period i . At the risk of repetition it should be pointed out that this is not surprising since our solution corresponds conceptually to Kydland and Prescott's discretionary consistent solution in their example on pp. 477-480.

The basis for imperfect credibility here is the policymaker's advantage over the public that is due to shifting objectives, and noisy control. He knows his stochastically changing objectives, but the public does not. The best the public can do is to form expectations, allowing for this noise, and use all the information available each period to infer current and future money growth.

6. THE POLITICALLY OPTIMAL LEVEL OF AMBIGUITY

To this point we considered the level of noise in the control of money as a technologically given parameter. Suppose however that technology only puts a lower bound σ_ψ^2 on the variance of the control error. The policymaker can choose any $\sigma_\psi^2 \geq \sigma_\psi^2$. We assume for simplicity $\sigma_\psi^2 = 0$.

In this section the policymaker sets the value of σ_ψ^2 once and for all so as to maximize the long-run expected value of his objective function in (12). The choice of this variance determines the politically optimal level of ambiguity in the

conduct of monetary policy, since a higher choice of σ_ψ^2 conveys more ambiguous signals to the public.

For a given level of σ_ψ^2 the optimized value of the objective function of the government is obtained by substituting the optimal choice of monetary growth (equation (15)) into the objective function of the government in (12). Abstracting from the conditional expected value operator in (12), this yields

$$(19) \quad J(\{\psi_i\}_{-\infty}^{\infty}, \{p_i\}_{-\infty}^{\infty}) = \sum_{i=0}^{\infty} \beta^i \left[\left\{ B_0 A + B p_i + \psi_i - \frac{1-\rho}{1-\lambda} B_0 A \right. \right. \\ \left. \left. - (\rho - \lambda) \sum_{j=0}^{\infty} \lambda^j (B_0 A + B p_{i-1-j} + \psi_{i-1-j}) \right\} x_i - \frac{(m_i^e)^2}{2} \right]$$

where $\{\psi_i\}_{-\infty}^{\infty}$ and $\{p_i\}_{-\infty}^{\infty}$ denote the sequences of ψ and p .

Since the policymaker sets σ_ψ^2 on the basis of the long-run value of his objective function rather than on the basis of particular recent realizations of x_i , the relevant objective function for the choice of σ_ψ^2 is the unconditional expected value of $J(\circ)$. It is shown in Part 3 of the Appendix that this expected value is

$$(20) \quad G'(\sigma_\psi^2) = EJ(\circ) = \frac{\sigma_\psi^2}{1-\beta} \left[\frac{1-\beta\rho^2}{(1-\beta\rho\lambda)(1-\rho\lambda)} \right. \\ \left. - \frac{(1-\beta\rho^2)^2}{2(1-\beta\rho\lambda)^2(1-\rho^2)} - K^2 \frac{(1-\beta\rho)^2}{2(1-\beta\lambda)^2} \right]$$

where

$$K \equiv \frac{A}{\sqrt{\sigma_\psi^2/(1-\rho^2)}} = \frac{A}{\sigma_p}$$

The first term in brackets on the right-hand side of (20) represents the mean (positive) contribution of economic stimulation to governmental objectives. The mean value of economic stimulation through surprise creation is zero since negative and positive surprises cancel each other on average. But the contribution of monetary surprises to governmental objectives is positive on average. The reason is that the rate of money growth is positively related to the marginal benefit of surprise creation to the government. As can be seen from (15), when the marginal benefit of a surprise is higher than average ($x_i > A \leftrightarrow p_i > 0$) government chooses a higher than average rate of monetary growth and when the marginal benefit of a surprise is lower than average ($x_i < A \leftrightarrow p_i < 0$) the government chooses a lower than average rate of monetary growth. Consequently, when government cares more than on average about economic stimulation, surprises are positive on average, and when it cares less than on average about economic stimulation surprises are negative on average, making the unconditional expected value of the benefits from surprise creation positive. The government derives a positive gain, on average, from the ability to create surprises because it can allocate large positive surprises to periods in which x_i is relatively high and leave the inevitable negative surprises for periods with relatively low values of x_i . More

formally, by using (7) and (11a), it can be shown that

$$(21) \quad E \sum_{i=0}^{\infty} \beta^i e_i x_i = \frac{B}{1-\beta} E p_i (p_i - p_i^*)$$

where

$$p_i^* = (\rho - \lambda) \sum_{j=0}^{\infty} \lambda^j p_{i-1-j}.$$

Obviously $E(p_i - p_i^*) = 0$. But $E p_i (p_i - p_i^*) > 0$ since there is a positive correlation between p_i and $p_i - p_i^*$. This positive correlation is created by the government's attempt to maximize its objective function which leads to the contingent behavior summarized in (15). By contrast when the government does not possess an information advantage, $\sigma_v^2 = 0$ and p_i is identically zero. This implies that the expression in (21) is zero as well. Thus the existence of asymmetric information makes it possible for government to attain a higher value for its objectives through surprise creation in a time consistent equilibrium.

