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Bipolar Neutrosophic Minimum Spanning Tree

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Abstract**—** The aim of this article is to introduce a matrix algorithm for finding minimum spanning tree (MST) in the environment of undirected bipolar neutrosophic connected graphs (UBNCG). Some weights are assigned to each edge in the form of bipolar neutrosophic number (BNN). The new algorithm is described by a flow chart and a numerical example by considering some hypothetical graph. By a comparison, the advantage of proposed matrix algorithm over some existing algorithms are also discussed.

Keywords—Neutrosophic sets, bipolar neutrosophic sets, spanning tree problem, score function.

I. INTRODUCTION

The concept of neutrosophic set (NS) in 1998 was proposed by Smarandache [1], from the philosophical point of view, to represent uncertain, imprecise, incomplete, inconsistent, and indeterminate information that are exist in the real world. The concept of the classic set, fuzzy set and intuitionistic fuzzy set (IFS) is generalized by the concept of neutrosophic set. Within the real standard or non-standard unit interval $]^{-}0, 1^{+}[$, the neutrosophic sets are categorized into three membership functions called truth-membership function (t), an indeterminate-membership function (i) and a false-membership function (f). For the first, Smarandache [1] introduced the single valued neutrosophic set (SVNS) to apply in science and engineering applications. Later on, some properties related to single valued neutrosophic sets was studied by Wang et al.[2].For dealing with real, scientific, and engineering applications, the neutrosophic set model is an important tool because it can handle not only incomplete information but also the inconsistent information and indeterminate information. One may refer to regarding the basic theory of NS, SVNS and their extensions with applications in several fields. Many researches making particularizations on the T, I, F components which leads to define particular case of neutrosophic sets such as simplified neutrosophic sets [20], interval valued neutrosophic sets [22], bipolar neutrosophic sets [23], trapezoidal neutrosophic set [24], rough neutrosophic set [25] and so on. As a special case of NSs, Ye[24] introduced the concept of single–valued trapezoidal neutrosophic set. In addition, a new ranking method to define the concept of cut sets for SVTNNs were proposed by Deli and Subas [26]. The authors applied it for solving MCDM problem. Mumtaz et al.[28] defined the concept of bipolar neutrosophic soft sets and applied it to decision making problem.

Prim and Kruskal algorithm are the common algorithms for searching the minimum spanning tree including in classical graph theory. A new theory is developed and called single valued neutrosophic graph theory (SVNGT) by applying the concept of single valued neutrosophic sets on graph theory. The concept of SVNGT and their extensions finds its applications in diverse fields [6-19]. To search the minimum spanning tree in neutrosophic environment recently few researchers have used neutrosophic methods. Ye [4] developped a method to find minimum spanning tree of a graph where nodes (samples) are represented in the form of SVNS and distance between two nodes which represents the dissimilarity between the corresponding samples has been derived. To cluster the data represented by double-valued neutrosophic information, Kandasamy [3] proposed a double-valued Neutrosophic Minimum Spanning Tree (DVN-MST) clustering algorithm.A solution approach of the optimum spanning tree problems considering the inconsistency, incompleteness and indeterminacy of the information, which was proposed by Mandal and Basu [5]. The authors consider a network problem with multiple criteria, which are represented by weight of each edge in neutrosophic set. The approach proposed by the authors is based on similarity measure. In another paper, Mullai [20] discussed the MST problem on a graph in which a bipolar neutrosopnic number is associated to each edge as its edge length, and illustrated it by a numerical example.

The main objective of this paper is to present a neutrosophic version of Kruskal algorithm for searching the cost minimum spanning tree of an undirected graph in which a bipolar neutrosophic number is associated to each edge as its edge length.

The rest of the paper is organized as follows. The concepts of neutrosophic sets, single valued neutrosophic sets, bipolar neutrosophic sets and the score function of bipolar neutrosophic number are briefly presented in section 2. A novel approach for finding the minimum spanning tree of neutrosophic undirected graph is proposed in section 3. A numerical example is presented to illustrate the proposed method in Section 4. A comparative study with existing methods is proposed in section 5, Finally, the main conclusion is presented in section 6.

II. Prliminaries

Some of the important background knowledge for the materials that are presented in this paper is presented in this section. These results can be found in [1, 2, 23].

