GRANGER CAUSALITY IN VOLATILITY BETWEEN AUSTRALIAN EQUITY AND DEBT MARKETS: A BAYESIAN ANALYSIS

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Abstract

This paper is concerned with identifying Granger causality in the volatilities of returns between the Australian equity and debt markets. Using a bivariate stochastic volatility model previously described by Yu and Renate (2006), we estimate and compare four causal models between equity market volatility, and the short term and long term debt market volatilities. The causal models are compared with two non-causal, bivariate stochastic volatility models. Models comparisons are performed using the Deviance Information Criteria (DIC). Modelling results suggest that bond market volatility Granger causes equity market volatility. Equity volatility and money market volatility show evidence of Granger causality between the two, but no dominate causal direction is identified suggesting causal feedback between the two market volatilities.

Keywords: Granger Causality, Financial Markets, Stochastic Volatility, Bayesian Analysis

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Introduction

Why is it important to understand the causal relationships between markets? How one market responds to changes in another is important information for investors and risk managers. Although these concerns can usually be satisfied with measures of correlation, knowing how our society and its social constructs relate causally can be valuable from purely a scientific or intellectual perspective, and more practically from a policy perspective. A better empirical understanding of the direction of market influence, supplies both support or otherwise, to the theories of how asset prices are achieved or influenced across different markets. Do changes in equity market prices or volatility have a dominant impact on money market prices or volatility, or does the influence flow in the opposite direction? Do changes in the prices or volatilities of long term debt instruments influence equity prices or volatility? Often, research studies that investigate the interaction of financial markets are only interested in those linkages as measured by covariance or correlation. Although these measures are important, they leave the story only partially told. What is lacking often in market linkage research is a discussion of directional influences. This deficiency is in part due to the controversial nature of causality as compared with correlation. Causality is more often associated with research that involves controlled experimentation, than with non-experimental fields such as financial economics. A less controversial view of causal knowledge, and one to which most subscribe, is that causal knowledge at very least involves the knowledge of causal direction, or causal antecedent. Although events A and B may be shown to be correlated, an investigator or theorist may well hold the belief that event A causes event B, but event B does not cause event A. In this paper we initiate an investigation seeking to identify causal relationships between volatilities in various Australian asset markets, where causation is defined under the conceptual framework of Granger causality.

In the following sections, we will review existing linkage research that covers markets analysed in this paper. We then review the concept of Granger causality, before describing the models used in the analysis. Our methodology is then described, followed by a discussion of the results and conclusions.

Literature Review

There are many studies that investigate linkages between various markets. Here we discuss two that cover such linkages in the Australian context, and which are closely aligned with the study in this paper.

Kim, In and Viney (2001) is the most similar to the investigation described in this paper. Unlike our own, the authors apply a classical framework using a Generalized Method of Moments (GMM) estimation approach. Using a data series, which spans from January 1988 to December 1999, containing futures prices in equities, money and bond market instruments, they conclude that Australia's futures market price return volatilities are highly correlated. They find that the equity series is dominated by asymmetric negative shocks, whilst the money and bond markets are dominated by positive shocks. The 'bad news' arrivals in the equities, were found to increase the volatility in the money and bond markets, whilst larger shocks in the bond market decreased volatility in the equity and money markets. No significant day-of-the-week effects on returns, other than in the money market, were found whilst monetary policy was found to have a simultaneous effect on all market volatilities.

Fleisher (2003) paper is based on the earlier work of Feming, Kirby and Ostdiek (1998), who modelled two sources of volatility linkages, one being new information common to the equity, money and bond markets and which affects them simultaneously, and the other, information spill-over effects between markets where investor's portfolio re-balancing impacts different markets. Volatility linkages between equity, money and bond markets both within and between Australian and US markets, is investigated using a rational expectations framework. Statistical analysis is performed using GMM estimation. Fleisher's study concludes that the model fits the data extremely well. Correlations between markets based on raw return proxies for volatility such as absolute and squared lagged returns are found to under represent the true high levels of correlations based on the estimated model.

Although the paper by Wang (2009) is an investigation of US market linkages rather than Australian, he uses implied volatilities to carry out measure of correlations between the US equity, bond and money markets. More importantly however, is his discussion of the problems of spurious regressions and their control. Wang (2009) notes that the presence of highly persistent implied volatilities can raise concerns regarding spurious regression results. Spurious regression results can become a concern when highly persistent independent variables and dependent variables lead to autocorrelation in the error terms. This autocorrelation in the error terms can result in biased standard errors and spurious conclusions regarding model parameters. The problem of spurious regressions can help explain Granger causality test results that change when further independent lagged variables are included in extended models. Granger, Hyung and Jeon (2001) recommended that the only approach to avoiding spurious regression is to incorporate further independent or lagged variables.