The last two terms on the right-hand side of (20) represent the mean (negative) contribution of inflation to governmental objectives. The effect of σ_v^2 on $G'(\cdot)$ comes from its effect on λ . Since λ is monotonically increasing in σ_v^2 and all the other elements that affect λ are fixed parameters, the choice of σ_v^2 is equivalent to a choice of λ . Hence the choice of the tightness of control procedures by the government can be expressed formally as

$$(22) \quad \max_{\lambda} G(\lambda) \equiv \max_{\sigma_v^2} G'(\sigma_v^2)$$

where $G'(\cdot)$ is given in (20).

If the policymaker chooses perfect control, $\sigma_v^2 \rightarrow 0$ and from (10b) $\lambda \rightarrow 0$. At the other extreme when $\sigma_v^2 \rightarrow \infty$, $\lambda \rightarrow \rho$ from below.²³ Hence the range of choice open for λ is from a minimum of zero (which corresponds to perfect control) to a maximum of ρ (which corresponds to the minimum possible amount of control).

A sufficient condition for a positive (politically) optimal level of ambiguity ($\sigma_v^2 > 0$) is that $G(\lambda)$ be an increasing function of λ at $\lambda = 0$. Using (20) and differentiating (22) with respect to λ ,

$$(23) \quad F(\lambda) \equiv \frac{\partial G}{\partial \lambda} = \frac{\sigma_v^2}{1-\beta} \left[\frac{\rho(1-\beta\rho^2)(1+\beta-2\beta\rho\lambda)}{(1-\beta\rho\lambda)^2(1-\rho\lambda)^2} - \frac{\beta\rho(1-\beta\rho^2)^2}{(1-\rho^2)(1-\beta\rho\lambda)^3} - \beta K^2 \frac{(1-\beta\rho)^2}{(1-\rho^2)(1-\beta\lambda)^3} \right].$$

By manipulating the monetary control parameter, σ_v^2 , the government affects the speed with which the public becomes aware of changes in governmental objectives and therefore the average value of benefits from surprise creation. In particular an increase in ambiguity increases the mean value of benefits from

²³ For $\sigma_v^2 \rightarrow \infty$, $r \rightarrow 0$ which implies $\lambda \rightarrow \rho$. See also footnote 19.

surprise creation²⁴ but it also increases the average rate of inflation which is bad from government's point of view. The level of monetary ambiguity is chosen by weighting the effects of those two conflicting elements on governmental objectives.

At $\lambda = 0$ the expression in (23) reduces to

$$(24) \quad F(0) \equiv \frac{\sigma_v^2}{1-\beta} \left[\rho(1-\beta\rho^2)(1+\beta) - \beta \left\{ \frac{\rho(1-\beta\rho^2)^2}{1-\rho^2} + K^2 \frac{(1-\beta\rho)^2}{1-\rho^2} \right\} \right].$$

The expression in (24) is positive if $\rho > 0$ and β is sufficiently small. Hence a strong degree of time preference on the part of government leads to a choice of institutions which produce loose control of the money supply even if perfect control is technologically feasible. For a sufficiently high value of β , G is decreasing at $\lambda = 0$, so the politically optimal level of control may be perfect. In other words a negative value of (24) (which obtains for sufficiently high β) is a necessary but not sufficient condition for the optimal value of σ_ψ^2 to be at zero.

At the other extreme when $\lambda = \rho$, (23) becomes

$$F(\rho) \equiv \frac{\sigma_v^2}{(1-\beta)(1-\rho^2)} \left[\frac{\rho(1+\beta-2\beta\rho^2)}{(1-\beta\rho^2)(1-\rho^2)} - \beta \left[\frac{\rho}{1-\beta\rho^2} + K^2 \frac{1}{1-\beta\rho} \right] \right].$$

This expression is negative for a sufficiently large β and ρ bounded away from one. For this case a sufficiently low degree of time preference on the part of government is necessary for a finite optimal level of ambiguity, $\sigma_\psi^2 < \infty$.

Let λ^* be the value of λ that maximizes the expression in equation (22). The previous discussion suggests that λ^* could be equal to either ρ , zero, or to some intermediate value between ρ and zero. Correspondingly the politically optimal value of σ_ψ^2 is infinite, zero, or some positive but finite value. For all $\beta < 1$ and $\sigma_v^2 > 0$, equation (23) implies that necessary conditions for $\lambda^* = \rho$, $0 < \lambda^* < \rho$, and $\lambda^* = 0$ are $F(\rho) \geq 0$, $F(\lambda^*) = 0$, and $F(0) \leq 0$ respectively. The following proposition provides a characterization of the three types of solution in terms of the parameters (ρ , β , and K^2) by providing a sufficient condition for each type of solution.

