ADJACENT VERTEX-DISTINGUISHING TOTAL
COLORING OF GENERAL MYCIELSKI OF P_n AND C_n

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Abstract: In this paper, we obtain the adjacent strong edge chromatic numbers of some General Mycielski $M_n(G)$ ($n \geq 1$) of special graphs G .

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1. Introduction

Definition 1. (see [1]) $\mu(G)$ is called Mycielski Graph of G , if

$$V(\mu(G)) = V(G) \cup \{v \mid v \in V(G)\} \cup \{w\}, \text{ and } w \notin V(G)$$

and

$$E(\mu(G)) = E(G) \cup \{uv \mid uv \in E(G)\} \cup \{vw\}.$$

Definition 2. $M_n(G)$ is called the general Mycielski graph of G , n is natural number, if

$$V(M_n(G)) = \{v_{01}, v_{02}, \dots, v_{0p}; v_{11}, v_{12}, \dots, v_{1p}; v_{n1}, v_{n2}, \dots, v_{np}\},$$

$$E(M_n(G)) = E(G) \cup \{v_{ij}v_{(i+1)k} \mid v_{0j}v_{0k} \in E(G), 1 \leq j, k \leq p, i = 0, 1, \dots, n-1\},$$

where $V(G) = \{v_{0i} \mid i = 1, 2, \dots, p\}$.

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Definition 3. (see [2]) For a graph $G(V, E)$, $uv \in E(G)$, if a proper k -edge coloring f satisfy $C(u) \neq C(v)$, then f is called k -adjacent strong edge coloring of G , which is abbreviated k -ASEC of G , and

$$\chi_{as}(G) = \min\{k \mid k\text{-ASEC of } G\}$$

is called the adjacent strong edge chromatic number of G , where $C(u) = \{f(uv) \mid uv \in E\}$.

Conjecture 1. (see [2]) For any connected graph G with $|V(G)| \geq 3$ and $G \neq C_5$, then

$$\chi_{as}(G) \leq \Delta(G) + 2$$

where $\Delta(G)$ is maximum degree of G .

In [3]-[7] is studied the adjacent strong edge chromatic number of some special graph. In [8], is studied the adjacent strong edge chromatic number of Mycielski graph of some graph.

In this paper, we will examine the adjacent strong edge chromatic number of general Mycielski graph of complete graph with odd order, path, star, fan, wheel.

The terminology and notations can be found in [9]-[11].

2. Main Results

Lemma 1. (see [9]-[11]) For complete graph K_p with order p , we have

$$\chi(K_p) = \begin{cases} p-1 & \text{if } p \equiv 0 \pmod{2}; \\ p & \text{if } p \equiv 1 \pmod{2}, \end{cases}$$

where $\chi(G)$ denotes edge chromatic number of G .

Lemma 2. Let n be a number, $n \geq 4$. Then there exists of magic square

$$\begin{pmatrix} 1 & a_{12} & a_{13} & \cdots & a_{1n} \\ a_{21} & 2 & a_{23} & \cdots & a_{2n} \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ a_{n1} & a_{n2} & a_{n3} & \cdots & n \end{pmatrix}.$$

Here

$$\begin{matrix} 1 & a_{12} & a_{13} & \cdots & a_{1n} \\ a_{21} & 2 & a_{23} & \cdots & a_{2n} \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ a_{n1} & a_{n2} & a_{n3} & \cdots & n \end{matrix}$$

and

$$\begin{array}{cccccc} 1 & a_{21} & a_{31} & \cdots & a_{n1} \\ a_{12} & 2 & a_{32} & \cdots & a_{n2} \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ a_{1n} & a_{2n} & a_{3n} & \cdots & n \end{array}$$

are different permutations of $1, 2, \dots, n$.

Lemma 3. (see [9]-[11]) For k -regular bipartite graph G ,

$$\chi(G) = k.$$

Theorem 1. Let $p \geq 3$. Then $\chi_{as}(K_p) \geq p + 1$.

Proof. Suppose $V(K_p) = \{v_1, v_2, \dots, v_p\}$.