The Granger Causality Model

An operational definition of causality is provided in Granger (1980). Granger's particular concept of causality has become known in the econometrics literature as 'Granger Causality'. The first rule of Granger causality is that only the past and present can give rise to causal antecedents, and their effects must be temporally ordered with causal antecedents being either lagged or contemporaneous to the observed effect events. The second rule is that the conditioning information set is minimal. That is, information does not contain observations that are deterministic functions of other observations. Finally, causal relationships should be invariant, whereby causal relationships between events remains constant overtime. A general definition given by Granger (1980) for Granger causality is

$$Prob(X_{n+1} \in A|\Omega_n) \neq Prob(X_{n+1} \in A|\Omega_n - Y_n) . \qquad (1)$$

Where A is a subset of values in the support of X_{n+1}. This description states that the information set Y_n, Granger causes X_{n+1}, given that the probability of $X_n \in A$ changes when Y_n is excluded from the universal information set Ω_n. The concept of Granger causation is therefore a probabilistic interpretation of causation. In this investigation, the information set that is removed from the universal set is the lagged latent log-volatility of one asset market from the the latent log-volatility process of a second asset market.

The stochastic volatility model applied in this research is taken from that described in Yu and Meyer (2006). Yu et al (2006) referred to this model as the Granger Causality model (mGC). Under this model, the mean-centred log-return process is modelled as follows.

$$y_t = \Omega_t \varepsilon_t \qquad (2)$$
$$\varepsilon_t \sim NID(0, \Sigma_\varepsilon)$$

Here $y_t = (y_{1,t}, y_{2,t})'$ is an observation of the mean-centred log-return series at time *t*. The innovation $\varepsilon_t = (\varepsilon_{1,t}, \varepsilon_{2,t})'$ is generated by a standard bivariate normal process that has a covariance matrix Σ_ε, and with correlation of $Corr(\varepsilon_{1,t}, \varepsilon_{2,t}) = \rho_\varepsilon$, giving

$$\Sigma_\varepsilon = \begin{bmatrix} 1 & \rho_\varepsilon \\ \rho_\varepsilon & 1 \end{bmatrix}. \quad (3)$$

Whilst $\varepsilon_t = (\varepsilon_{1,t}, \varepsilon_{2,t})'$ and Ω_t is equal to

$$\Omega_t = \begin{bmatrix} exp(h_{1,t}/2) & 0 \\ 0 & exp(h_{2,t}/2) \end{bmatrix} \qquad (4)$$

The log-volatility process is modelled as a mean reverting process,

$$h_t = \mu + \Phi(h_{t-1} - \mu) + \eta_t \qquad (5)$$
$$\eta_t \sim NID(0, \Sigma_\eta).$$

Where $h_t = (h_{1,t}, h_{2,t})'$, $\mu = (\mu_1, \mu_2)'$ and $\eta_t = (\eta_{1,t}, \eta_{2,t})$. The covariance Σ_η is given by

$$\Sigma_\eta = \begin{bmatrix} \sigma^2_{\eta_1} & 0 \\ 0 & \sigma^2_{\eta_2} \end{bmatrix}. \qquad (6)$$

And

$$\Phi = \begin{bmatrix} \phi_{11} & 0 \\ \phi_{21} & \phi_{22} \end{bmatrix}. \qquad (7)$$

Under this model the univariate marginal and conditional distributions can be described as follows.

$$y_{2,t} | y_{1,t} \sim N\left(\rho_\varepsilon y_{1,t} exp\left(\frac{h_{2,t} - h_{1,t}}{2}\right), exp(h_{1,t})(1 - \rho_\varepsilon^2)\right) \qquad \textbf{(8)}$$
$$y_{1,t} \sim N(0, exp(h_{1,t})) \qquad \textbf{(9)}$$
$$h_{1,t} \sim N(\mu_1 + \phi_{11}(h_{1,t-1} - \mu_1), \sigma^2_{\eta_1}) \qquad \textbf{(10)}$$
$$h_{2,t} | h_{1,t-1}, h_{2,t-1} \sim N(\mu_2 + \phi_{21}(h_{1,t-1} - \mu_1) + \phi_{22}(h_{2,t-1} - \mu_2), \sigma^2_{\eta_2}) \qquad \textbf{(11)}$$
$$h_{1,t=0} \sim N(\mu_1, \sigma^2_{\eta_1}) \qquad (12)$$
$$h_{2,t=0} \sim N(\mu_2, \sigma^2_{\eta_2}) \qquad \textbf{(13)}$$

In the above description of the Granger Causality model (mGC), *asset market 2* volatility is partly Granger caused by volatility in *asset market 1* when the parameter ϕ_{21} is non-zero. The model that hypothesizes Granger causality in volatility from *asset market 1* to *asset market 2* is described as $mGC_{market\ 1 \to market\ 2}$.

To provide a benchmark comparison for the causal models, a non-causal model is also estimated. We take the mGC model and set its parameter $\phi_{21} = 0$. We call this model the non-causal bivariate stochastic volatility model and designate it $mBSV$.