PROPOSITION 4: Let

$$c_1 \equiv \frac{\rho}{\beta} \frac{1}{(1-\rho\beta)^2} \left[\frac{1}{1-\rho^2} + \frac{\beta}{1-\rho^2\beta} - \beta(1-\rho^2\beta)^2 \right],$$

$$c_H \equiv \frac{\rho}{\beta} \frac{1-\rho^2\beta}{(1-\rho\beta)^2} [1-\rho^2(1+\beta(1-\beta))],$$

$$c_L \equiv \frac{\rho}{\beta} \frac{1-\rho\beta}{1-\rho^2},$$

²⁴ That is, the first term inside the brackets on the right-hand side of (23) is positive since $1 + \beta - 2\beta\rho\lambda > 0$. The intuition again is that in periods with high x_t - the positive effect of postponing the adjustment of expectations on governmental objectives is larger than the negative effect of such a postponement when x_t is below its mean value.

$$c_2 \equiv \frac{\rho}{\beta} \frac{1-\rho\beta}{1-\rho^2\beta} [(1-\rho^2)(1-\rho^2\beta)(1+\beta) - \beta];$$

then for $K^2 \equiv A^2/\sigma_p^2$,

- (i) $c_2 \leq c_1$;
- (ii) if $K^2 < c_2$, $\lambda^* = \rho$;
- (iii) if $K^2 > c_1$, $\lambda^* = 0$;
- (iv) if $c_L < K^2 < c_H$, $0 < \lambda^* < \rho$; and $c_2 \leq c_i \leq c_1$ for $i = L, H$.

PROOF: See Part 4 of the Appendix.

Proposition 4 states that if K^2 is larger than c_1 the optimal level of ambiguity is zero. If K^2 is smaller than c_2 the optimal level of ambiguity is as large as possible given the underlying stochastic structure. Finally, if $c_L < K^2 < c_H$ there is an internal solution for the optimal level of ambiguity. Figure 1 illustrates the different ranges of K^2 and the corresponding solution types for λ^* . The range (c_H, c_L) may be empty depending on the values of the discount factor β and the degree of persistence in governmental objectives as measured by ρ . It is more likely to be nonempty for lower values of ρ and for higher values of β .²⁵

Proposition 4 and Figure 1 suggest that for given ρ and β the size of K is crucial for the determination of the solution type. K^2 increases directly with A^2 and inversely with σ_p^2 . The latter is a measure of the uncertainty about the government's objectives. The larger the uncertain part in governmental objectives in relation to the certain part, A , the smaller is K and the more likely that ambiguity is high. At the other extreme when K is very large, say $K \rightarrow \infty$, there is almost no uncertainty with respect to governmental objectives.²⁶

Proposition 4 suggests that there is a monotonic relation between instability in government's objectives and the optimal level of ambiguity. When the level of instability in governmental objectives is high, so that $K^2 < c_2$, the optimal level of ambiguity is high too. When the level of instability in governmental objectives is relatively low, so that $K^2 > c_1$, the optimal level of ambiguity is zero. This

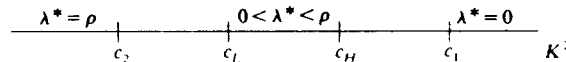


FIGURE 1.

²⁵ However, even when c_H and c_L are such that $c_H < c_L$ so that condition (iv) of Proposition 4 is not satisfied, λ^* may be internal if $c_2 < K^2 < c_1$. Condition (iv) is sufficient but not necessary for an internal solution.

²⁶ For a given nonzero A this is equivalent to the case $\sigma_v^2 = 0$ (and $\sigma_p^2 = 0$) discussed in Section 5.

monotonic relationship between the level of instability in governmental objectives and the degree of ambiguity embodied in monetary institutions holds also for internal solutions. Proposition 5 summarizes the effect of larger uncertainty in relative governmental objectives on the precision of monetary institutions chosen by government.

PROPOSITION 5: For given values of β and ρ , (i) if $0 < \lambda^* < \rho$ an increase in K^2 decreases λ^* and, (ii) if λ^* is at a corner (0 or ρ) an increase in K^2 does not increase λ^* .

PROOF: When $0 < \lambda^* < \rho$, $F(\lambda^*) = 0$. Using this fact, (23), and the implicit function theorem, we obtain

$$\frac{d\lambda^*}{dK^2} = \frac{1}{\frac{\partial^2 G}{\partial \lambda^2}} \left[\frac{\beta(1-\beta\rho)^2}{(1-\rho^2)(1-\beta\lambda^*)^3} \right].$$

This derivative is negative since $\partial^2 G / \partial \lambda^2 < 0$ by the second order condition for an internal maximum and the fact that β , ρ , and λ are all bounded between zero and one. This establishes (i).

When $\lambda^* = \rho$ an increase in K^2 may decrease λ^* but will not increase it, since λ^* is already at its maximal feasible value. When $\lambda^* = 0$ this implies that even if $F(\lambda)$ is positive for some λ in the range $0 < \lambda < \rho$ it is not sufficiently positive since $\lambda^* = 0$. Equation (23) implies that an increase in K^2 decreases $F(\lambda)$ more the higher the value of λ . Hence if for the original K^2 , $\lambda^* = 0$ it will, a fortiori, be zero for a larger K^2 . Q.E.D.

Proposition 5 has a useful, intuitive meaning. When governmental objectives are relatively unstable it pays a rational public to give a lot of weight to recent developments in forecasting the future rate of growth of money. For a given quality of monetary control (a given σ_ψ^2) individuals are more sensitive to recent developments in an economy with relatively unstable governmental objectives making it more difficult for government to exploit the benefits of monetary surprises. By increasing σ_ψ^2 the government, with relatively unstable objectives, can partially offset this effect by increasing the length of time it takes the public to detect a given shift in its objectives.

