Optical source transformations

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Abstract: Transformation optics is a recently appreciated methodology for the design of complex media that control the propagation of electromagnetic and other types of waves. The transformation optical technique involves the use of coordinate transformations applied to some region of space, providing a conceptual means to redirect the flow of waves. Successfully designed devices to date have made use of transformations acting on passive space only; however, the technique can also be applied when source distributions (e.g., current and charge) are included within the space being transformed. In this paper we present examples of source transformations that illustrate the potential of these expanded transformation optical methods. In particular, using finite-element full-wave simulations, we confirm the restoration of dipole radiation patterns from both a distorted 'pin-wheel' antenna and a bent dipole partially occluded by a cylindrical scatterer. We propose the technique of source transformations as a powerful approach for antenna design, especially in relation to conformal antennas.

OCIS codes: (070.4690) Morphological transformations;(160.3918) Metamaterials.

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1. Introduction

The technique of transformation optics was recently introduced as a conceptual means of designing complex electromagnetic media having unique functionality [1]. Underlying the method, a coordinate transformation is generated and applied to some region of space where the trajectories and propagation characteristics of electromagnetic waves are to be modified. [2] In the transformed coordinates the light appears to bend, and there are simple tools which utilize the underlying symmetries of Maxwell's equations to allow one to translate this apparent bending of the light into actual behavior through the introduction of materials.[3]-[5]. This symmetry of Maxwell's equations under coordinate transformations is well understood and has been used previously, for example, as a computational technique in numerical simulations [6]. However, given the relatively complex material specifications that arise from the transformations (the required materials are generally anisotropic with independently spatially varying permittivity and permeability tensor elements) an actual physical implementation might have been deemed unfeasible. The emergence of metamaterials has dramatically changed that perspective. Metamaterials are artificially structured materials that offer a much broader range of electromagnetic response than naturally available materials, and which can be engineered with great precision [7]. The implementation of a metamaterial "invisibility cloak" was demonstrated at microwave frequencies in 2006, illustrating the potential for metamaterials to be the enabling technology in realizing transformation optical designs [8].

Though there has been considerable interest in the transformation optical approach for the design of structures that can conceal objects from detection by electromagnetic waves [9]-[14], a much broader range of devices can be formed using such complex media [15]-[17]. In all of these reports, the transformation of a region of passive space (usually vacuum) yields the prescription for an optical device with unique functionality. Indeed, given the dispersive nature of metamaterials, electromagnetic cloaking is likely not the most probable application of transformation optics. Rather, the integration of transformation optical media into antenna or other communication systems seems a more reasonable prospect. For such applications, the radiating elements (or, more fundamentally, the current density distributions) with the transformation optical medium should be considered together in the overall design. The underlying symmetries of Maxwell's equations on which transformation optics is based extend to regions that contain current distributions.

In cloaking transformations, a singularity is introduced (the concealment region) that maps to the inner surface of the cloaking structure under the transformation. A current density can thus naturally be imposed along the singularity, which will then radiate. In a recent report, Zhang et al. [18] make use of an elliptical transformation that maps a line current to a surface current on a sphere. Combining the transformed current distribution with the appropriate transformation optical medium produces similar radiation characteristics as the original line current. In this paper, we generalize the idea of transforming current density distributions, providing several examples that demonstrate the concept. While these examples are simply to illustrate the methods and are not of practical importance, the relevance to conformal antenna systems

