

# An Improved ICI Reduction Method in OFDM Communication System

Heung-Gyoon Ryu, *Member, IEEE*, Yingshan Li, and Jin-Soo Park, *Member, IEEE*

**Abstract**—Orthogonal frequency division multiplexing (OFDM) is a promising technique for the broadband wireless communication system. However, the inter-sub-carrier-interference (ICI) produced by the phase noise of transceiver local oscillator is a serious problem. Bit error rate (BER) performance is degraded because the orthogonal properties between the sub-carriers are broken down. In this paper, ICI self-cancellation of data-conjugate method is studied to reduce ICI effectively. CPE (common phase error), ICI and CIR (carrier to interference power ratio) are derived and discussed by the linear approximation of the phase noise. Then, the system performance of the data-conjugate method is compared with those of the original OFDM and the conventional data-conversion method. As results, it can be shown that CPE becomes zero in the OFDM of the data-conjugate method. Besides, in the OFDM system with phase noise, the data-conjugate method can make remarkable improvement of the BER performance and it is better than the data-conversion method and the original OFDM with or without convolution coding.

**Index Terms**—Data-conjugate, data-conversion, ICI, ICI self-cancellation and OFDM.

## I. INTRODUCTION

ORTHOGONAL frequency division multiplexing (OFDM) is widely known as the promising communication technique in the current broadband wireless mobile communication system due to the high spectral efficiency and robustness to the multi-path interference. Currently, OFDM has been adapted to the digital audio and video broadcasting (DAB/DVB) system, high-speed wireless local area networks (WLAN) such as IEEE802.11x, HIPERLAN II and multimedia mobile access communications (MMAC), ADSL, digital multi-media broadcasting (DMB) system and multi-band OFDM type ultra-wideband (MB-OFDM UWB) system, etc. In single carrier communication system, phase noise basically produces the rotation of signal constellation. However, in multi-carrier OFDM system, OFDM system is very vulnerable to the phase noise or frequency offset. The serious inter-sub-carrier-interference (ICI) component results from the phase noise. So, the orthogonal characteristics between sub-carriers are easily broken down by this ICI so that system performance may be considerably degraded.

There have been many previous studies on the phase noise and ICI. In [1], C. Muschallik analyzed the phase rotation and

ICI effect in OFDM system. In [2], the common phase error (CPE) and ICI caused by phase noise were analyzed in detail. In [3] and [4], ICI self-cancellation of the data-conversion method was proposed to cancel the ICI caused by frequency offset in the OFDM system, and carrier-to-interference ratio (CIR) and bit error rate (BER) were analyzed. However, the ICI caused by phase noise was not discussed. In [5], ICI self-cancellation of the data-conjugate method was proposed to minimize the ICI caused by frequency offset and it could reduce the peak average to power ratio (PAPR) than the data-conversion method. But, the analysis of ICI caused by phase noise was not covered, either. In [6], it was shown that ICI caused by phase noise could be removed by data-conversion method. Actually, some techniques to estimate and compensate common phase error (CPE) caused by phase noise have been already studied, but ICI compensation is still under consideration.

At beginning, the data-conversion method and the data-conjugate method were proposed to reduce the ICI caused by frequency offset. In this paper, it is our goal to minimize the ICI caused by the phase noise of transceiver local oscillators. Therefore, the phase noise compensation effect of the data-conjugate method is newly analyzed, and CPE, ICI and CIR are analyzed by linear approximation of the phase noise. Furthermore, BER performances of data-conversion method and data-conjugate method the original OFDM with or without convolution coding are compared with each other.