The same effect can be seen slightly differently by noting that, ceteris paribus, the larger the variance of the innovation of governmental objectives, σ_v^2 , the higher the optimal level of σ_ψ^2 . For given values of ρ , β , and K^2 there is an optimal level of credibility that is represented by the learning parameter λ^* . This parameter induces through equation (10b) an optimal level of r that is denoted r^* . Given r^* , the value of σ_ψ^2 chosen by government is larger, or at least not smaller, the larger σ_v^2 .²⁷ This leads to the following proposition.

²⁷ The reason is that an increase in $a = \sigma_v^2 / \sigma_\psi^2$, taking into account the resulting change in B , causes an increase in r . This follows from the fact that $dr/da = B[B + 2aB^2\beta\rho(\partial\lambda/\partial r)] / (1 - \beta\rho\lambda - 2aB^2\beta\rho(\partial\lambda/\partial r))$ is nonnegative since $1 - \beta\rho\lambda \geq 0$. Hence the larger σ_v^2 the larger the value of σ_ψ^2 needed to attain the optimal values of r^* and λ^* . This interpretation was suggested by Robert Hetzel.

PROPOSITION 6: For given values of ρ , β , and K^2 , the politically optimal level of σ_v^2 is a nondecreasing function of σ_p^2 .

Equation (23) implies that two countries with identical values of the parameters ρ , β , and K^2 will have identical levels of credibility since λ^* is the same in both. But Proposition 6 implies that the country with the higher σ_p^2 will achieve this common credibility level by choosing a higher level of noise in the control of money.

A large body of empirical evidence (Okun, 1971; Logue and Willet, 1976; Jaffe and Kleiman, 1977) suggests that the level and the variability of inflation are positively related across countries. Our framework links this positive relationship to differences in the relative instability of governmental objectives across countries. More precisely, for a given A , a country with a higher σ_p^2 will have a lower K^2 and, by Proposition 5, a higher σ_v^2 . Propositions 1 and 2 imply that, ceteris paribus, a higher σ_v^2 implies both higher average monetary growth as well as higher monetary variability.²⁸ Hence if political instability varies across countries, this variation will produce a positive relation between average money growth and the variability of money growth. This, in turn, induces a positive relationship between the average level and the variability of inflation across countries.

We now consider the effect of the policymaker's discount factor β on the optimal level of ambiguity. Since for given σ_p^2 , ρ , and A the optimal level of credibility, λ^* , and the optimal degree of ambiguity as embodied in σ_v^2 are monotonically related, it is enough to find the effect of β on λ^* . For internal solutions the direction of this effect depends on the sign of the partial derivative of $F(\lambda^*)$ (from (23)) with respect to β . The sign is ambiguous in general. For corner solutions at 0 or ρ , the sign depends on the effect of β on $F(\lambda)$ in the range $0 \leq \lambda \leq \rho$. This sign is also ambiguous. In spite of the ambiguities, however, numerical simulations for a wide range of values of ρ , β , and K^2 suggest that in more than 99 per cent of the cases examined λ^* is weakly decreasing in β .²⁹ Moreover the few cases in which λ^* turned out to be strictly increasing in β all occurred for $\beta > 0.8$. We conclude on the basis of these simulations that, except for a relatively small subset of parameters, the optimal level of ambiguity increases, or at least does not decrease, when the degree of time preference increases (β declines). Policymakers with short horizons and high time preference are likely to prefer more ambiguity and are, therefore, less credible.

We conclude this section by demonstrating that there are cases in which the unconditional expected value of government's objective function is larger under discretion than under a Friedman (1960) type money growth rule which constrains the policymaker to set m_i^e , for all i , at zero. With a Friedman rule the policymaker does not have the ability to choose the timing of surprises. To the extent that surprises occur they are due to imperfect monetary control and are completely

²⁸ $V(m_i)$ is larger also because of the direct effect of σ_p^2 . See equation (18).

²⁹ Altogether 19,796 different combinations of the parameters ρ , β , and K^2 were examined. For each combination λ^* was evaluated by numerical search. The combinations were formed from 7 values of ρ between 0 and 1, 28 values of K^2 between 0 and 10, and 101 equally spaced values of β in the $[0, 1]$ interval. λ^* turned out to be increasing in β in only 117 cases.

unrelated to changes in the objectives of the policymaker. As a consequence the unconditional expected value of government's objective function is zero in this case.

Under discretion with $\lambda = \rho$ ³⁰ the unconditional expected value of governmental objectives from equation (20) reduces to

$$(25) \quad \frac{\sigma_v^2}{2(1-\beta)(1-\rho^2)}(1-K^2)$$

which is positive for all $K < 1$. Hence if governmental objectives are sufficiently unstable the greater "natural" ability of the government to create surprises at the politically right time yields a better value to government's objectives under discretion than under a Friedman rule. This example, chosen mostly for its simplicity, illustrates a fundamental difference between symmetric information and asymmetric information with persistence in objectives. It relates directly to claims made in Section 5 (see footnote 20).