If $p \equiv 1 \pmod{2}$ and $p \geq 3$, according to [2], the conclusion is true.

Let $p \equiv 0 \pmod{2}$ and $p \geq 4$.

First, we have to prove $\chi_{as}(K_p) \geq p + 1$. Otherwise, if $\chi_{as}(K_p) = p$, then for all $v \in V(K_p)$, have $|C(v_i)| = p - 1$ and $1 \leq i < j \leq p, \overline{C}(v_i) \neq \overline{C}(v_j)$. Therefore, every color colored $\frac{p-2}{2}$ edges at most. But

$$\frac{p(p-2)}{2} < \frac{p(p-1)}{2} = |E(K_p)|,$$

it is contradiction.

So $\chi_{as}(K_p) \geq p + 1$.

We give a $(p + 1)$ -ASEC of K_p as following:

Let $C = \{1, 2, \dots, p, 0\}$, $V(K_p) = \{v_i \mid i = 1, 2, \dots, p\}$, $p \geq 4$ and $p \equiv 0 \pmod{2}$.

If $p = 4$, let f be defined as follows:

$$f(v_1v_i) = i + 1 \pmod{5}, i = 2, 3, 4; \quad f(v_2v_3) = 1, \quad f(v_2v_4) = 4, \quad f(v_3v_4) = 2.$$

If $p = 6$, let:

$$f(v_1v_i) = i + 1 \pmod{7}, \quad i = 2, 3, 4, 5, 6;$$

$$f(v_2v_3) = 0, \quad f(v_2v_4) = 4, \quad f(v_2v_5) = 1, \quad f(v_2v_6) = 5;$$

$$f(v_3v_4) = 1, \quad f(v_3v_5) = 5, \quad f(v_3v_6) = 2, \quad f(v_4v_5) = 2, \quad f(v_4v_6) = 6, \quad f(v_5v_6) = 3.$$

If $p \geq 8$, let:

$$f(v_1v_i) = i + 1 \pmod{p+1}, \quad i = 2, 3, \dots, p;$$

$$f(v_i v_{i+1}) = \frac{p}{2} + 2 + i, \quad f(v_i v_{i+2}) = i + 2,$$

$$f(v_i v_{i+3}) = \frac{p}{2} + 3 + i, \quad f(v_i v_{i+4}) = i + 3, \dots, f(v_i v_p) = \frac{p+i}{2} + 1, \quad i = 2, 4, \dots, p-2;$$

$$f(v_i v_{i+1}) = \frac{p}{2} + 2 + i, \quad f(v_i v_{i+2}) = i + 2,$$

$$f(v_i v_{i+3}) = \frac{p}{2} + 3 + i, \quad f(v_i v_{i+4}) = i + 3, \quad f(v_i v_p) = \frac{i+1}{2}, \quad i = 3, 5, \dots, p-1.$$

Obviously, f is $(p+1)$ -ASEC of K_p .

The theorem has been proven. $\square$

Theorem 2. For complete graph K_p $p \equiv 1 \pmod{2}$ and $p \geq 3$, we have

$$\chi_{as}(M_n(K_p)) = 2p - 1 \quad (n \geq 1).$$

Proof. Let $C = \{1, 2, \dots, 2p-1\}$ be a color set. By theorem 1, color the edges of K_p by $1, 2, \dots, p$, s.t.

$$C(v_{0i}) = \{1, 2, \dots, p\} \setminus \{i\} = \{f_0(v_{0i} v_{0j}) \mid i \neq j, j = 1, 2, \dots, p\}, \quad i = 1, 2, \dots, p;$$

Mark this coloring method with f_0 .