## II. ICI SELF-CANCELLATION OF DATA-CONJUGATE METHOD

Fig. 1 shows the block diagram of the OFDM system using data-conjugate method. The high-speed information data pass through the serial to parallel converter and become parallel data streams of  $N/2$  branch. Then, they are converted into  $N$  branch parallel data by the data-conjugate method. The conversion process is as follows. After serial to parallel converter, the parallel data streams are remapped as the form of  $X'_{2k} = X_k$ ,  $X'_{2k+1} = -X_k^*$ , ( $k = 0, \dots, N-1$ ). Here,  $X_k$  is the information data to the  $k$ -th branch before data-conjugate conversion, and  $X'_{2k}$  is the information data to the  $2k$ -th branch after ICI cancellation mapping. Likewise, every information data is mapped into a pair of adjacent sub-carriers by data-conjugate method, so the  $N/2$  branch data are extended to map onto the  $N$  sub-carriers.

Even though OFDM symbol is actually added by the cyclic prefix in order to cope with multi-path delay spread, this prefix is not considered for the simplicity of analysis in this paper. In the receiver, after serial to parallel converter and fast Fourier transform (FFT), the  $N$  carrier data are converted back into the  $N/2$  branch data by the de-mapping. The original data can be recovered from the simple relation of  $Z'_k = (Y_{2k} - Y_{2k+1}^*)/2$ . Here,  $Y_{2k}$  is the  $2k$ -th sub-carrier data,  $Z'_k$  is the  $k$ th branch

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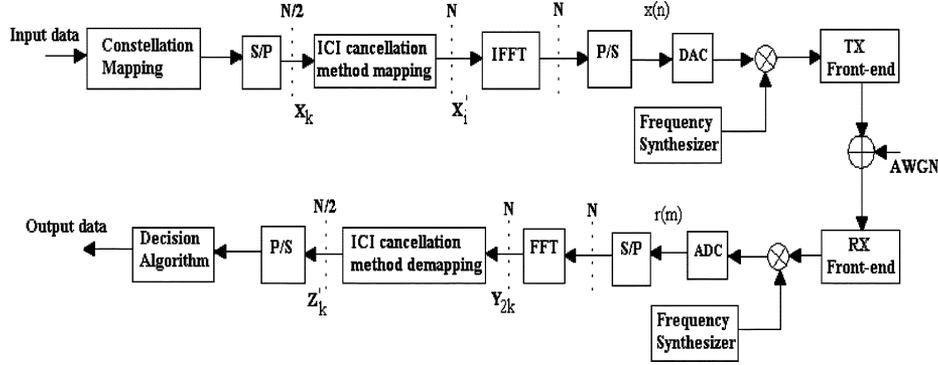


Fig. 1. Block diagram of the OFDM system using the data conjugate method.

information data after de-mapping. Finally, the information data can be found through the detection process.

The complex base-band OFDM signal after data conjugate mapping is as follows.

$$\begin{aligned} x(n) &= \sum_{i=0}^{N-1} X'_i \cdot e^{j\frac{2\pi}{N}in} \\ &= \sum_{k=0}^{\frac{N}{2}-1} \left[ X_k \cdot e^{j\frac{2\pi}{N}2kn} - X_k^* \cdot e^{j\frac{2\pi}{N}(2k+1)n} \right] \\ &\quad \text{for } 0 \leq n < N \end{aligned} \quad (1)$$

where  $j = \sqrt{-1}$ ,  $N$  is the total number of sub-carriers,  $X_k$  is data symbol for the  $k$ -th parallel branch and  $X'_i$  is the  $i$ -th sub-carrier data symbol after data-conjugate mapping.  $x(n)$  is corrupted by the phase noise in the transmitter (TX) local oscillator. Furthermore, the received signal is influenced by the phase noise of receiver (RX) local oscillator. So, it is expressed as

$$r(t) = \left\{ \left( x(t) \cdot e^{j\phi_{TX}(t)} \right) \otimes h(t) + n(t) \right\} \cdot e^{j\phi_{RX}(t)}, \quad (2)$$