To sum up the main finding of this section is that the politically optimal level of control over monetary growth is not necessarily the minimum level. It may be in the government's interest to pick institutions with loose control over the money supply even if better control is technologically feasible.³¹

7. CONCLUDING COMMENTS

This paper develops a politically based theory of credibility, monetary growth, and ambiguity for a monetary system in which control is imperfect and the policymaker has an information advantage about his own objectives. For a technologically determined level of control the credibility of newly instituted disinflationary policies depends on the quality of monetary control. With tight control, a few periods of determined slowdown in the rate of monetary expansion suffice to convince the public that money growth is permanently lower. As a consequence, expectations of inflation fall quickly. Unexpected rates of monetary growth remain negative for a relatively brief period, and the accompanying unemployment is relatively small. In this case a "cold turkey" disinflationary policy is preferable to "gradualism" since a large decrease in monetary expansion generates credibility relatively quickly. If the policymaker has poor control of money growth, disinflationary policy takes substantially longer to become credible, however. The interim period of unemployment is longer and unemployment is larger. The costs of disinflation are higher. A gradual approach, that permits the public to adjust anticipations, seems preferable in these circumstances.³²

³⁰ The optimal value of λ may be lower than ρ . But if it is, the expected value of government's objectives will be at least as large as in equation (25) and therefore still better than the value achieved under a Friedman rule.

³¹ If the policymaker's objectives are a function of actual money growth (as suggested at the end of Section 3) the incentive for ambiguity would be smaller, since a higher σ_v^2 increases the cost of inflation by increasing the unconditional expected values of m_t^2 .

³² In Cukierman and Meltzer (1986) the analysis is extended to the case of mandatory announcements of monetary targets.

Taylor (1980b) and Meyer and Webster (1982) analyze the response of inflation when learning is gradual. Using a nominal contracts framework, Fischer (1984) provides estimates of the costs of disinflation for two alternative assumptions about the reaction of expectations to changes in policy. In one case there is no change in expectations regarding money growth and in another expectations are adaptive. His analysis suggests that the costs of disinflation are quite sensitive to the way expectations adjust. Our paper relates this critical speed of adjustment to some underlying factors like the quality of monetary control and the degree of instability in the objectives of the policymaker. Fischer's analysis in conjunction with ours creates a link between the costs of disinflation on one hand and the quality of monetary control and the degree of instability in the objectives of the policymaker on the other.

A main result of the paper is that the policymaker does not necessarily choose the most efficient control procedure available. Instead, he may choose to increase ambiguity. We find that there is a politically optimal level of ambiguity.

Our finding that the government may prefer a higher to a lower level of ambiguity suggests an explanation for the Federal Reserve's inclination for secrecy, documented in Goodfriend (1986). The intuitive reason is that a certain degree of ambiguity provides the policymaker with greater control of the timing of monetary surprises. When there is ambiguity about policy, he can create large positive surprises when he cares most about stimulation and leave the inevitable negative surprises for periods in which he is relatively more concerned about inflation.

The policymaker determines the level of ambiguity by choosing the quality of monetary control. This choice determines, in turn, the speed with which individuals become convinced that the policymaker's objectives have changed when this is the case. Credibility depends on the speed with which the public learns; actions that delay learning lower credibility. We show that policymakers with relatively unstable objectives tend to be more ambiguous and less credible. Since both the mean and the variance of monetary growth are positively related to the level of noise in monetary control this implies the existence of a positive cross-sectional relationship between the mean and the variance of inflation. The existence of such a relation is widely documented (Okun, 1971; Logue and Willet, 1976; Jaffe and Kleiman, 1977).

The level of ambiguity chosen by the policymaker is also systematically related to his degree of time preference. Our results suggest that more often than not policymakers with stronger time preference choose less precise control procedures. In addition, for a given precision of control procedures, monetary uncertainty is larger the larger is the degree of time preference of the policymaker.

Although we did not submit the theory to empirical verification we believe it is consistent with a number of additional observations. First, rates of monetary growth vary considerably both over time within a given country and between countries. Second, the actual conduct of monetary policy in many countries, including the U.S., corresponds more to discretion than to rules. Third, if the policymaker's rate of time preference rises during emergencies, the theory predicts

higher monetary uncertainty and higher monetary variability at such times. This seems to be consistent with numerous war time episodes.

The model of this paper identifies some of the determinants of credibility. The paper also suggests that the so called "credibility hypothesis" and the rational expectations paradigm should be viewed as complements rather than substitutes.³³

Dept. of Economics, Tel Aviv University, Ramat Aviv 69978, Israel
and
GSIA, Carnegie-Mellon University, Pittsburgh, Pennsylvania 15213

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APPENDIX

1. DERIVATION OF THE OPTIMAL PREDICTOR IN EQUATION (10) OF THE TEXT

Define the dummy stochastic variable $\varepsilon_t = \psi_t/B$. Substituting this relation into equation (8),

$$y_t = B(p_t + \varepsilon_t) = Bz_t.$$

Since the public knows the parameter B , an observation on y_t is equivalent to an observation on z_t . Hence the expected value of p_t conditioned on past values of y is equal to this expected value conditioned on past values of z . We turn now to the calculation of this expected value.