Definition 2.1 [1] Le ξ be a universal set. The neutrosophic set A on the universal set ξ categorized into three membership functions called the true T_A(x), indeterminate I_A(x) and false F_A(x) contained in real standard or non-standard subset of]-0, 1+[respectively.

$$-0 \leq \sup \mathrm{T_A}(\mathrm{x}) + \sup \mathrm{I_A}(\mathrm{x}) + \sup \mathrm{F_A}(\mathrm{x}) \leq 3+ \quad (1)$$

Definition 2.2 [2] Let ξ be a universal set. The single valued neutrosophic sets (SVNs) A on the universal ξ is denoted as following

$$\mathrm{A} = \{< x: \mathrm{T_A}(\mathrm{x}), \mathrm{I_A}(\mathrm{x}), \mathrm{F_A}(\mathrm{x}) > x \in \xi\} \quad (2)$$

The functions $\mathrm{T_A}(\mathrm{x}) \in [0.\ 1]$, $\mathrm{I_A}(\mathrm{x}) \in [0.\ 1]$ and $\mathrm{F_A}(\mathrm{x}) \in [0.\ 1]$ are named degree of true, indeterminate and false membership of x in A, satisfy the following condition:

$$0 \leq T_A(\mathrm{x}) + I_A(\mathrm{x}) + F_A(\mathrm{x}) \leq 3 \quad (3)$$

Definition 2.3 [23]. A bipolar neutrosophic set A in ξ is defined as an object of the form A={<x, $T^p(x)$, $I^p(x)$, $F^p(x)$, $T^n(x)$, $I^n(x)$, $F^n(x)$>: x ∈ X}, where T^p, I^p, $F^p: \xi \rightarrow$ [1, 0] and T^n, I^n, $F^n: \xi \rightarrow$ [-1, 0] .The positive membership degree $T^p(x)$, $I^p(x)$, $F^p(x)$ denotes the true membership, indeterminate membership and false membership of an element ∈ ξ corresponding to a bipolar neutrosophic set A and the negative membership degree $T^n(x)$, $I^n(x)$, $F^n(x)$ denotes the true membership, indeterminate membership and false membership of an element ∈ ξ to some implicit counter-property corresponding to a bipolar neutrosophic set A.

To compare two Bipolar neutrosophic numbesr (abbr.BNNs), Deli et al.[23] introduced the concept of score function and in case where the score value of two BNNs are same, they can be distinguished by using accuracy function and certainty function as follow

Definition 2.4[23]. Let $\tilde{A} =< T^p, \mathrm{I}^p, \mathrm{F}^p, T^n, \mathrm{I}^n, \mathrm{F}^n >$ be a bipolar neutrosophic number, then, the score function $s(\tilde{A})$, accuracy function $a(\tilde{A})$ and certainty function $c(\tilde{A})$ of a BNN are defined as follows:

(i) $$s(\tilde{A}) = \left(\frac{1}{6}\right) \times \left[T^p + 1 - I^p + 1 - F^p + 1 + T^n - I^n - F^n\right] \quad (4)$$

(ii) $$a(\tilde{A}) = T^p - F^p + T^n - F^n \quad (5)$$

(iii) $$c(\tilde{A}) = T^p - F^n \quad (6)$$

For any two BNNs $\tilde{A}_1$ and $\tilde{A}_2$:

i. If $s(\tilde{A}_1) \succ s(\tilde{A}_2)$, then $\tilde{A}_1$ is greater than $\tilde{A}_2$, that is, $\tilde{A}_1$ is superior to $\tilde{A}_2$, denoted by $\tilde{A}_1 \succ \tilde{A}_2$

ii. If $s(\tilde{A}_1) = s(\tilde{A}_2)$ and $a(\tilde{A}_1) \succ a(\tilde{A}_2)$, then $\tilde{A}_1$ is greater than $\tilde{A}_2$, that is, $\tilde{A}_1$ is superior to $\tilde{A}_2$, denoted by $\tilde{A}_1 \succ \tilde{A}_2$