The Methodology

Bayesian analysis, including model estimation and comparisons, of each model is carried out using the freely available Bayesian software, WinBUGS[15,16]. This tool is the windows version of the original 'Bayesian Analysis Using Gibbs Sampling' tool known as BUGS, which implements, Gibbs sample and other Markov Chain Monte Carlo (MCMC) inferential techniques. In our results generation, we produce three sets of results for each combination of markets. These combinations are the equity and money markets, and the equity and bond markets. Of each of the three result sets per market combinations, one result is from the bivariate stochastic volatility model, and the other two from the causal models, where the direction of causality runs in one direction and is then reversed. Each model is defined in Table 1, showing the hypothesized direction of Granger causality in volatility implicit in the model.

[15] WinBUGS is available for free download from http://www.mrc-bsu.cam.ac.uk/bugs/

[16] Other statistical analysis is carried out using R, and the R package 'coda'.

Table 1. Bivariate Stochastic Volatility and Granger Causality Model Definitions

Model	Volatility in Asset Market 1 $\overset{GC}{\rightarrow}$ Volatility in Asset Market 2
$mBSV_{E-B}$	Equity – Bond
$mGC_{E\rightarrow B}$	Equity $\overset{GC}{\rightarrow}$ Bond
$mGC_{B\rightarrow E}$	Bond $\overset{GC}{\rightarrow}$ Equity
$mBSV_{E-M}$	Equity – Money
$mGC_{E\rightarrow M}$	Equity $\overset{GC}{\rightarrow}$ Money
$mGC_{M\rightarrow E}$	Money $\overset{GC}{\rightarrow}$ Equity

When using MCMC simulation to estimate model parameters, it is necessary to monitor convergence of the chain of iterates samples being generated. This ensures that the samples drawn are drawn from a stationary markov chain. To monitor convergence, Geweke's Z-score (Geweke, 1992) is used.[17] To overcome any problems associated with poor mixing around the mode of support, we use a single, long sample chain, for each of the parameter estimations. Poor mixing around the mode of support may occur in situations where the posterior is multi-modal, or the MCMC algorithm moves only slowly around the mode. In our estimation process, in some situations, chain lengths exceed 2 million iterations, and in one model estimation, the chain was run up to 3 million iterations. For each chain, only the 10th iteration is recorded to reduce demands on computer memory storage. The length of each chain is determined by reference to Geweke's Z-Score, with each MCMC chain continued until all parameters achieve a Z-score within modulus 2. All Z-Scores are calculated based on the recommended default settings of 0.1 and 0.5 for the first window and second window respectively. In most model estimations, the default settings are sufficient. In a few situations, which have been identified in the subsequent results, first windows settings of 0.2 or 0.3 are required however, these were only accepted when the runs are already very long, and most other parameters have already achieved a Z-Score within modulus 2.

Identification of Granger Causality proceeds based on the foundations of the concept described earlier, particularly as in equation (1). In a slight modification of this, as we do not determine the posterior predictive distributions as a measure of this condition, instead measure each models performance using the Deviance Information Criteria (DIC). The DIC measure, much like the Akaike Information Criteria (AIC) or Bayesian (Schwarz) Information Criteria (BIC) measures, uses a prescribed measure of model 'fit' to the observed data, usually based on some deviance criteria, and a complexity measure that penalizes an increasingly complex model. In the classical sense, model complexity is usually measured by the number of free parameters available within the model, as models with increasing numbers of free parameters achieve a better fit. The DIC measure is particularly suited for hierarchical models, where the number of parameters can exceed the number of observations. This is the situation with the models estimated in this study. Because we have a latent log-volatility h_t present, estimation of the latent log-volatility is also required. Due to the inclusion of these latent factors in the estimation results, we have more free 'parameters' than observations.

In testing for Granger Causality, we compare each models adequacy relative to the reversed causal model and non-causal model counterpart. Models that achieve a lower DIC score have a better, complexity adjusted, fit to the observed data than those with a comparatively higher DIC score. DIC measures can be either negative or positive. Although the DIC measure is available under the WinBUGS tool, its calculation is straightforward, and any MCMC output can have a DIC calculated. A comprehensive discussion of DIC and its use in comparing stochastic volatility models can be found in Berg, Meyer and Yu (2004).

As part of a Bayesian analysis, it is necessary to define prior probability distributions over the parameters of the models. The priors represent the prior beliefs of the investigator, before new data is observed. These priors are detailed in the next section.

Priors

The proper priors are specified over the model parameters. The intention is to provide dispersed distributions, reflecting uninformative prior beliefs. The low informativeness of the priors allows the information contained within the observed return data to dominate the estimation of parameter values. Specific distributions were selected based on the theoretical range of the parameters to be estimated. These are,

[17] Calculation of Geweke's Z-Score is performed using the function 'geweke.diag' available under the 'coda' add-in package under R.

$$\mu_1 \sim N(0, 25), \qquad \mu_2 \sim N(0, 25)$$

$$\tau_{\eta_1} \sim Gamma(2.5, 0.025), \quad \tau_{\eta_2} \sim Gamma(2.5, 0.025)$$

Where,

$$\sigma^2_{\eta_1} = \frac{1}{\tau_{\eta_1}}, \quad \sigma^2_{\eta_1} = \frac{1}{\tau_{\eta_1}}.$$

$$\phi_{11} \sim Beta(20, 1.5), \qquad \phi_{21} \sim N(0, 10), \qquad \phi_{22} \sim Beta(20, 1.5)$$

$$\rho_\varepsilon \sim U(-1, 1)$$

$$h_{1,t=0} \sim N\left(\mu_1, \frac{1}{\tau_1}\right), h_{2,t=0} \sim N\left(\mu_2, \frac{1}{\tau_2}\right)$$

We see from the prior distributions that the parameter ϕ_{21} is centred on zero, reflecting a prior belief that log-volatility in the second asset market of the mGC model, is not caused by a one period lagged volatility in the first asset market. Likewise, the dispersion around zero is large, reflecting uncertainty regarding this proposition.