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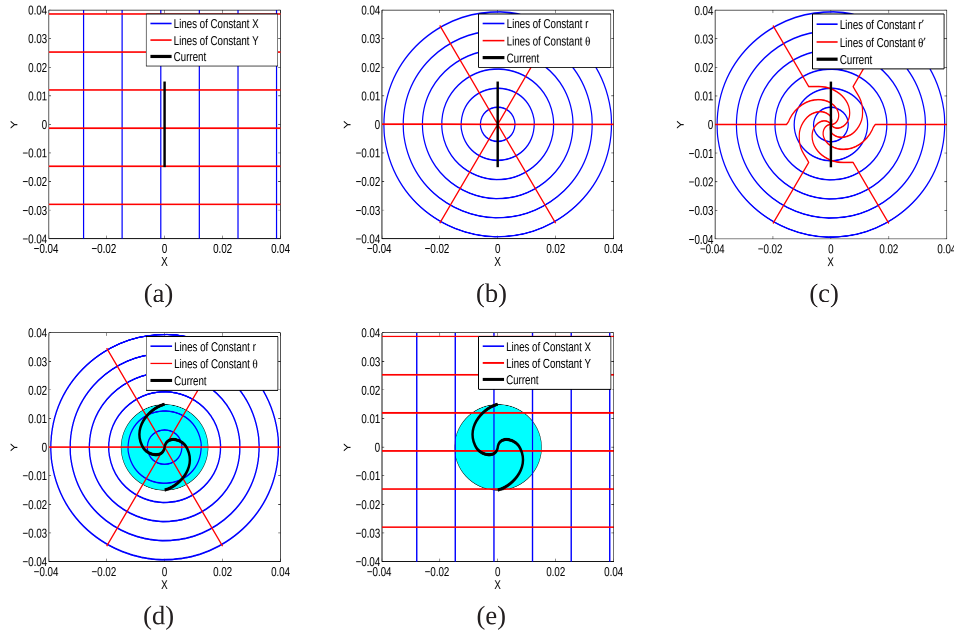

Fig. 1. The process for the ‘pinwheel’ transformation is shown schematically. (a) The current is defined in Cartesian coordinates. (b) The current is transformed into cylindrical coordinates. (c) The current is transformed into ‘pinwheel’ coordinates. Note that to this point that when shown on a cartesian axis the current has not changed, it is simply expressed differently. (d) The expression for the current from (c) is used, but $\theta' \rightarrow \theta$ and $r' \rightarrow r$. (e) The current distribution is expressed in a Cartesian basis. In (d) and (e) there is material introduced in the shaded region.

should be clear. The transformation of both passive space as well as source distributions provides enormous flexibility in the physical layout of an antenna element while retaining desired performance characteristics.

2. Extending transformation optics to sources

In order to describe the inclusion of sources into transformation optics, it is instructive to outline the process used for performing transformations. This process is shown schematically for the ‘pinwheel’ transformation[19][20] in Fig. 1.

Transformation optics makes use of the fact that Maxwell’s equations can be written such that they are ‘form invariant’ under coordinate transformations[4]. Improper rotations, which switch between right-handed and left-handed coordinates, are generally excluded from the space of allowed transformations. In general this technique can be used for arbitrary changes in the metric [3], but we focus on those induced by a coordinate change. In this formulation of Maxwell’s equations, rather than re-write the curl and divergence for each set of coordinates, all of the coordinate information is carried in the transformation of the material parameters and sources.

When engineering a device using transformation optics it is asked that one set of coordinates be used to span the space, while the fields behave as though a different set of coordinates had been used. The set of coordinates used to span the space is often chosen to be Cartesian, but is arbitrary in general. For example, cylindrical coordinates were used for the first cloaks[21][1].

To achieve this behavior, a straightforward coordinate transform is performed from Cartesian

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coordinates to a coordinate set in which the fields behave as desired (i.e. have the desired functional form). One then replaces the coordinates in which the fields behave as desired with those that will be used to span the space. This means that the functional form of the fields, material parameters and currents found is retained, but we associate them with the different coordinates. This process is often referred to as ‘dropping the primes’ and is shown schematically in Fig. 1. Finally, it is necessary to use the transform rules to find all of the material parameters and currents in a Cartesian coordinate set, where the ‘form invariant’ Maxwell’s equations can be identified with our more familiar form.

In order to perform such transformations it is critical to understand the manner in which material parameters and sources transform to satisfy the form invariant Maxwell’s equations. Under such coordinate transformations the material parameters transform as 2nd rank tensors of weight +1.

$$\varepsilon_r^{m'n'} = \frac{1}{|A|} A_m^{m'} A_n^{n'} \varepsilon_r^{mn} \tag{1}$$

$$\mu_r^{m'n'} = \frac{1}{|A|} A_m^{m'} A_n^{n'} \mu_r^{mn} \tag{2}$$

where

$$A_m^{m'} = \frac{\partial x^{m'}}{\partial x^m} \tag{3}$$

With one exception[18] these transformations have always been demonstrated in a passive space (no sources). Here these capabilities are extended to the inclusion of currents within the transformation region. The current transforms as vector density of weight +1[4].

$$j^{m'} = \frac{1}{|A|} A_m^{m'} j^m \tag{4}$$

The current being carried by a wire will be conserved by a coordinate transform. It is important to note, however, that when integrating over a wire to find the total current one should not use the Jacobian. All of the information about the coordinate change is contained within the material parameters and the current density. This ensures that when one ‘drops the primes’, abruptly altering the Jacobian by using a different set of coordinates, total current is conserved.