iii. If $s(\tilde{A}_1) = s(\tilde{A}_2)$, $a(\tilde{A}_1) = a(\tilde{A}_2)$, and $c(\tilde{A}_1) \succ c(\tilde{A}_2)$ then $\tilde{A}_1$ is greater than $\tilde{A}_2$, that is, $\tilde{A}_1$ is superior to $\tilde{A}_2$, denoted by $\tilde{A}_1 \succ \tilde{A}_2$

iv. If $s(\tilde{A}_1) = s(\tilde{A}_2)$, $a(\tilde{A}_1) = a(\tilde{A}_2)$, and $c(\tilde{A}_1) = c(\tilde{A}_2)$ then $\tilde{A}_1$ is equal to $\tilde{A}_2$, that is, $\tilde{A}_1$ is indifferent to $\tilde{A}_2$, denoted by $\tilde{A}_1 = \tilde{A}_2$

III. MINIMUM SPANNIG TREE ALGORITHM OF BN- UNDIRECTED GRAPH

In this section, a neutrosophic version of Kruskal algorithm is proposed to handle minimum spanning tree in a bipolar neutrosophic environment. In the following, we propose a bipolar neutrosophic minimum spanning tree algorithm, whose steps are defined below:

Algorithm:

Input: The weight matrix $M = \left[W_{ij}\right]_{n \times n}$ for the undirected weighted neutrosophic graph G.

Output: Minimum cost Spanning tree T of G.

Step 1: Input neutrosophic adjacency matrix A.

Step 2: Translate the BN-matrix into score matrix $\left[S_{ij}\right]_{n \times n}$ by using score of bipolar neutrosophic number.

Step 3: Iterate step 4 and step 5 until all $(n-1)$ entries matrix of S are either marked or set to zero or other words all the nonzero elements are marked.

Step 4: Find the score matrix S either columns-wise or row-wise to find the unmarked minimum entries S_{ij}, which is the weight of the corresponding edge e_{ij} in S.

Step 5: If the corresponding edge e_{ij} of selected S_{ij} produce a cycle with the previous marked entries of the score matrix S then set $S_{ij}=0$ else mark S_{ij}.

Step 6: Building the graph T including only the marked entries from the score matrix S which shall be desired minimum cost spanning tree of G.

Step 7: Stop.

An illustrative flow chart of the given algorithm is presented in fig. 3.

IV. NUMERICAL EXAMPLE

In this section, a numerical example is explained based on the above algorithm. Consider a hypothetical graph with edge values are given in the table below.

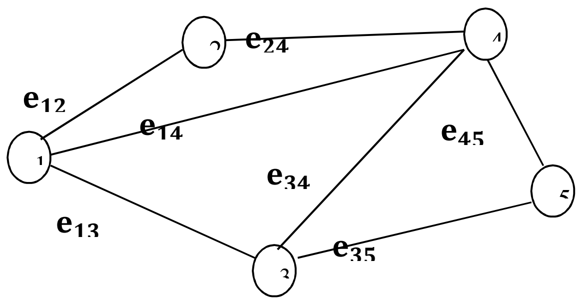

Fig 2. Undirected bipolar neutrosophic- graphs

E	Edge length
e_{12}	$<0.3, 0.1, 0.2, -0.8, -0.5, -0.1>$
e_{13}	$<0.4, 0.5, 0.4, -0.2, -0.4, -0.5>$
e_{14}	$<0.6, 0.7, 0.8, -0.6, -0.4, -0.4>$
e_{24}	$<0.4, 0.8, 0.3, -0.2, -0.5, -0.7>$
e_{34}	$<0.2, 0.3, 0.7, -0.2, -0.4, -0.4>$
e_{35}	$<0.4, 0.6, 0.5, -0.4, -0.4, -0.3>$
e_{45}	$<0.5, 0.4, 0.3, -0.4, -0.5, -0.8>$