Although deviating from the nomenclature of the models described in the previous sections, we specify priors against the inverse of $\sigma^2_{\eta_1}$ and $\sigma^2_{\eta_2}$, which are the precision parameters $\tau_{\eta_1}, \tau_{\eta_2}$. Precision parameters, as opposed to variance parameters, are more readily assumed in the WinBUGS software.

Although prior specifications are a necessary component of a Bayesian analysis, when defining priors there is always a risk that some informativeness will be included. Given the length of the data series involved however, it is expected that most of the prior information will be overwhelmed by information contained within the observed data. This of course can be readily confirmed by altering the priors and re-estimating the models to check for robustness.

The Data

We use price returns on the Australian All Ordinaries and S&P200 Index Futures contracts, 90 day Bank Bill contracts and 10 Year Government Bond Futures contracts as proxies for the returns in the equity, money and bond markets respectively. Data is taken from Datastream International sources, and comprises daily closing prices starting from March 2001 to January 2008. This covers 1,782 observations for each of the three series.

In a similar manner to Fleischer (2003), closing prices are adjusted using the Sydney Futures Exchange guide to pricing conventions. Prices for 90 day Bank Bill futures contracts are calculated using,

$$P_t = \frac{1,000,000 \times 365}{365 + \left(\frac{90 \times yield_t}{100}\right)}.$$

Where $yield_t$ = 100 – Futures Price Quote (at time *t*).

The price of a single Australian Commonwealth Government 10 year Bond Futures contract is given by

$$P_t = 1000 \times \left[\frac{\frac{\$6.00}{2} \times \left(1 - \left(1 + \frac{yield_t}{200}\right)\right)^{-20}}{\left(\frac{yield_t}{200}\right)} + \$100 \times \left(1 + \frac{yield_t}{200}\right)^{-20} \right]$$

Here also, the annual yield is calculated by deducting the quoted Futures price from 100. The coupon rate on the theoretical 10-year Government Bonds is 6% p.a.

The S&P SPI Futures only began trading on 2nd May 2000, so to construct the longer series we combine the previous All Ordinaries SPI Futures contracts with the available S&P/ASX200 SPI Futures.

The return series is calculated as the mean centred natural log of price differentials at time t for each market i.

$$r_{i,t} = log_e\left(\frac{P_{i,t}}{P_{i,t-1}}\right)$$

$$y_{i,t} = r_{i,t} - \left\{\frac{1}{T}\sum_{t=1}^{T} r_{i,t}\right\}$$

Although the intra-day price movements provide an opportunity to compare causal relationships between asset markets at a finer level of detail, the daily prices are considered to provide sufficient frequency so as to avoid the risk of missing causal events that can occur when longer inter-event durations are used.

Summaries of the return data and proxies for volatility are provided in Table 2 and Table 3. The correlations in Table 3 suggest that the bond market returns and volatility proxies are more correlated with equity market returns than the money market returns.

Table 2. Descriptive statistics for the daily holding period log-return (r_t) equity, money and bond market Series

Asset Market	Mean **(%)**	Std dev. **(%)**	Skewness	Kurtosis
Equity	0.03369	0.79881	-0.2183	3.258
Money	-0.00038	0.01042	-2.459	52.105
Bond	-0.00448	0.44375	-0.2100	1.770

Table 3. Correlations with the Equity Market

	y_t	$\|y_t\|$	y_t^2	Δy_t
Money	-0.07079	0.05968	0.04029	-0.05996
Bond	-0.16938	0.13207	0.09099	-0.16073

Similar to Kim, In and Viney (2001), our study sample also presents return asymmetries for the equity, money and bond returns. Over the period of the data series, innovations in observed equity returns showed a marginal predominance of down ticks that averaged a daily return of -0.85%, whilst daily up tick returns averaged 0.98%. Money market returns were marginally dominated by upward ticks in returns, with positive daily returns averaging 0.009% and down returns of -0.01%. The bond market also had a marginal dominance of positive returns over the series period, with positive daily returns averaging 0.47% and negative daily returns averaging -0.5%. Skewness and Kurtosis for both series for the equity and bond series appear quite conservative, however returns on the money market has fat tails as indicated by its high kurtosis, and it is negative, or left skewed, having relatively fewer negative returns than positive returns.