We note that it is possible define the current as being colinear with the wire without the defining an inner product on our manifold. This ensures that when we abruptly change the metric by ‘dropping the primes’ we are not left with a system in which the current has components transverse to the wire.

3. Examples

In previous work related to transforming sources, transforms were chosen which did not significantly alter the fields[18]. In order to demonstrate the success of this method for transforming sources, the transformation of a dipole in two examples in 2D were considered; a ‘pinwheel’ transformation and a ‘cloak’. In both of these transformations familiar cylindrical coordinates were used to span the space, while fields were asked to behave as though separate coordinates were used.

The Jacobian for the transformation from Cartesian coordinates to cylindrical coordinates, A_1, is

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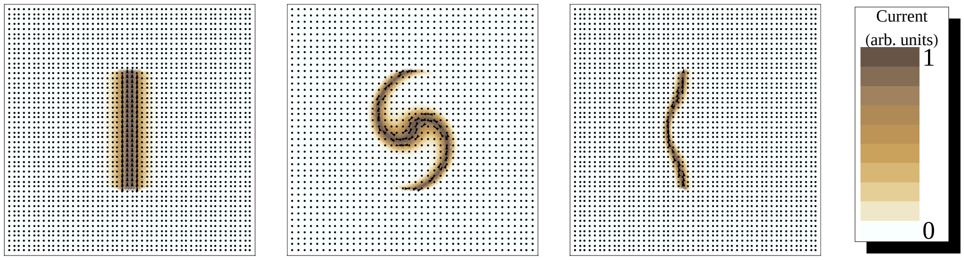

(a) (b) (c)

Fig. 2. Vector field plots of transformed currents are shown. (a) Original current (b) Current after ‘pinwheel’ transformation with a rotation of $\Delta\theta = \pi$ (c)Current after a cloaking transform NOTE: The initial current used was shifted slightly to the left to avoid current in a region with diverging material parameters.

$$A_1 = \begin{pmatrix} \cos(\theta) & \sin(\theta) & 0 \\ -\frac{\sin(\theta)}{r} & \frac{\cos(\theta)}{r} & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad (5)$$

In the first, ‘pinwheel’, transformation the coordinates are rotated by some angle, $\Delta\theta$, between the origin and $r = R_1$. Current sources that remain in the region $r < R_1$ were chosen. The following calculations are specific to this region.

$$r' = r$$

$$\theta' = \begin{cases} \theta & r > R_1 \\ \theta + \Delta\theta\left(1 - \frac{r}{R_1}\right) & r < R_1 \end{cases} \quad (6)$$

The Jacobian for this transformation, A_2, is

$$A_2 = \begin{pmatrix} 1 & 0 & 0 \\ \frac{-\Delta\theta}{R1} & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad (7)$$

An analytic expression for the current distribution to approximate a wire along $x = 0$ was chosen.

$$j = \begin{pmatrix} 0 \\ \frac{1}{\sqrt{\delta * \pi}} e^{-\frac{(x)^2}{\delta}} \\ 0 \end{pmatrix} \quad (8)$$

where δ is chosen to be small compared to the length of the wire.

Using Eq. (3) one finds the expression for the current in cylindrical coordinates, j_1, is given by

$$j_1 = \frac{1}{\sqrt{\delta * \pi}} \exp\left(-\frac{(r * \cos(\theta))^2}{\delta}\right) * \begin{pmatrix} r\sin(\theta) \\ \cos(\theta) \\ 0 \end{pmatrix} \quad (9)$$

Again using Eq. (3) to express the current in our pinwheel coordinates one finds

$$j_2 = \frac{1}{R_1\sqrt{\delta*\pi}} \exp\left(-\frac{(r'\cos(u'))^2}{\delta}\right) * \begin{pmatrix} -R_1 r' \sin(u') \\ R_1\cos(u') - r'\Delta\theta\sin(u') \\ 0 \end{pmatrix} \tag{10}$$
$$u' = \Delta\theta\left(1-\frac{r'}{R_1}\right) - \theta'$$