For $\Delta(M_n(K_p)) = 2(p-1)$ and the maximum degrees are adjacent, then $\chi_{as}(M_n(K_p)) \geq 2p-1$. Now, we prove $\chi_{as}(M_n(K_p)) \leq 2p-1$. We only prove the existence of $(2p-1)$ -ASEC of $M_n(K_p)$. Let

$$f(v_{ij} v_{(i+1)k}) = f_0(v_{0j} v_{0k}), \quad j \neq k; \quad j, k = 1, 2, \dots, p, \quad i \equiv 1 \pmod{2}$$

If $i \equiv 0 \pmod{2}$, the subgraph induced by all the edges between $v_{i1} v_{i2} \dots v_{ip}$ and $v_{(i+1)1} v_{(i+1)2} \dots v_{(i+1)p}$ is $(p-1)$ -regular graph in $M_n(K_p)$, by Lemma 3, we can color the edges by $p+1, p+2, \dots, 2p-1$.

In $f_0 \cup f$, the color j will disappear in set of edges color of vertex v_{ij} with $(i = 0, 1, 2, \dots, n-1; j = 1, 2, \dots, p)$, so that the $f_0 \cup f$ is a f of $(2p-1)$ -ASEC of $M_n(K_p)$, thus proving the theorem. $\square$

Theorem 3. For $m \geq 3$, then

$$\chi_{as}(M_n(P_m)) = 5.$$

Proof. Let $C = \{1, 2, 3, 4, 5\}$ be a color set, $P_m = v_{01} v_{02} \dots v_{0m}$. For $\Delta(M_n(P_m)) = 4$ and the maximum degrees are adjacent, then $\chi_{as}(M_n(P_m)) \geq$

5. Now, we prove

$\chi_{as}(M_n(P_m)) \leq 5$. We only prove the existence of 5-ASEC of $M_n(P_m)$. Let

$$\text{when } j \equiv 1, 2 \pmod{3}, f(v_{0j}v_{0(j+1)}) = j \pmod{3};$$

$$\text{when } j \equiv 0 \pmod{3}, f(v_{0j}v_{0(j+1)}) = 3, j = 1, 2, \dots, m-1;$$

For $i = 0, 1, 2, \dots, n-1$, when $i \equiv 0 \pmod{2}$,

$$f(v_{ij}v_{(i+1)(j+1)}) = \begin{cases} 4 & \text{if } j \equiv 1 \pmod{2}; \\ 5 & \text{if } j \equiv 0 \pmod{2}. \end{cases}$$

$$f(v_{i(j+1)}v_{(i+1)j}) = \begin{cases} 4 & \text{if } j \equiv 1 \pmod{2}; \\ 5 & \text{if } j \equiv 0 \pmod{2}. \end{cases}$$

Where $j = 1, 2, \dots, m-1$

For $i = 0, 1, 2, \dots, n$, when $i \equiv 1 \pmod{2}$,

$$f(v_{ij}v_{(i+1)(j+1)}) = \begin{cases} j \pmod{3} & \text{if } j \equiv 1, 2 \pmod{3}; \\ 3 & \text{if } j \equiv 0 \pmod{3}. \end{cases}$$

$$f(v_{i(j+1)}v_{(i+1)j}) = \begin{cases} j \pmod{3} & \text{if } j \equiv 1, 2 \pmod{3}; \\ 3 & \text{if } j \equiv 0 \pmod{3}. \end{cases}$$

Obviously, for any edge uv of graph $M_n(P_m)$, $C(u) \neq C(v)$.

So f is a 5-ASEC of $M_n(P_m)$. □

Theorem 4. For S_m ($m \geq 3$), then

$$\chi_{as}(M_n(S_m)) = 2m.$$

Let $C = \{1, 2, 3, \dots, 2m\}$ be a color set, $V(S_m) = v_{01}, v_{02}, \dots, v_{0(m+1)}$. For $\Delta(M_n(S_m)) = 2m$ and the maximum degrees are not adjacent, then

$$\chi_{as}(M_n(S_m)) \geq 2m.$$

Now, we prove $\chi_{as}(M_n(S_m)) \leq 2m$. We only prove the existence of 2m-ASEC of $M_n(S_m)$. Let f be:

$$f(v_{01}v_{0j})j - 1, j = 2, 3, \dots, m+1.$$

For $i = 0, 1, 2, \dots, n$, when $i \equiv 0 \pmod{2}$,

$$f(v_{i1}v_{(i+1)j}) = j + m - 1, j = 2, \dots, m+1;$$

$$f(v_{ij}v_{(i+1)1}) = j + m - 1, j = 2, \dots, m + 1.$$

When $i \equiv 1 \pmod{2}$,

$$f(v_{i1}v_{(i+1)j}) = j - 1, j = 2, \dots, m + 1; f(v_{ij}v_{(i+1)1}) = j - 1, j = 2, 3, \dots, m + 1.$$

Obviously, for any edge uv of graph $M_n(S_m)$, $C(u) \neq C(v)$.