Since p_t and z_t are normally distributed the expected value of p_t conditioned on $z_{t-1}, i \geq 1$, is a linear function with fixed coefficients of the observations on $z_{t-i}, i \geq 1$. That is,

$$(A1) \quad E[p_t | z_{t-1}, z_{t-2}, \dots] = \sum_{i=1}^{\infty} a_i z_{t-i}.$$

Since this conditional expected value is also the point estimate of p_t which minimizes the mean square error around this estimate, it follows that $\{a_i\}_{i=1}^{\infty}$ are to be chosen so as to minimize

$$(A2) \quad Q = E \left[p_t - \sum_{i=1}^{\infty} a_i z_{t-i} \right]^2.$$

Substituting the relation between y and z into (A2), using the fact that ε and v are mutually independent and passing the expectation operator through, Q can be written

$$(A3) \quad Q = [1 + (\rho - a_1)^2 + (\rho^2 - \rho a_1 - a_2)^2 + \dots + (\rho^i - \rho^{i-1} a_1 - \dots - a_i)^2 + \dots] \sigma_v^2 + \sum_{i=1}^{\infty} a_i^2 \sigma_\varepsilon^2 \sigma_v^2$$

where

$$\sigma_\varepsilon^2 = \frac{\sigma_\psi^2}{B^2}.$$

The necessary first-order conditions for an extremum of Q are:

$$(A4) \quad \frac{\partial Q}{\partial a_i} = -2[(\rho^i - \rho^{i-1} a_1 - \dots - a_i) + \rho(\rho^{i+1} - \rho^i a_1 - \dots - a_{i+1}) \\ + \rho^2(\rho^{i+2} - \rho^{i+1} a_1 - \dots - a_{i+2}) + \dots] + 2\sigma_\varepsilon^2 a_i = 0, \quad i \geq 1.$$

³³ Fellner and Haberler, who have introduced the term "credibility hypothesis," seem to imply that they are at least partial substitutes. See Fellner (1976, p. 170; 1979). Haberler (1980, p. 280), writes: "Thus the credibility approach does not accept the assumption made by the rational expectations school that government actions fall nearly into two extreme categories—systematic, wholly predictable policies and unsystematic, entirely unpredictable shocks. Especially after a long period of inflation, which has shaken the public's confidence that the government will carry out its anti-inflationary policy, a sustained and deliberate effort must be made to restore credibility." The analysis presented in our paper suggests that it is possible to accept Haberler's second statement without having to accept the first one. High credibility will quicken and ease the process of disinflation even in a rational expectations model in which policy actions can be decomposed—as is the case here—into predictable and unpredictable components. Fellner softened his position in Fellner (1980).

Leading (A4) by one period, multiplying by ρ , and subtracting (A4) from the resulting expression,

$$(A5) \quad (\rho^i - \rho^{i-1}a_1 - \dots - a_i)\sigma_v^2 + (\rho a_{i+1} - a_i)\sigma_r^2 = 0, \quad i \geq 1.$$

Multiplying (A5) by ρ , subtracting the resulting expression from (A5) led by one period, using the relation $\sigma_r^2 = \sigma_v^2/B^2$ and rearranging,

$$(A6) \quad a_{i+2} - \left(\frac{1+r}{\rho} + \rho\right)a_{i+1} + a_i = 0, \quad i \geq 1,$$

where

$$r = \frac{\sigma_v^2}{\sigma_r^2} = \frac{B^2 \sigma_v^2}{\sigma_w^2}.$$

This is a second order homogeneous difference equation whose general solution is

$$(A7) \quad a_i = C\lambda^i$$

where C is a constant to be determined by initial conditions and λ is the root of the quadratic

$$(A8) \quad u^2 - \left(\frac{1+r}{\rho} + \rho\right)u + 1 = 0.$$

The roots of this equation are given by

$$(A9) \quad u_{1,2} = \frac{1}{2} \left(\frac{1+r}{\rho} + \rho\right) \pm \sqrt{\left\{\frac{1}{2} \left(\frac{1+r}{\rho} + \rho\right)\right\}^2 - 1}.$$

The positive root in (A12) is larger than one and the negative root is bounded between zero and one. Thus a_i does not diverge only if the smaller root is substituted for λ in (A10). Since a_i has to yield a minimum for Q it cannot diverge. Hence

$$\lambda = \frac{1}{2} \left(\frac{1+r}{\rho} + \rho\right) - \sqrt{\left\{\frac{1}{2} \left(\frac{1+r}{\rho} + \rho\right)\right\}^2 - 1}.$$

For $i = 1$ (A5) implies

$$(A10) \quad (\rho - a_1)\sigma_v^2 + (\rho a_2 - a_1)\sigma_r^2 = 0.$$

Using (A7) to express a_1 and a_2 in terms of C and λ , substituting into (A10), and rearranging,

$$(A11) \quad C = \frac{\rho \sigma_v^2}{\lambda [\sigma_v^2 + \sigma_r^2 - \rho \lambda \sigma_r^2]} = \frac{\rho}{\lambda} \frac{r}{1+r-\rho \lambda}.$$

Since λ is a root of the quadratic in (8) it satisfies

$$\lambda^2 - \left(\frac{1}{\rho} + \rho + \frac{r}{\rho}\right)\lambda + 1 = 0$$

which implies

$$(A12) \quad r = \rho \lambda + \frac{\rho}{\lambda} - (1 + \rho^2).$$

Substituting (A12) into (A11) and rearranging

$$(A13) \quad C = \frac{\rho - \lambda}{\lambda}.$$

Substituting (A13) into (A7), substituting the resulting expression into (A1), and rearranging, we obtain

$$\begin{aligned} E[p_t | z_{t-1}, z_{t-2}, \dots] &= (\rho - \lambda) \sum_{j=0}^{\infty} \lambda^j z_{t-1-j} \\ &= \frac{\rho - \lambda}{B} \sum_{j=0}^{\infty} \lambda^j (B p_{t-1-j} + \psi_{t-1-j}) = \frac{\rho - \lambda}{B} \sum_{j=0}^{\infty} \lambda^j y_{t-1-j}. \end{aligned}$$