Here the primes are dropped and inverse of A_1 is used to retrieve the current expressed in a Cartesian basis

$$j_3 = \frac{1}{R_1\sqrt{\delta*\pi}} \exp\left(-\frac{(r*\cos(u))^2}{\delta}\right) * \begin{pmatrix} -R_1\sin(u+\theta) + r\Delta\theta\sin(\theta)\sin\left(\Delta\theta\left(\frac{r}{R_1}-1\right)+\theta\right) \\ -R_1\cos(u+\theta) + r\Delta\theta\cos(\theta)\sin(u) \\ 0 \end{pmatrix}$$
$$u = \Delta\theta\left(1-\frac{r}{R_1}\right) - \theta$$
$$r = \sqrt{x^2+y^2}$$
$$\theta = \arctan\left(\frac{y}{x}\right) \tag{11}$$

The material parameters ε and μ can be calculated in the same manner. The specifics have been excluded for the sake of brevity.

The current distributions for the original dipole, the 'Pinwheel' transformation and a 'Cloak' transformation are shown in Fig. 2.

4. Simulations

To demonstrate the success of this transform, simulations were performed in commercially available finite-element simulation software (COMSOL). A two-dimensional TM mode wave was used. Taking the analytic expression for the current to the limit $\delta \to 0$ allows the current distribution calculated in Eq. 10 to be replaced by a discontinuity in the magnetic field using the Biot-Savart Law. This discontinuity can be implemented as a source in simulations.

It was mentioned earlier that current must be conserved under these transformations. To demonstrate this explicitly for the 'Pinwheel' transformation a line cut of the current in the y direction was taken along the y=0 axis. Numerical integrals were performed as shown in Fig. 3. The total current carried throught each segment is exactly 1. While total current will always be conserved, it is noted that the current density can fluctuate in general. For instance, if the current had been chosen to be in the z-direction initially the current distribution would not be constant. However, this process of using an analytic form to perform the transformation and then integrating to find the total current in a sheet or line source, can be used for ANY source transformation.

Simulation results from a dipole with a length of one free-space wavelength are shown in Fig. 4(a). If the dipole is twisted without compensation from the correct material parameters a distinctly different field pattern is found (Fig. 4(b)). When the correct material is in place the field pattern of a dipole is retrieved in the region outside of the transformation media (Fig. 4(c)).

One potential use of this technique is to design an antenna very close to an object while limiting the interference of the object. To demonstrate this possibility we embed an antenna into a standard cloaking transform, defined below.

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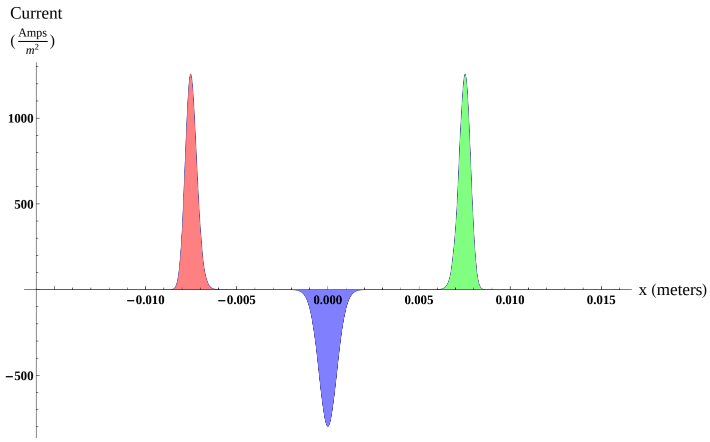

Fig. 3. A line cut along y=0 of the current in the y direction is shown. Despite the stretching induced by the transformation, each region still carries a total current of exactly one $Amp-m^{-1}$. This property will always be true under these transformations and allows the analytic current expression shown in Eq. 10 to be replaced by a discontinuity in the magnetic field in the limit $\delta \rightarrow 0$.

$$r' = \begin{cases} r & r > R_2 \\ \frac{(r-\alpha)(R_2-R_1)}{R_2-\alpha} + R_1 & \alpha < r < R_2 \\ \frac{R_1}{\alpha} r & r < \alpha \end{cases} \qquad \theta' = \theta \tag{12}$$

The cloaked region is the area $R < R_1$. The cloaking material extends to $r = R_2$. While the parameter α is generally set to 0 for cloaking, resulting in a topology change in the system, here it has been included so that the limit may be taken analytically. We note that it is preferable to move the source away from $r = R_1$. If the current is left on the boundary, any physical cloaking transform ($\alpha > 0$) will result in current everywhere in the space $r < R_1$. Neglecting this will lead to inaccuracies in simulation and experiment. Furthermore, in the ($\alpha \rightarrow 0$) limit, embedding the current directly on the boundary requires the use of sources in a region with diverging material parameters. Beginning with a dipole slightly offset from the center is sufficient to retrieve good agreement with the fields expected outside of the transformed media, even in the $\alpha \rightarrow 0$ limit.