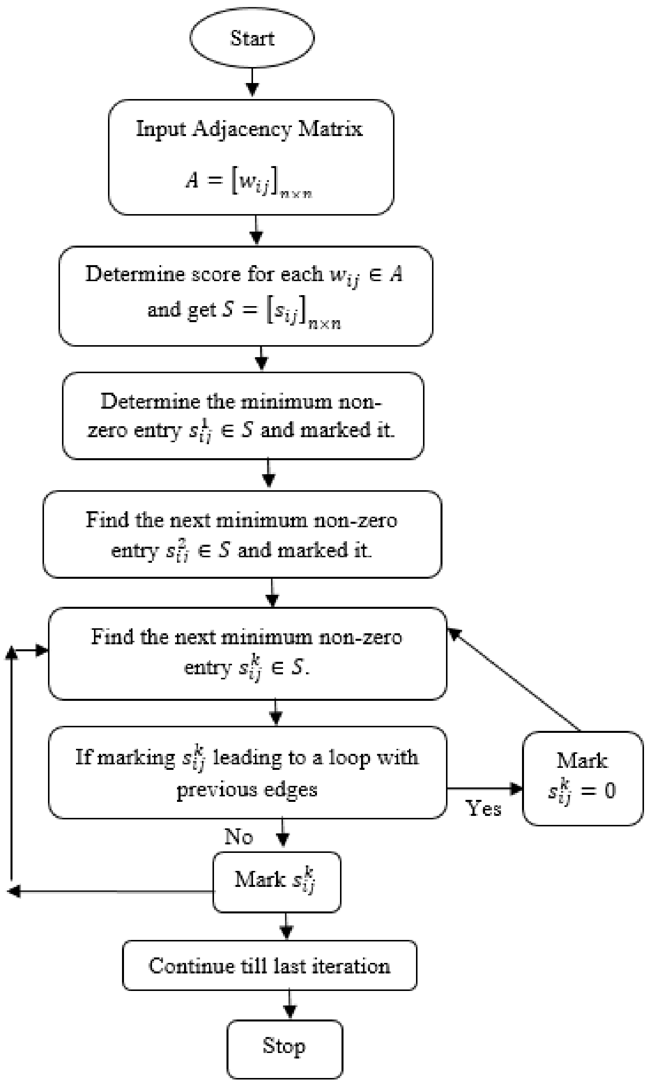

Fig. 3 (Flow chart describing proposed algorithm)

The BN- adjacency matrix A is given below:

$$A = \begin{bmatrix} 0 & <0.3,0.1,0.2,-0.8,-0.5,-0.1> & <0.4,0.5,0.4,-0.2,-0.4,-0.5> & <0.6,0.7,0.8,-0.6,-0.4,-0.4> & 0 \\ <0.3,0.1,0.2,-0.8,-0.5,-0.1> & 0 & 0 & <0.4,0.8,0.3,-0.2,-0.5,-0.7> & 0 \\ <0.4,0.5,0.4,-0.2,-0.4,-0.5> & 0 & 0 & <0.2,0.3,0.7,-0.2,-0.4,-0.4> & <0.4,0.6,0.5,-0.4,-0.4,-0.3> \\ <0.6,0.7,0.8,-0.6,-0.4,-0.4> & <0.4,0.8,0.3,-0.2,-0.5,-0.7> & <0.2,0.3,0.7,-0.2,-0.4,-0.4> & 0 & <0.5,0.4,0.3,-0.4,-0.5,-0.8> \\ 0 & 0 & <0.4,0.6,0.5,-0.4,-0.4,-0.3> & <0.5,0.4,0.3,-0.4,-0.5,-0.8> & 0 \end{bmatrix}$$

Hence, using the score function introduced in definition 2.1, we get the score matrix

$$S = \begin{bmatrix} 0 & 0.47 & 0.53 & 0.38 & 0 \\ 0.47 & 0 & 0 & 0.55 & 0 \\ 0.53 & 0 & 0 & 0.52 & 0.43 \\ 0.38 & 0.55 & 0.52 & 0 & 0.62 \\ 0 & 0 & 0.43 & 0.62 & 0 \end{bmatrix}$$

Fig. 4. Score matrix

Clearly from figure 4, it is observed that 0.38 is the least value so edge (1, 4) is marked as red as shown in figure 5. This process shall be continued until last iteration.