Results and Discussion

Modelling results are presented in Table 4 through to Table 9. Parameter estimates are presented for each model. Each table contains the parameter estimates drawn from the posterior distribution of the parameter, conditional on the observed return data. The details in each table are the summary statistics of the generated sample from the parameter posterior. Included in the summary statistics is the central location of the parameter's posterior, represented by its mean. The standard deviation of the sample is also provided, describing the dispersion associated with the posterior distribution. The standard error of the mean is provided as the naive standard error. The samples taken are drawn using a Markov Chain Monte Carlo simulation, where a Metropolis-Hastings algorithm is used to draw samples from the posterior distribution. Under this approach, the sample iterates drawn are correlated, requiring an adjustment to the naive standard error calculation, which results in the time-series adjusted standard error. Quantile values are also shown for each parameter estimate. In accordance with our Bayesian analysis approach, the 5% Credible Interval for each of the parameters is calculated. The 5% Credible Interval identifies the lower and upper bound values of the parameter. These bounds contain the range for the parameter value where 95% of the posterior probability density is located. This is sometimes referred to in the Bayesian literature as the 95% Highest Probability Density region.

In order for volatility in *asset market 1* to Granger cause volatility in *asset market 2*, two conditions must be met. Firstly, the ϕ_{21} parameter which relates the log-volatility of *asset market 1* to *asset market 2* in equation (11) must be significantly different from zero. To be significantly different from zero, a parameter must not have zero within its 5% Credible Interval region. Secondly, its DIC score must be lower than that of the non-causal model's DIC score, indicating that a better complexity adjusted fit is obtained by the causal model over the non-causal model.

Reviewing results for the non-causal model $mBSV_{E-B}$ in Table 4 shows that all parameters are significantly different to zero, other than μ_1. We can see that there is a high persistence in log-volatilities for both the equity and bond markets, with values of ϕ_{11} and ϕ_{22} being 0.9199 and 0.9958 respectively. Correlation in the bivariate innovations on the return equation (2), represented by ρ_ε, is small at -0.0322. Finally, variance in the log-volatility process of equities appears to be significantly larger than for the log-volatility of the bond market, with $\sigma^2_{\eta_1}$ being 3.339 compared to and 0.0053 for $\sigma^2_{\eta_2}$.

All parameters for the first causal model $mGC_{E\to B}$ in Table 5, are significantly different than zero except for μ_1 and ϕ_{22}. Importantly the parameter ϕ_{21}, which includes the effect of the equity market's log-volatility in the bond market's log-volatility is significant with a value of 0.0315. The inclusion of ϕ_{21} has resulted in a reduction in the persistence parameter, ϕ_{22}, of the bond markets log-volatility which has declined from 0.9958, for the non-causal model, to 0.1693 for the causal model. Interestingly, the comparatively high log-volatility volatility, $\sigma^2_{\eta_1}$, found in the non-causal model has reduced with the inclusion of equity market log-volatility into the log-volatility process for the bond market for $mGC_{E\to B}$. Granger causality can be said to be present when on the inclusion of a lagged or contemporaneous variable, model prediction improves. When we compare the DIC of $mGC_{E\to B}$ with the non-causal $mBSV_{E-B}$, we find that the non-causal model has a lower DIC, indicating a better fit. This suggests that volatility in the equity market does not Granger cause bond market volatility, despite the presence of a significant ϕ_{21}, as model fit performance decreases.

Table 4. Summary of posterior distributions for the non-causal bivariate stochastic volatility model for Equity (1) and Bond (2) markets

						Quantiles					*Geweke's*
$mBSV_{E-B}$	*Mean*	*CI (95%)*	*Std.dev*	*Naive SE*	*T-S SE*	*2.5%*	*25%*	*50%*	*75%*	*97.5%*	*Z Score*
μ_1	-0.0001	(-0.3872, 0.3900)	0.1986	0.0006	0.0076	-0.3897	-0.1347	0.0002	0.1340	0.3880	-0.5460
μ_2	-0.4136	(-0.7882, -0.0342)	0.1918	0.0006	0.0064	-0.8002	-0.5401	-0.4109	-0.2834	-0.0440	-1.0150
ϕ_{11}	0.9199	(0.8646, 0.9667)	0.0274	0.0001	0.0031	0.8545	0.9046	0.9244	0.9397	0.9613	1.9020
ϕ_{21}	n/a	n/a	n/a	n/a	n/a	n/a	n/a	n/a	n/a	n/a	n/a
ϕ_{22}	0.9958	(0.9906,0.9996)	0.0026	0.0000	0.0001	0.9892	0.9945	0.9962	0.9976	0.9993	-1.7440
ρ_ε	-0.0322	(-0.0561,-0.0092)	0.0126	0.0000	0.0015	-0.0590	-0.0406	-0.0311	-0.0226	-0.0110	0.7455
$\sigma^2_{\eta_1}$	3.339	(1.929,4.864)	0.7718	0.0022	0.0713	2.072	2.779	3.261	3.809	5.108	-0.9710
$\sigma^2_{\eta_2}$	0.0053	(0.0028,0.0084)	0.0015	0.0000	0.00011	0.0031	0.0042	0.0051	0.0062	0.0090	1.1120
DIC	5886.5										
Burn In	60,000										
#Iterations	180,000										