where  $n(t)$  is the complex Gaussian noise and  $h(t)$  is the channel impulse response.  $\phi_{TX}(t)$  and  $\phi_{RX}(t)$  are the time varying phase noise processes generated in the transceiver oscillators. Here, it is assumed that  $\phi_{TX}(t) = \phi_{RX}(t) = \phi(t)$ , and  $\phi_{tot}(t) = \phi_{TX}(t) + \phi_{RX}(t)$  for simple analysis. In the original OFDM system without ICI self-cancellation method, the  $k$ -th sub-carrier signal after FFT can be written as

$$Y_k = \frac{1}{N} \sum_{l=0}^{N-1} X_l \cdot H_l \sum_{m=0}^{N-1} e^{j\left(\frac{2\pi}{N}(l-k)m + \phi_{tot}(m)\right)} + N_k. \quad (3)$$

$H_l$  is the frequency domain expression of channel impulse response of the  $l$ -th sub-carrier.

However, in the data-conjugate method, the sub-carrier data is mapped in the form of  $X_{2k} = X_k$ ,  $X'_{2k+1} = -X_k^*$ . Therefore, the  $2k$ th sub-carrier data after FFT in the receiver is arranged as

$$Y_{2k} = \sum_{l=0}^{\frac{N}{2}-1} [X_l H_{2l} Q_{2l-2k} - X_l^* H_{2l+1} Q_{2l+1-2k}] + N_{2k}. \quad (4)$$

$$Q_L = \frac{1}{N} \sum_{m=0}^{N-1} e^{j\left(\frac{2\pi}{N}Lm + \phi_{tot}(m)\right)}. \quad (5)$$

$N_{2k}$  is a sampled FFT version of the complex AWGN multiplied by the phase noise of RX local oscillator, and random phase noise process  $\phi_{tot}[m]$  is equal to  $\phi_{TX}[m] + \phi_{RX}[m]$ .

Similarly, the  $2k+1$ -th sub-carrier signal is expressed as

$$Y_{2k+1} = \sum_{l=0}^{\frac{N}{2}-1} [X_l H_{2l} Q_{2l-2k-1} - X_l^* H_{2l+1} Q_{2l-2k}] + N_{2k+1}. \quad (6)$$

In the (4) and (6),  $l = k$  corresponds to the original signal with CPE, and  $l \neq k$  corresponds to the ICI component.

In the receiver, the decision variable  $Z'_k$  of the  $k$ -th symbol is found from the difference of adjacent sub-carrier signals affected by phase noise. That is,

$$\begin{aligned} Z'_k &= \frac{(Y_{2k} - Y_{2k+1}^*)}{2} \\ &= \frac{1}{2} X_k (H_{2k} Q_0 + H_{2k+1}^* Q_0^*) \\ &\quad - \frac{1}{2} X_k^* (H_{2k+1} Q_1 + H_{2l}^* Q_{-1}^*) \\ &\quad + \frac{1}{2} \sum_{\substack{l=0 \\ l \neq k}}^{\frac{N}{2}-1} \left\{ X_l [H_{2l} Q_{2l-2k} + H_{2l+1}^* Q_{2l-2k}^*] \right. \\ &\quad \left. - X_l^* [H_{2l+1} Q_{2l+1-2k} + H_{2l}^* Q_{2l-2k-1}^*] \right\} \\ &\quad + N_k \end{aligned} \quad (7)$$

where  $N_k = (1/2)(N_{2k} - N_{2k+1}^*)$  is the AWGN of the  $k$ -th parallel branch data in the receiver.

When channel is flat, frequency response of channel  $H(k)$  equals 1.  $Z'_k$  is as follows.

$$\begin{aligned} Z'_k &= X_k + \frac{1}{2} \sum_{\substack{l=0 \\ l \neq k}}^{\frac{N}{2}-1} \left\{ X_l [Q_{2l-2k} + Q_{2l-2k}^*] \right. \\ &\quad \left. - X_l^* [Q_{2l+1-2k} + Q_{2l-2k-1}^*] \right\} + N_k. \end{aligned} \quad (8)$$