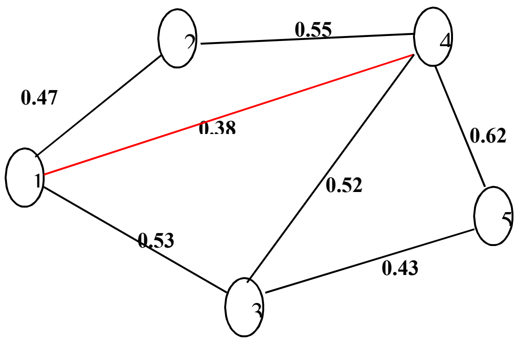

Fig. 5

Clearly from the figure 6, the next non zero minimum entries 0.43 is marked and colored corresponding edge (3, 5) is given in figure 7.

$$S = \begin{bmatrix} 0 & 0.47 & 0.53 & 0.38 & 0 \\ 0.47 & 0 & 0 & 0.55 & 0 \\ 0.53 & 0 & 0 & 0.52 & 0.43 \\ 0.38 & 0.55 & 0.52 & 0 & 0.62 \\ 0 & 0 & 0.43 & 0.62 & 0 \end{bmatrix}$$

Fig. 6

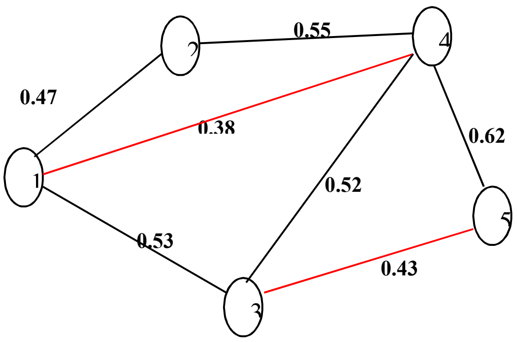

Fig 7

$$S = \begin{bmatrix} 0 & 0.47 & 0.53 & 0.38 & 0 \\ 0.47 & 0 & 0 & 0.55 & 0 \\ 0.53 & 0 & 0 & 0.52 & 0.43 \\ 0.38 & 0.55 & 0.52 & 0 & 0.62 \\ 0 & 0 & 0.43 & 0.62 & 0 \end{bmatrix}$$

Fig.8

Clearly from the figure 8, the next minimum non-zero element 0.47 is marked and the colored corresponding edge is given in figure 9.

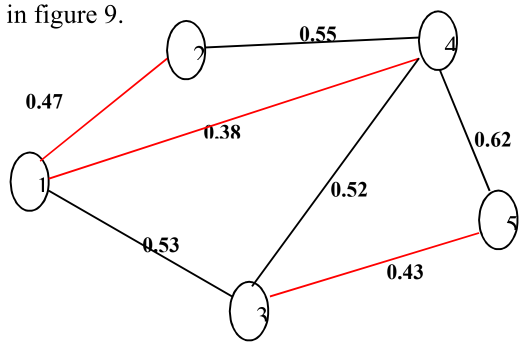

Fig 9

Clearly from the figure 10. The next minimum non-zero element 0.52 is marked, and colored corresponding edge (3, 4) is given in figure 11.

$$S = \begin{bmatrix} 0 & 0.47 & 0.53 & 0.38 & 0 \\ 0.47 & 0 & 0 & 0.55 & 0 \\ 0.53 & 0 & 0 & 0.52 & 0.43 \\ 0.38 & 0.55 & 0.52 & 0 & 0.62 \\ 0 & 0 & 0.43 & 0.62 & 0 \end{bmatrix}$$

Fig.10

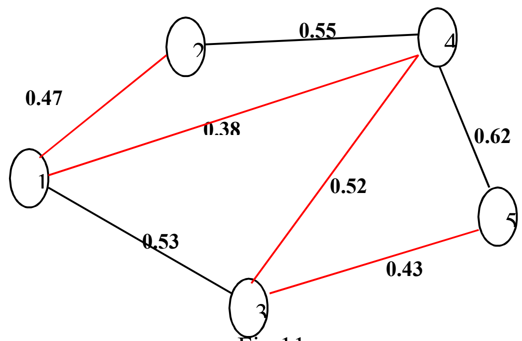

Fig.11

Clearly from the figure 12. The next minimum non-zero element 0.53 is marked. But while drawing the edges it produces the cycle. So we reject and mark it as 0 instead of 0.53

$$S = \begin{bmatrix} 0 & 0.47 & 0.53 & 0.38 & 0 \\ 0.47 & 0 & 0 & 0.55 & 0 \\ \cancel{0.53}\ 0 & 0 & 0 & 0.52 & 0.43 \\ 0.38 & 0.55 & 0.52 & 0 & 0.62 \\ 0 & 0 & 0.43 & 0.62 & 0 \end{bmatrix}$$