So f is a $2m$ -ASEC of $M_n(S_m)$. □

Theorem 5. For F_m ($m \geq 4$), then

$$\chi_{as}(M_n(F_m)) = 2m.$$

Let $C = \{1, 2, 3, \dots, 2m\}$ be a color set, $V(F_m) = v_{01}, v_{02}, \dots, v_{0(m+1)}$. For $\Delta(M_n(F_m)) = 2m$ and the maximum degrees are not adjacent, then

$$\chi_{as}(M_n(F_m)) \geq 2m.$$

Now, we prove $\chi_{as}(M_n(F_m)) \leq 2m$. We only prove the existence of $2m$ -ASEC of $M_n(F_m)$. Let f be:

$$f(v_{01}v_{0j}) = j - 1, j = 2, \dots, m + 1;$$

$$f(v_{0j}v_{0(j+1)}) = \begin{cases} m & j + 1 \equiv 0 \pmod{m}; \\ (j + 1) \pmod{m} & \text{others.} \end{cases}$$

For $i = 0, 1, 2, \dots, n$, when $i \equiv 0 \pmod{2}$,

$$f(v_{i1}v_{(i+1)j}) = \begin{cases} 2m & j - 1 + m \equiv 0 \pmod{2m}; \\ (j - 1 + m) \pmod{2m} & \text{others.} \end{cases}$$

Where, $j = 2, \dots, m + 1$.

$$f(v_{ij}v_{(i+1)(j+1)}) = \begin{cases} 2m & j + 1 + m \equiv 0 \pmod{2m}; \\ (j + 1 + m) \pmod{2m} & \text{others.} \end{cases}$$

Where, $j = 2, \dots, m$.

$$f(v_{ij}v_{(i+1)1}) = \begin{cases} 2m & j - 1 + m \equiv 0 \pmod{2m}; \\ (j - 1 + m) \pmod{2m} & \text{others.} \end{cases}$$

Where, $j = 2, \dots, m+1$.

$$f(v_{i(j+1)}v_{(i+1)j}) = \begin{cases} 2m & j+1+m \equiv 0 \pmod{2m}; \\ (j+1+m) \pmod{2m} & \text{others.} \end{cases}$$

Where, $j = 2, \dots, m$.
when $i \equiv 1 \pmod{2}$,

$$f(v_{i1}v_{(i+1)j}) = \begin{cases} m & j-1 \equiv 0 \pmod{m}; \\ (j-1) \pmod{m} & \text{others.} \end{cases}$$

Where, $j = 2, \dots, m+1$.

$$f(v_{ij}v_{(i+1)(j+1)}) = \begin{cases} m & j+1 \equiv 0 \pmod{m}; \\ (j+1) \pmod{m} & \text{others.} \end{cases}$$

Where, $j = 2, \dots, m$.

$$f(v_{ij}v_{(i+1)1}) = \begin{cases} m & j-1 \equiv 0 \pmod{m}; \\ (j-1) \pmod{m} & \text{others.} \end{cases}$$

Where, $j = 2, \dots, m+1$.

$$f(v_{i(j+1)}v_{(i+1)j}) = \begin{cases} m & j+1 \equiv 0 \pmod{m}; \\ (j+1) \pmod{m} & \text{others.} \end{cases}$$

Where, $j = 2, \dots, m$.

Obviously, for any edge uv of graph $M_n(F_m)$, $C(u) \neq C(v)$.

So f is a $2m$ -ASEC of $M_n(F_m)$.