### III. CPE, ICI AND CIR ANALYSIS

#### A. Original OFDM

In the original OFDM, the  $k$ -th sub-carrier signal after FFT is as follows:

$$Y_k = X_k + X_k [Q_0 - 1] + \sum_{\substack{l=0 \\ l \neq k}}^{N-1} X_l \cdot Q_{l-k} + N_k. \quad (9)$$

The received desired signal power on the  $k$ -th sub-carrier is

$$E[|Y_{k1}|^2] = E[|X_k Q_0|^2]. \quad (10)$$

ICI power is

$$E [Y_{k2}^2] = E \left[ \left| \sum_{\substack{l=0 \\ l \neq k}}^{N-1} X_l Q_{l-k} \right|^2 \right]. \quad (11)$$

Transmitted signal is supposed to have zero mean and statistically independence. So, the CIR of the original OFDM transmission method is as follows:

$$CIR = \frac{|Q_0|^2}{\sum_{\substack{l=0 \\ l \neq k}}^{N-1} |Q_{l-k}|^2} = \frac{|Q_0|^2}{\sum_{l=1}^{N-1} |Q_l|^2}. \quad (12)$$

### B. Data-Conversion Method

In the data-conversion ICI self-cancellation method, the data are remapped in the form of  $X'_{2k} = X_k$ ,  $X'_{2k+1} = -X_k$  [4]. So, the desired signal is recovered in the receiver as follows:

$$\begin{aligned} Z'_k &= \frac{(Y_{2k} - Y_{2k+1})}{2} \\ &= X_k + \frac{1}{2} X_k [-Q_{-1} + 2(Q_0 - 1) - Q_1] \\ &\quad + \frac{1}{2} \sum_{\substack{l=0 \\ l \neq k}}^{\frac{N}{2}-1} X_l [-Q_{2l-2k-1} + 2Q_{2l-2k} - Q_{2l-2k+1}] \\ &\quad + N_k. \end{aligned} \quad (13)$$

CPE is as follows:

$$CPE = \frac{j2X_k}{N} \sum_{m=0}^{N-1} \sin^2 \left( \frac{\pi m}{N} \right) \phi_{tot}(m). \quad (14)$$

ICI component of the  $k$ -th sub-carrier is as follows:

$$\begin{aligned} ICI &= \frac{2j}{N} \sum_{\substack{l=0 \\ l \neq k}}^{\frac{N}{2}-1} X_l \sum_{m=0}^{N-1} \sin^2 \left( \frac{\pi m}{N} \right) \\ &\quad \cdot \exp \left( \frac{j4\pi m(l-k)}{N} \right) \phi_{tot}(m). \end{aligned} \quad (15)$$

So

$$\begin{aligned} CIR &= \frac{|-Q_{-1} + 2Q_0 - Q_1|^2}{\sum_{\substack{l=0 \\ l \neq k}}^{\frac{N}{2}-1} |-Q_{2l-2k-1} + 2Q_{2l-2k} - Q_{2l-2k+1}|^2} \\ &= \frac{|-Q_{-1} + 2Q_0 - Q_1|^2}{\sum_{l=1}^{\frac{N}{2}-1} |-Q_{2l-1} + 2Q_{2l} - Q_{2l+1}|^2}. \end{aligned} \quad (16)$$

### C. Data-Conjugate Method

In the data conjugate method, the decision variable can be written as follows:

$$\begin{aligned} Z'_k &= X_k + \frac{1}{2} \sum_{\substack{l=0 \\ l \neq k}}^{\frac{N}{2}-1} \{ X_l [Q_{2l-2k} + Q_{2l-2k}^*] \\ &\quad - X_l^* [Q_{2l+1-2k} + Q_{2l-2k-1}^*] \} + N_k. \end{aligned} \quad (17)$$

TABLE I  
COMPARISON OF CODING EFFICIENCY

Method	Coding efficiency ( $C_r$ )
Original OFDM method	1
OFDM with Convolution coding	1/2
Data conversion method	1/2
Data conjugate method	1/2

Through the same calculation, CPE, ICI and CIR of the data-conjugate method are found.

$$CPE = 0. \quad (18)$$

The fact CPE is zero is completely different from the data-conversion method whose CPE is not zero like (14).