Fig .12

The next least value is 0.55 but including this edge results in the formation of a cycle. So this value is marked as zero as shown in the figure 13.

$$S=\begin{bmatrix} 0 & 0.47 & 0.53 & 0.38 & 0 \\ 0.47 & 0 & 0 & \cancel{0.55}\ 0 & 0 \\ \cancel{0.53}\ 0 & 0 & 0 & 0.52 & 0.43 \\ 0.38 & 0.55 & 0.52 & 0 & 0.62 \\ 0 & 0 & 0.43 & 0.62 & 0 \end{bmatrix}$$

Fig .13

Clearly from the figure 14. The next minimum non-zero element 0.62 is marked. But while drawing the edges it produces the cycle. So we reject and mark it as 0 instead of 0.62.

$$S=\begin{bmatrix} 0 & 0.47 & 0.53 & 0.38 & 0 \\ 0.47 & 0 & 0 & \cancel{0.55}\ 0 & 0 \\ \cancel{0.53}\ 0 & 0 & 0 & 0.52 & 0.43 \\ 0.38 & 0.55 & 0.52 & 0 & \cancel{0.62}\ 0 \\ 0 & 0 & 0.43 & 0.62 & 0 \end{bmatrix}$$

Fig .14

After the above steps, the final path of minimum cost of spanning tree of G is given in figure 15.

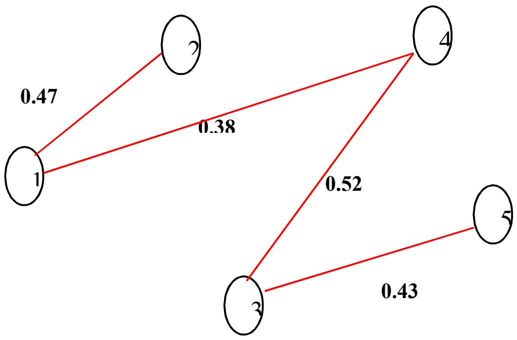

Fig .15. Final path of minimum cost of spanning tree of G .

Following the steps of proposed algorithm presented in section 3. Therefore the crisp minimum cost spanning tree is 1,8 and the final path of minimum cost of spanning tree is{2,1},{1, 4},{4, 3},{3, 5}.

V. COMPARATIVE STUDY

In this section, the same process is carried out by the algorithm of Mullai et al [20]. The results obtained in different iterations by this existing algorithm are illustrated below.

Let $C_1=\{1\}$ and $\overline{C_1}=\{2,3,4,5\}$

Iteration 2:

Let $C_2=\{1,4\}$ and $\overline{C_2}=\{2,3,5\}$

Iteration 3:

Let $C_3=\{1,4,3\}$ and $\overline{C_3}=\{2,5\}$

Iteration 4:

Let $C_4=\{1,4,5,3\}$ and $\overline{C_4}=\{2\}$

Based on these iterations of Mullai's algorithm, we have the following MST.

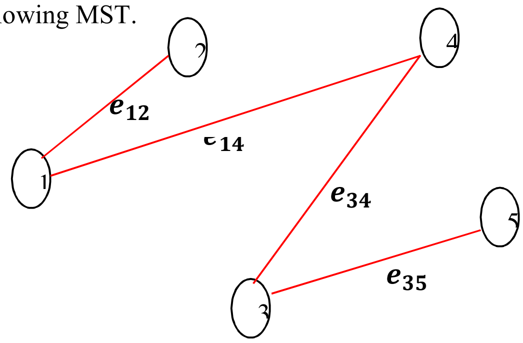

Fig .16.MST obtained by Mullai's Algorithm

This comparison makes the point that that both the existing and new algorithm leads to the same results.

The advantage of new algorithm over existing algorithm is that the new algorithm is matrix based and can be easily performed in MATLAB while Mullai's algorithm is based on edge comparison and is difficult to be performed.

VI. CONCLUSION

This paper considers a minimum spanning tree problem under the situation where the weights of edges are represented by BNNs. It is discussed how proposed algorithm is better in formulation and implementation. This work can be extended to the case of directed neutrosophic graphs and other types of neutrosophic graphs such as interval valued bipolar neutrosophic graphs

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