Theorem 6. For W_m ($m \geq 4$), then

$$\chi_{as}(M_n(W_m)) = 2m.$$

Proof. Let $C = \{1, 2, 3, \dots, 2m\}$ be a color set, $V(W_m) = v_{01}, v_{02}, \dots, v_{0(m+1)}$.
For $\Delta(M_n(W_m)) = 2m$ and the maximum degrees are not adjacent, then

$$\chi_{as}(M_n(W_m)) \geq 2m.$$

Now, we prove $\chi_{as}(M_n(W_m)) \leq 2m$. We only prove the existence of $2m$ -ASEC of $M_n(W_m)$. Let

$$f(v_{01}v_{0j}) = j-1, j = 2, \dots, m+1; f(v_{0(m+1)}v_{02}) = 2$$

$$f(v_{0j}v_{0(j+1)}) = \begin{cases} m & j+1 \equiv 0 \pmod{m}; \\ (j+1) \pmod{m} & \text{others.} \end{cases}$$

For $i = 0, 1, 2, \dots, n$, when $i \equiv 0 \pmod{2}$,

$$f(v_{i1}v_{(i+1)j}) = \begin{cases} 2m & j-1+m \equiv 0 \pmod{2m}; \\ (j-1+m) \pmod{2m} & \text{others.} \end{cases}$$

Where, $j = 2, \dots, m+1$.

$$f(v_{ij}v_{(i+1)(j+1)}) = \begin{cases} 2m & j+1+m \equiv 0 \pmod{2m}; \\ (j+1+m) \pmod{2m} & \text{others.} \end{cases}$$

$f(v_{i2}v_{(i+1)(m+1)}) = m+2$. Where, $j = 2, \dots, m$.

$$f(v_{ij}v_{(i+1)1}) = \begin{cases} 2m & j-1+m \equiv 0 \pmod{2m}; \\ (j-1+m) \pmod{2m} & \text{others.} \end{cases}$$

Where, $j = 2, \dots, m+1$.

$$f(v_{i(j+1)}v_{(i+1)j}) = \begin{cases} 2m & j+1+m \equiv 0 \pmod{2m}; \\ (j+1+m) \pmod{2m} & \text{others.} \end{cases}$$

$f(v_{i(m+1)}v_{(i+1)2}) = m+2$. Where, $j = 2, \dots, m$.
when $i \equiv 1 \pmod{2}$,

$$f(v_{i1}v_{(i+1)j}) = \begin{cases} m & j-1 \equiv 0 \pmod{m}; \\ (j-1) \pmod{m} & \text{others.} \end{cases}$$

Where, $j = 2, \dots, m+1$.

$$f(v_{ij}v_{(i+1)(j+1)}) = \begin{cases} m & j+1 \equiv 0 \pmod{m}; \\ (j+1) \pmod{m} & \text{others.} \end{cases}$$

$f(v_{i2}v_{(i+1)(m+1)}) = 2$. Where, $j = 2, \dots, m$.

$$f(v_{ij}v_{(i+1)1}) = \begin{cases} m & j-1 \equiv 0 \pmod{m}; \\ (j-1) \pmod{m} & \text{others.} \end{cases}$$

Where, $j = 2, \dots, m+1$.

$$f(v_{i(j+1)}v_{(i+1)j}) = \begin{cases} m & j+1 \equiv 0 \pmod{m}; \\ (j+1) \pmod{m} & \text{others.} \end{cases}$$

$f(v_{i(m+1)}v_{(i+1)2}) = 2$ Where, $j = 2, \dots, m$.

Obviously, for any edge uv of graph $M_n(W_m)$, $C(u) \neq C(v)$.

So f is a $2m$ -ASEC of $M_n(W_m)$. □

Conjecture 2. For simple graph G , $V(G) \geq 2$ and $n \geq 1$, then

$$\chi_{as}(M_n(G)) = \begin{cases} \Delta(M_n(G)) + 1 & \text{if maximum degree vertices is adjacent;} \\ \Delta(M_n(G)) & \text{others.} \end{cases}$$

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