Then, ICI of data conjugate method is

$$\begin{aligned} ICI &= \frac{1}{N} \sum_{\substack{l=0 \\ l \neq k}}^{\frac{N}{2}-1} \sum_{m=0}^{N-1} \sin \left( \frac{4\pi}{N} (l-k)m \right) \\ &\quad \cdot [X_l^* \cdot e^{j\frac{2\pi}{N}m} - X_l] \cdot \phi_{tot}(m). \end{aligned} \quad (19)$$

The above term is the summation of the signal of the other sub-carriers multiplied by some complex number resulted from an average of phase noise with spectral shift. This component is added into the  $k$ -th branch data  $Z'_k$ . It may break down the orthogonalities between sub-carriers. So, CIR is

$$CIR = \frac{4}{\sum_{l=2}^{\frac{N}{2}-1} [ |Q_{2l} + Q_{2l}^*|^2 + |Q_{2l+1} + Q_{2l-1}^*|^2 ]}. \quad (20)$$

## IV. SIMULATION RESULTS AND DISCUSSION

BERs are found to discuss the system performance affected by phase noise in the original OFDM system with or without the convolution coding, the data-conversion method, and the data-conjugate method, respectively. QPSK modulation and 16QAM modulations are used, the numbers of OFDM sub-carriers is 64 with 4 times over-sampling. AWGN channel and quasistatic channel with Rician K-factor of 10 are considered respectively [7], [8]. BER versus the required-transmitted-signal-to-noise-ratio  $E_b/C_r N_0$  ( $C_r$  is the coding efficiency in each method, listed in Table I;  $N_0$  is the spectral density coefficient for the white noise) is considered [9]. The PLL (phase locked loop) frequency synthesizer in the [10]–[12] is considered for the phase noise model.

In a certain offset frequency range ( $\pm b$ ), the phase noise variance can be calculated as

$$\sigma_\phi^2 = \int_{-b}^{+b} \left( \frac{N_{op}}{C} \right)_f df = \int_0^b \left( \frac{2N_{op}}{C} \right)_f df \text{ rads}^2 \quad (21)$$

where  $\sigma_\phi^2$  (denoted as pn.var in the Fig. 2–6) is the variance of  $\phi_{TX}(t)$  or  $\phi_{RX}(t)$ , and  $(N_{op}/C)_f$  [dBc] is the power spectral

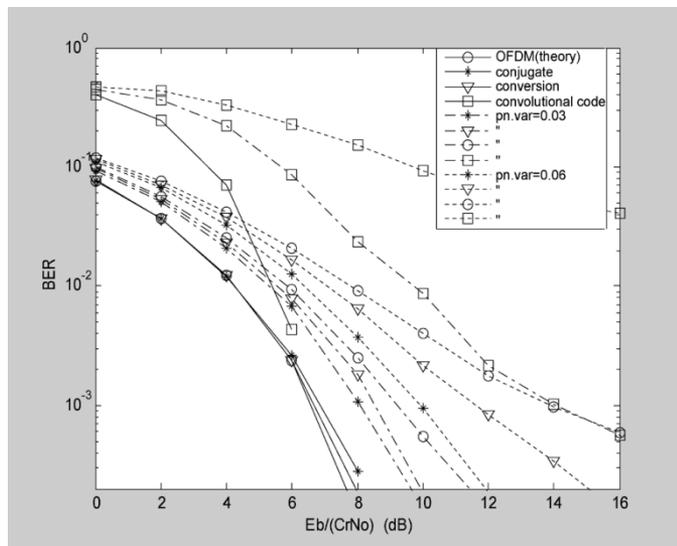


Fig. 2. BER comparison in AWGN channel (QPSK, N = 64).