Table 5. Summary of posterior distributions for the Granger Causality model of volatility in equity market $\xrightarrow{GC}$ volatility in bond market

						Quantiles					*Geweke's*
$mGC_{E\to B}$	*Mean*	*CI (95%)*	*Std.dev*	*Naive SE*	*T-S SE*	*2.5%*	*Z Score*	*50%*	*75%*	*97.5%*	*Z Score*
μ_1	0.0297	(-0.3601, 0.4236)	0.1997	0.0005	0.0012	-0.3619	-0.1049	0.0299	0.1645	0.4220	-1.099
μ_2	-1.0889	(-1.214, -0.9613)	0.0644	0.0002	0.0065	-1.2170	-1.1320	-1.0890	-1.0450	-0.9634	1.237
ϕ_{11}	0.9936	(0.9856, 0.9994)	0.0042	0.0000	0.0004	0.9830	0.9920	0.9944	0.9964	0.9988	2.009
ϕ_{21}	0.0315	(0.0131, 0.0517)	0.0101	0.0000	0.0011	0.0145	0.0243	0.0305	0.0375	0.05400	-0.7931
ϕ_{22}	0.1693	(-0.0526, 0.4070)	0.1182	0.0003	0.0038	-0.0432	0.0880	0.1624	0.2430	0.4198	-0.6437
ρ_ε	-0.05812	(-0.1098, -0.0153)	0.0256	0.0000	0.0020	-0.1155	-0.0740	-0.0547	-0.0389	-0.0186	-0.3698
$\sigma^2_{\eta_1}$	0.9573	(0.3897, 1.6184)	0.3332	0.0008	0.0266	0.4476	0.7215	0.9091	1.1423	1.7331	-1.166
$\sigma^2_{\eta_2}$	0.3561	(0.2357, 0.4755)	0.0610	0.0001	0.0014	0.2366	0.3157	0.3557	0.3962	0.4766	0.2321
DIC	5943.9										
Burn In	20,000										
#Iterations	180,000										

Table 6. Summary of posterior distributions for the Granger Causality model of volatility in bond market $\xrightarrow{GC}$ volatility in equity market

						Quantiles					Geweke's
$mGC_{B\to E}$	*Mean*	*CI (95%)*	*Std.dev*	*Naive SE*	*T-S SE*	*2.5%*	*25%*	*50%*	*75%*	*97.5%*	*Z Score*
μ_1	-0.1108	(-0.5059, 0.2760)	0.1981	0.0004	0.0026	-0.4985	-0.2450	-0.1109	0.0230	0.2767	-0.6270
μ_2	-0.0030	(-0.1835, 0.2094)	0.0928	0.0002	0.0066	-0.1754	-0.0670	-0.0068	0.0573	0.1888	-2.0350
ϕ_{11}	0.9993	(0.9970, 0.9998)	0.0008	0.0000	0.0000	0.9968	0.9982	0.9988	0.9992	0.9998	1.550†
ϕ_{21}	0.0770	(0.0396, 0.1311)	0.0204	0.0000	0.0020	0.0446	0.0620	0.0744	0.0896	0.1231	-1.330
ϕ_{22}	0.0717	(-0.1300, 0.2820)	0.1053	0.0002	0.0016	-0.1232	-0.0004	0.0672	0.1392	0.2914	1.494
ρ_ε	-0.8331	(-0.8523, -0.8133)	0.0100	0.0000	0.0002	-0.8518	-0.8401	-0.8334	-0.8266	-0.8127	-1.120
$\sigma^2_{\eta_1}$	7.3867	(2.1280, 16.040)	3.2365	0.0069	0.3111	2.8390	5.0190	6.8110	9.0930	15.3000	0.7619
$\sigma^2_{\eta_2}$	3.1723	(2.2850, 4.1770)	0.5029	0.0010	0.0058	2.3730	2.8210	3.1120	3.453	4.3320	0.8732
DIC	5854.0										
Burn In	60,000										
#Iterations	220,000										

* Fraction in first window 0.2, fraction in second window 0.5.

Table 7. Summary of posterior distributions for the non-causal bivariate stochastic volatility model for equity and money markets

						Quantiles					Geweke's
$mBSV_{E-M}$	*Mean*	*CI (95%)*	*Std.dev*	*Naive SE*	*T-S SE*	*2.5%*	*25%*	*50%*	*75%*	*97.5%*	*Z Score*
μ_1	0.0348	(-0.3582,0.4405)	0.1974	0.0005	0.0248	-0.3323	-0.0951	0.0313	0.1469	0.4771	-1.987
μ_2	-1.967	(-2.086, -1.858)	0.0582	0.0001	0.0002	2.081	2.006	1.967	1.928	-1.853	0.1102
ϕ_{11}	0.8780	(0.6506,0.9986)	0.1118	0.0003	0.0107	0.5903	0.8234	0.9104	0.9656	0.9933	-1.560
ϕ_{21}	n/a	n/a	n/a	n/a	n/a	n/a	n/a	n/a	n/a	n/a	n/a
ϕ_{22}	0.1982	(0.1252,0.2729)	0.0377	0.0001	0.0001	0.1243	0.1729	0.1982	0.2235	0.2721	-0.2181
ρ_ε	0.0027	(-0.0410, 0.0458)	0.0219	0.0001	0.0003	-0.0414	-0.0113	0.0026	0.0170	0.0455	0.9780
$\sigma^2_{\eta_1}$	0.0182	(0.0020, 0.0538)	0.0189	0.0001	0.0019	0.0039	0.0078	0.0120	0.0207	0.0731	-0.6957
$\sigma^2_{\eta_2}$	2.897	(2.507, 3.276)	0.1959	0.0005	0.0007	2.523	2.762	2.893	3.026	3.293	0.9102
DIC	3574.1										
Burn In	20,000										
#Iterations	180,000										

