ISSN 1063-7796, Physics of Particles and Nuclei, 2011, Vol. 42, No. 1, pp. 79–92. © Pleiades Publishing, Ltd., 2011.

Some QED Processes: Light-by-Light and Moeller Scattering[1]

A. B. Arbuzov[a], V. V. Bytev[a], E. A. Kuraev[a], E. Tomasi-Gustafsson[b], and Yu. M. Bystritskiy[a]
[a]Joint Institute for Nuclear Research
[b]IPN, Orsay, France

Abstract—We consider several applications of the simplest nonlinear QED phenomena described by the light-by-light (LBL) scattering tensor. Among the relevant processes we present the splitting of high energy photon in a Coulomb field, calculate the asymptotics of differential photon photon elastic scattering. We show that LBL mechanism of the four photon mode of neutral pion decay have a dominant role compared, for instance, with the quark loop Feynman amplitude contribution. The mechanisms of creation of two and three gluon jets at colliding electro-positron beams is analyzed. We calculate also the contribution of LBL mechanism to the ortho-positronium decay width. One Of the important application is the analytic calculation of the QED contribution to the anomalous magnetic moment of the muon arising from LBL mechanism realized through electron positron loops, which is enhanced by the logarithm of the ratio of muon to electron masses. The modification of the QED kernel, which takes into account the QED polarization operator is used to extract the pure strong interaction contribution. We consider as well the problem of the Coulomb law modification. At second part of review we consider Moeller scattering process and RC to it. We show that RC are in agreement with renormalization group approach and could be taken into account in form of Drell–Yan process cross–section.

DOI: 10.1134/S1063779611010047

CONTENTS

[1] The article is published in the original.

1. INTRODUCTION

To the blessed memory of Teachers Alexandr Ilich Akhiezer, Sergej Semenovich Sannikov, and Vladimir Naumovich Gribov

Some applications of LBL scattering tensor which contribute from electron–positron closed loop are considered. Among them the total cross section of LBL scattering at high energies [4](and Refs. therein), contribution to muon anomalous magnetic moment [5–7] four photon channel of neutral pion decay [8], splitting of photon to two photons in Coulomb field [9] and the crossing process of photons fusion [10]. In modern experiments the contribution of LBL mechanisms to ortho-positronium total width [11] become important. Creation of two and three gluon jets in electron–positron collisions [12] can be investigated. We mention as well the Delbruck process-scattering of photon on Coulomb field started from [13] with further development in [7], [32]. In spite of rather cumbersome form of LBL tensor the processes considered in this section are described in a compact form.

At small photon energies, where the using of high intensity sources can be applied the cross section of photon on photon elastic scattering is very small $\sigma_{\gamma\gamma} \sim (0.1\alpha^2 r_0^2/\pi)(\omega/m)^6$, with m, r_0, ω-mass, classical radii of electron and center–of mass photon energy. So for $\omega = 1$ MeV cross section have an order $\sigma_{\gamma\gamma} \sim 10^{-65}$ cm^2. Cross section is maximal $\sigma_{\gamma\gamma} \sim 1.2\pi\alpha^2 r_0^2 \sim 1.6\times 10^{-30}$ cm^2 for $\omega \sim m$. For large values of photon energies $\omega \gg m$

cross section (in lowest order of perturbation theory) decreases as $\sigma_{\gamma\gamma} \sim (20\alpha^2 r_0^2)(m/\omega)^2$.

Additional process of annihilation of e^+e^- pair through one virtual photon to three real photons can not be directly measured due large background of direct annihilation to three photons,which mechanism dominate. Nevertheless it's contribution to the width of ortho-positronium can in principle be measured. The similar process in frames QCD-annihilation to three gluons in region of energies without narrow resonances as well can be used to investigate nonlinear effects. The problem of calculation of anomalous magnetic moment of muon as well require the knowledge of LBL scattering tensor in 6 and 8 orders of PT due to the corresponding contribution is now in frames of experimental accuracy.

A lot of attention was paid to calculations LBL tensor and investigation of manifestations of nonlinear phenomena. We send the reader to the paper of Costantini, De Tollis and Pistoni [4] with almost complete list of relevant literature. We do not pretend on the complete description of this problem. Some applications to questions mentioned above are given below.