The results of this transformation with and without compensating materials are shown in Figs. 4(d) and 4(e) respectively.

5. Conclusions

By successfully demonstrating the inclusion of sources in the tool-box of transformation optics we believe we have opened up the door for many potentially useful applications. These include embedding antennas into transformed regions to avoid interference, such as the cloak. Antennas designed in this way may also make use of the inherent properties of metamaterials to meet unique design parameters. For instance, a transformation designed antenna may have less overall metal than a standard antenna. The natural dispersion of metamaterials may then result in an antenna that interferes only very weakly at frequencies away from its frequency of operation. Under certain circumstances the over-all weight of the antenna may also be reduced.

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Finally, we expect that this tool will be particularly useful in the design of antennas with arbitrary geometry that may be used as conformal elements in larger devices, structures or even vehicles.

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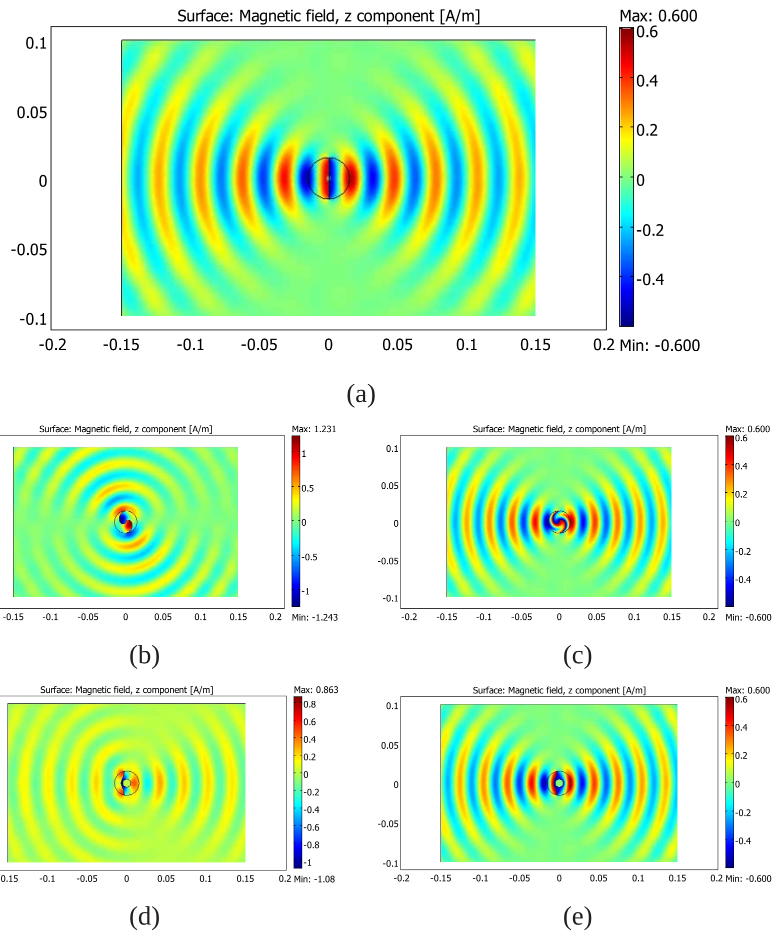

Fig. 4. The z-component of the magnetic field is plotted for (Media 1)(a) a pure dipole of lenth L=$\lambda/2$ (Media 2)(b) A dipole that has undergone a pinwheel rotation of π without material compensation. (Media 3)(c) A dipole that has undergone a pinwheel rotation of π with proper material compensation (Media 4)(d) A dipole that has been bent around a PEC scatterer (Media 5)(e) A dipole that has been bent around a PEC scatterer with proper 'cloaking' compensation. In each image there is a circle around the region in which the transformation has taken place. The current is carried on the wire inside of this region.

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