Table 8. Summary of posterior distributions for the Granger Causality model of volatility in equity market $\xrightarrow{GC}$ volatility in money market

						Quantiles					Geweke's
$mGC_{E\to M}$	*Mean*	*CI (95%)*	*Std.dev*	*Naive SE*	*T-S SE*	*2.5%*	*25%*	*50%*	*75%*	*97.5%*	*Z Score*
μ_1	0.0298	(-0.3560, 0.5404)	0.1927	0.0004	0.0117	-0.3445	-0.1005	0.0288	0.1589	0.4104	0.2436
μ_2	-0.4165	(-0.8719, -0.0514)	0.1865	0.0004	0.0138	-0.7842	-0.5416	-0.4158	-0.2913	-0.0522	-1.2730
ϕ_{11}	0.9980	(0.9956, 1.0000)	0.0012	0.0000	0.0000	0.9952	0.9974	0.9982	0.9988	0.9998	-0.0351
ϕ_{21}	0.6822	(0.4247, 1.1650)	0.1485	0.0003	0.0179	0.4302	0.5701	0.6703	0.7870	0.9876	-0.2107
ϕ_{22}	0.0722	(-0.0034, 0.1494)	0.0389	0.0001	0.0007	-0.0042	0.0458	0.0722	0.0986	0.1480	0.3026
ρ_ε	0.0009	(-0.0441, 0.0473)	0.0234	0.0001	0.0001	-0.0451	-0.0149	0.0009	0.01667	0.0468	-1.3630
$\sigma^2_{\eta_1}$	0.0195	(0.0036, 0.0367)	0.0111	0.0000	0.0011	0.0075	0.0122	0.0169	0.0236	0.0479	-1.4750
$\sigma^2_{\eta_2}$	2.5671	(2.2040, 2.9280)	0.1848	0.0004	0.0025	2.2170	2.4400	2.5630	2.6900	2.9400	0.1255
DIC	3544.4										
Burn In	80,000										
#Iterations	240,000										

Table 9. Summary of posterior distributions for the Granger Causality model of volatility in money market$\xrightarrow{GC}$volatility in equity market

$mGC_{M\rightarrow E}$	*Mean*	*CI (95%)*	*Std.dev*	*Naive SE*	*T-S SE*	*Quantiles* 2.5%	25%	50%	75%	97.5%	*Geweke's Z Score*
μ_1	-0.0060	(-0.3961, 0.3865)	0.2001	0.0004	0.0118	-0.3917	-0.1422	-0.0083	0.1286	0.3911	0.2526†
μ_2	-0.3971	(-0.7691, -0.0141)	0.1931	0.0004	0.0157	-0.7692	-0.5293	-0.3988	-0.2670	-0.0141	1.2490
ϕ_{11}	0.9980	(0.9960, 0.9998)	0.0011	0.0000	0.0000	0.9954	0.9974	0.9982	0.9990	0.9998	-0.1859
ϕ_{21}	-0.6320	(-0.9194, -0.4156)	0.1316	0.0002	0.0168	-0.9674	-0.6949	-0.6073	-0.5404	-0.4418	1.0060
ϕ_{22}	0.0726	(-0.0052, 0.1478)	0.0391	0.0000	0.0008	-0.0042	0.0462	0.0726	0.0990	0.1488	-1.8790
ρ_ε	0.0000	(-0.0012, 0.0012)	0.0006	0.0000	0.0000	-0.0012	-0.0003	0.0000	0.0003	0.0012	-1.1890
$\sigma^2_{\eta_1}$	0.0209	(0.0055, 0.0406)	0.0101	0.0000	0.0009	0.0074	0.0137	0.0191	0.0256	0.0459	1.8000
$\sigma^2_{\eta_2}$	2.5640	(2.2090, 2.9310)	0.1844	0.0003	0.0024	2.2140	2.4370	2.5600	2.6850	2.9370	-1.6030
DIC	3547.8										
Burn In	20,000										
Iterations	300,000										

† Fraction in first window 0.2, fraction in second window 0.5.