The optimal predictor in equation (10) of the text follows by recalling that past values of z and y carry the same information. Hence

$$E[p_t | z_{t-1}, z_{t-2}, \dots] = E[p_t | y_{t-1}, y_{t-2}, \dots] = \frac{\rho - \lambda}{B} \sum_{j=0}^{\infty} \lambda^j y_{t-1-j}$$

which is equation (10a) in the text.

Note that for $\rho = 1$ and $B = 1$, p_t becomes a random walk and this predictor reduces to the Muth (1960) optimal predictor.

2. PROOF OF PROPOSITION 3

Substituting (7) and (9) into the expression for $V(e)$ that precedes Proposition 3, and rearranging,

$$(A14) \quad V(e) = \sigma_\psi^2 + B^2 E[p_t - E[p_t | y_{t-1}, y_{t-2}, \dots]]^2.$$

Since $E[p_t | y_{t-1}, y_{t-2}, \dots]$ is a minimum mean square error predictor,

$$(A15) \quad E[p_t - E[p_t | y_{t-1}, y_{t-2}, \dots]] = \min_{\{a_j\}} E \left[p_t - \sum_{j=1}^{\infty} a_j y_{t-j} \right]^2.$$

Substituting (A15) into (A14), passing B to the right of the min operator, and redefining variables,

$$(A16) \quad V(e) = \sigma_\psi^2 + \min_{\{a_j\}} E \left[q_t - \sum_{j=1}^{\infty} a_j (q_{t-j} + \psi_{t-j}) \right]^2; \quad q_j = B p_j \quad \text{for all } j.$$

The definition of q_j in (A16) in conjunction with (5) implies

$$(A17) \quad E q = 0, \quad \sigma_q^2 = B^2 \sigma_p^2, \quad E q_t q_{t-j} = \rho^j \sigma_q^2.$$

Expanding the square term on the right-hand side of (A16), using (A17) and the fact that q_j and ψ_j are mutually and serially uncorrelated, equation (A16) can be rewritten

$$(A18) \quad V(e) = \sigma_\psi^2 + \min_{\{a_j\}} \left[\sum_{j=1}^{\infty} a_j^2 \sigma_\psi^2 + Q[\{a_j\}] \sigma_q^2 \right]$$

where

$$(A19) \quad Q[\{a_j\}] = \left(1 - 2 \sum_{j=1}^{\infty} a_j \rho^j + \sum_{j=1}^{\infty} \sum_{i=1}^{\infty} a_j a_i \rho^{j+i} \right).$$

The difference between the minimization problems in (A15) and (A16) is only that in the later case the constant B has been absorbed into the minimization problem. Hence the minimizing values of $\{a_j\}$ are identical for the two problems and are (see (10)) given by

$$(A20) \quad a_j = (\rho - \lambda) \lambda^{j-1};$$

substituting (A20) into (A19) and rearranging,

$$(A21) \quad Q[\{a_j\}] = \frac{1 - \rho^2}{1 - \lambda^2} > 0.$$

By the envelope theorem the sign of the total effect of a change in σ_q^2 on $V(e)$ in (A18) is the same as that of the direct effect of σ_q^2 which is given by $Q[\cdot]$ in (A21). Since $Q[\cdot]$ is positive this implies that the total effect of a decrease in σ_q^2 is to decrease $V(e)$. But we saw in footnote 17 that an increase in β reduces B which reduces σ_q^2 through (A17). It follows that an increase in β reduces $V(e)$.

3. DERIVATION OF EQUATION (20)

Collecting all the terms involving $B_0 A$ in (19) it is easily seen that their sum is zero. Substituting (4) and (6) into the remaining expression and taking its unconditional expected value, we obtain

$$(A22) \quad E J(\cdot) = \sum_{i=0}^{\infty} \beta^i \left[B E \left\{ p_t - (\rho - \lambda) \sum_{j=0}^{\infty} \lambda^j p_{t-j-1} \right\} (A + p_t) - \frac{1}{2} E (B_0 A + B p_t)^2 \right].$$

In deriving (A22) use has been made of the fact that ψ has a zero expected value and is statistically independent of p . Passing the expectation operator in (A22) through and recalling that $Ep_i = 0$, $Ep_i^2 = \sigma_p^2$, and $Ep_i p_{i-j} = \rho^j \sigma_p^2$ we obtain

$$(A23) \quad EJ(\circ) = \sum_{i=0}^{\infty} \beta^i [B\sigma_p^2 \{1 - (\rho - \lambda)(\rho + \lambda\rho^2 + \lambda^2\rho^3 + \dots)\} - \frac{1}{2}(B_0^2 A^2 + B^2 \sigma_p^2)].$$

Summing the infinite sums involving β and $\lambda\rho$ and using (16), (A23) can be rewritten

$$(A24) \quad \frac{1}{1-\beta} \left[\frac{1-\beta\rho^2}{1-\beta\rho\lambda} \frac{1-\rho^2}{1-\lambda\rho} \sigma_p^2 - \left(\frac{1-\beta\rho^2}{1-\beta\rho\lambda} \right)^2 \frac{\sigma_p^2}{2} - \frac{1}{2} \left(\frac{1-\beta\rho}{1-\beta\lambda} \right)^2 A^2 \right].$$

Equation (20) in the text follows by noting that $\sigma_p^2 = \sigma_0^2 / (1 - \rho^2)$, $K^2 = A^2 / \sigma_p^2$, and by rearranging.