At second part of rewiew we consider Moeller scattering process and RC to it. In Section 3 we calculate the contribution of additional hard photon emission and by using the well-known result for RC from soft photon emission and virtual RC we show that all corrections are in agreement with renormalization group approach and could be taken into account in form of Drell–Yan process. Also we put the explicit form of non-leading terms (including the compensation term form additional hard photon emission) in form of so-called K-factor.

Throughout our paper we use the next designations:

FD—Feynman diagram

LBL—light–by–light

QCD—Quantum Chromodynamics

QED—Quantum Electrodynamics

RC—radiative corrections

SM—Standard Model

2. LIGHT–LIGHT SCATTERING TENSOR AND VACUUM POLARIZATION

2.1. Photon Splitting in a Coulomb Field. Photon–Photon Elastic Scattering

Consider first the splitting of photon on an atomic electron [9]

$$\gamma(k_1, \lambda_1) + Y(p) \to Y(p') + \gamma(k_3, \lambda_3) + \gamma(k_4, \lambda_4). \quad (2.1)$$

Cross section in Weizsaecker–Williams approximation will be:

$$d\sigma_{\gamma\to\gamma\gamma} = \frac{\alpha}{\pi}\frac{dr}{r} L d\sigma_{\gamma\gamma\to\gamma\gamma}, \quad (2.2)$$

with differential photon–photon elastic scattering cross section

$$d\sigma_{\gamma\gamma\to\gamma\gamma} = \frac{\alpha^4}{2\pi r}|M|^2 \frac{d^3k_3}{\omega_3}\frac{d^3k_4}{\omega_4}\delta^4(k_1+k_2-k_3-k_4), \quad (2.3)$$

and

$$L = \int_{q^2_{\min}}^{q^2_{\max}} \frac{dz}{z}(1-F(z))^2, \quad k_2 = p - p', \quad (2.4)$$

$q^2_{\min,\max}$-minimal and maximal transversal momentum squared to the nuclei, which are determined by experiment; $F(z)$ is the atomic form–factor. The kinematical invariants are defined as

$$\begin{aligned} r &= k_3k_4 = \frac{\vec{k}_3^2}{2y(1-y)} = \omega_1^2\theta_3^2\frac{y}{2(1-y)}, \\ s &= -k_1k_3 = -\frac{1}{3}\omega_1^2\theta_3^2 y, \quad (2.5) \\ t &= -k_1k_4 = -\omega_1^2\theta_3^2\frac{y^2}{2(1-y)}, \end{aligned}$$

with $\omega_3 = y\omega_1$, $\omega_4 = (1-y)\omega_1$, $\omega_3 = \widehat{\vec{k}_1, \vec{k}_3}$., and ω_i are the energies of corresponding photons. We use here the normalization accepted in [4].

Cross section for the case of unpolarized photons can be written in form

$$\frac{d\sigma_{\gamma\to\gamma\gamma}}{d\omega_3 d\Omega_3} = \frac{4Z^2\alpha^5(1-y)}{y\pi^3}\frac{L}{\omega_3^3\theta_3^4}|\overline{M}^2|, \quad (2.6)$$

Total photon splitting cross section in the case of full screening is (we use the numerical estimation of the integral $\int_0^\infty (dr/r)\sigma^{\rm tot}_{\gamma\gamma\to\gamma\gamma}(r) = 5\times10^{-30}$ cm^2):

$$\begin{aligned} d\sigma_{\gamma\to\gamma\gamma} &= \frac{2Z^2\alpha}{\pi}\ln(183Z^{1/3})\int_0^\infty \frac{dr}{r}\sigma_{\gamma\gamma\to\gamma\gamma} \\ &= \frac{Z^2\alpha}{\pi}\ln(183Z^{1/3})10^{-29}\ \text{cm}^2. \end{aligned} \quad (2.7)$$

For large invariants case $r \sim -t \sim -s \gg m^2$ and unpolarized photons we have

$$\begin{aligned} |M|^2 \to |\overline{M}|^2 &= \frac{1}{2}[|M_{++++}|^2 + |M_{++--}|^2 \\ &+ |M_{+-+-}|^2 + |M_{+--+}|^2 + 4|M_{+++-}|^2], \end{aligned} \quad (2.8)$$