For the results of the reversed causal model $mGC_{B\rightarrow E}$, as shown in Table 6, we find that μ_1,μ_2 and ϕ_{22} are not significantly different from zero. The parameter $\phi_{21}$is significant at 0.0770, but its small value indicates that very little of the $h_{1,t-1}$ log-volatility in the bond market is translated to the $h_{2,t}$ log-volatility in the equity market. Interestingly, we see now that given ϕ_{21} is small but significant, and ϕ_{22} is insignificant, greater persistence of $h_{1,t}$ log-volatility for the bond market is recorded with ϕ_{11} estimated at 0.9993. With the reduction in the effects of ϕ_{21} and ϕ_{22}, the volatility of the log-volatility processes, $\sigma^2_{\eta_1}$ and $\sigma^2_{\eta_2}$ grows in magnitude. The correlation coefficient on the return series innovations becoming significantly more negatively correlated when compared with models $mBSV_{E-B}$ and $mGC_{E\rightarrow B}$. The DIC score for model $mGC_{B\rightarrow E}$, which hypothesizes Granger causality in volatility from the bond to equity market, is 5854. This suggests, prima facie, that the inclusion of the lagged bond volatility improves the model fit of observed returns in the equity market, therefore indicating that bond volatility Granger causes equity volatility.

Results for the equity and money market models are shown in Table 7 through to Table 9. Beginning with the non-causal model $nBSV_{E-M}$ in Table 7, we find that μ_1 and ρ_ε are not significantly different from zero. The model's results also show strong, but less persistence ϕ_{11} in the $h_{1,t}$ log-volatility for the equity market, but a comparatively smaller persistence ϕ_{22} for the $h_{2,t}$ log-volatility for the money market returns. Volatilities of the log-volatility process for equity and money market, $\sigma^2_{\eta_1}$ and $\sigma^2_{\eta_2}$, also vary being comparatively small for the equity market and much larger for the money market.

Results for the first of the equity and money market causal models, is shown in Table 8. These estimates show that parameters μ_1, ϕ_{22} and ρ_ε are not significantly different to zero at a 5% credibility interval. The volatility persistence is strong in the equity market with ϕ_{11} at 0.9980. The influence of equity volatility on money market volatility is quite strong, with ϕ_{21}registering a 0.6822 estimate. Once again, as shown in the non-causal bivariate stochastic volatility model, volatility in the equity market log-volatility is small at 0.0195, but much larger in the money market log-volatility process at 2.5671.

Results for the final causal model between money and equity markets are reported in Table 9. Interestingly, the results are very similar to those reported in Table 8. As with the causal $mGC_{E\rightarrow M}$, μ_1, ϕ_{22} and ρ_ε in the $mGC_{M\rightarrow E}$ are not significantly different than zero. The interesting result however is in the value of ϕ_{21}, which is of a similar magnitude as that in Table 8, but of reversed sign, suggesting a damping effect on the log-volatility of the equity market returns. This suggests that higher lagged log-volatilities in the money markets have a damping effect on later log-volatilities in the equity markets. This contrasts with the $mGC_{E\rightarrow M}$ model which suggests that lagged log-volatilities in the equity market influences larger log-volatilities in the money markets. Interpretation of such a result suggests that although higher volatility in the equity markets may cause higher volatility in the money markets, the higher resulting volatility in the money market may then proceed to dampen further volatility in the equity markets. Alternatively, proceeding from an initial exogenous volatility increase in the money market, higher volatility may well dampen volatility in the equity market, which in turn dampens volatility in the money market.

When comparing the DIC results for each of the equity and money market models, we find that both the $mGC_{E \to M}$ and $mGC_{M \to E}$ have similar DIC scores, 3544 and 3547 respectively, which are lower than that of the non-causal $mBSV_{E-M}$ model with 3574. Such results suggest that although Granger causality exists between the equity and money market, based on the lower DIC scores for the causal models, feedback between them overwhelms any dominate causal direction.

Conclusion

In the paper we have described a Bayesian analysis of Granger causality between volatilities in the Australian equity and short term and long term debt markets. The Bayesian approach has been found to be comparatively straightforward to implement using freely available Bayesian analysis software tools. Setting up of the models, and carrying out analysis on the MCMC samples has required only minimal programming, with many useful analytical functions being already available through the R statistical software and packages. A drawback however of using standard MCMC algorithms as contained within the tools is that simulation and convergence of the estimates can be somewhat prolonged if one is not using fast computing resources.

Analysis and interpretation of each model suggests that bond market volatilities Granger cause equity market volatilities. In the equity and money markets however, there is prima facie evidence that Granger causality may be at work between volatilities in each market, but no dominate direction can be found suggesting the existence of volatility feedback between each. Interestingly, estimated parameter results for ϕ_{21} in models $mGC_{M \to E}$ and $mGC_{E \to M}$ suggest a damping effect between volatilities.

Although our results provide interesting findings, care must be taken in their interpretation. This is due to the potential problems of spurious regression. Granger, Hyung and Jeon (2001) suggest an approach to reducing the risk of spurious regressions, by the inclusion of further lags of the dependent and independent variables. In this early stage of our causal research, we settle for making the problem known, and recommending care in interpreting results. Future research will look at adding further lag variables to the models, and comparing the Granger causality implied by these extensions.

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