4. PROOF OF PROPOSITION 4

It is convenient to start by rewriting equation (23) of the text as

$$(A25) \quad \frac{1-\beta}{\sigma_0^2} F(\lambda) = A_D(\lambda) - B_D(\lambda) - C_D(\lambda)K^2 \equiv F_1(\lambda)$$

where

$$(A26-1) \quad A_D(\lambda) = \frac{\rho(1-\beta\rho^2)(1+\beta-2\beta\rho\lambda)}{(1-\beta\rho\lambda)^2(1-\rho\lambda)^2},$$

$$(A26-2) \quad B_D(\lambda) = \frac{\beta\rho(1-\beta\rho^2)^2}{(1-\rho^2)(1-\beta\rho\lambda)^3},$$

$$(A26-3) \quad C_D(\lambda) = \frac{\beta(1-\beta\rho)^2}{(1-\rho^2)(1-\beta\lambda)^3}.$$

LEMMA: $A_D(\lambda)$, $B_D(\lambda)$, and $C_D(\lambda)$ are all monotonically increasing functions of λ .

PROOF OF LEMMA: The Lemma follows by inspection of (A26-2) and (A26-3) and by noting that the partial derivative of $A_D(\lambda)$ with respect to λ is

$$(A27) \quad \frac{\partial A_D(\lambda)}{\partial \lambda} = \frac{2\rho^2(1-\beta\rho^2)[(1-\rho\beta\lambda)^2 + (\beta(1-\rho\lambda))^2 + \beta(1-\rho\lambda)(1-\rho\beta\lambda)]}{[(1-\beta\rho\lambda)(1-\rho\lambda)]^3}$$

Since β , ρ , and λ are all between zero and one, the partial derivative in (A27) is positive. This completes the proof of the Lemma.

If $F(\lambda)$ is negative for all $0 \leq \lambda \leq \rho$, $G(\lambda)$ is decreasing in λ in all the relevant range and $\lambda^* = 0$. Since $\beta < 1$ and $\sigma_0^2 > 0$, $F(\lambda)$ and $F_1(\lambda)$ have the same sign. It follows that a sufficient condition for $\lambda^* = 0$ is $F_1(\lambda) < 0$ for all $0 \leq \lambda \leq \rho$. The Lemma implies

$$(A28) \quad F_1(\lambda) \leq A_D(\rho) - B_D(0) - C_D(0)K^2 \equiv H_1, \quad 0 \leq \lambda \leq \rho.$$

Hence a sufficient condition for $\lambda^* = 0$ is $H_1 < 0$ which, using (A28), is equivalent to

$$(A29) \quad K^2 > c_1.$$

Similarly a sufficient condition for $\lambda^* = \rho$ is $F_1(\lambda) > 0$ for all $0 \leq \lambda \leq \rho$. The Lemma implies

$$(A30) \quad F_1(\lambda) \geq A_D(0) - B_D(\rho) - C_D(\rho)K^2 \equiv H_2, \quad 0 \leq \lambda \leq \rho.$$

Hence a sufficient condition for $\lambda^* = \rho$ is $H_2 > 0$ which, using (A30), is equivalent to

$$(A31) \quad K^2 < c_2.$$

We prove that $c_1 \geq c_2$ by contradiction. Suppose $c_1 < c_2$; then there exists a value of K^2 such that $c_1 < K^2 < c_2$. Since $K^2 > c_1$ (A29) implies that $\lambda^* = 0$. But since $K^2 < c_2$ (A31) implies that $\lambda^* = \rho$, which is a contradiction. This establishes parts (i)-(iii) of the Proposition.

A sufficient condition for an internal value of λ^* is

$$(A32) \quad F_1(0) > 0 \text{ and } F_1(\rho) < 0.$$

Using (A25) in (A32) and rearranging, this sufficient condition can be reformulated as

$$(A33) \quad c_L < K^2 < c_H.$$

We prove that when $c_H > c_L$ both of those numbers must be between c_1 and c_2 by contradiction. Suppose (A33) is satisfied but that c_L is lower than c_2 . Then there exists a K^2 which satisfies both (A31) and (A33), implying that λ^* is internal and also equal to ρ , which is a contradiction. Alternatively suppose (A33) holds and $c_H > c_1$. Then there exists a K^2 that satisfies both (A29) and (A33), implying that λ^* is both internal and equal to 0, which is a contradiction. It follows that when $c_L < c_H$, both c_L and c_H must be bounded between c_2 and c_1 . This establishes part (iv) of the Proposition.

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