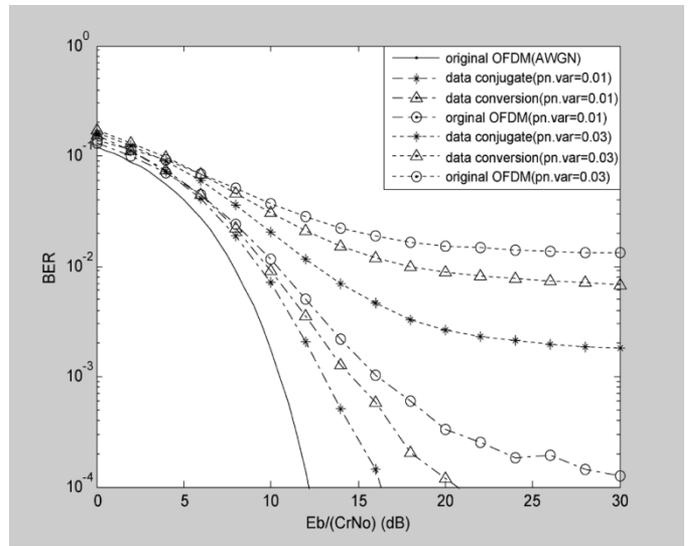


Fig. 5. BER comparison in quasistatic channel (16QAM, N = 64).

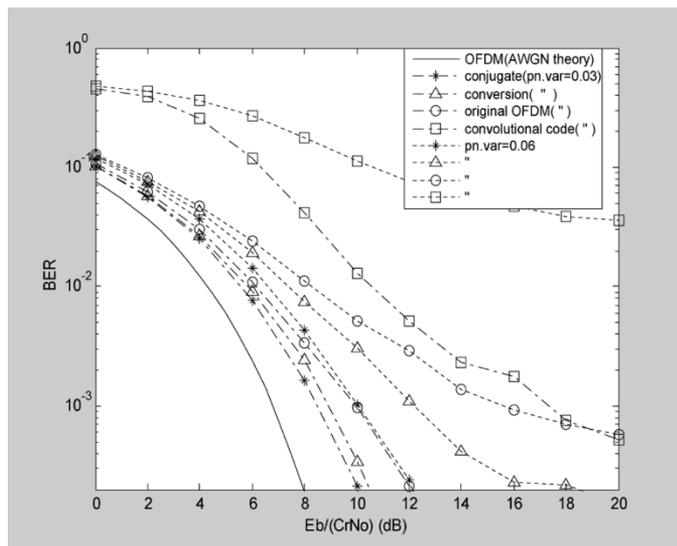


Fig. 3. BER comparison in quasistatic channel (QPSK, N = 64).

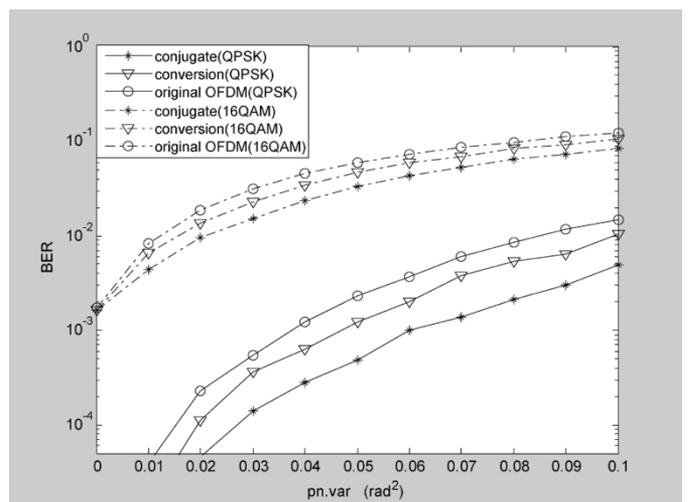


Fig. 6. BERs v.s. phase noise variance  $\sigma_\phi^2$  (SNR = 10 dB, N = 64).

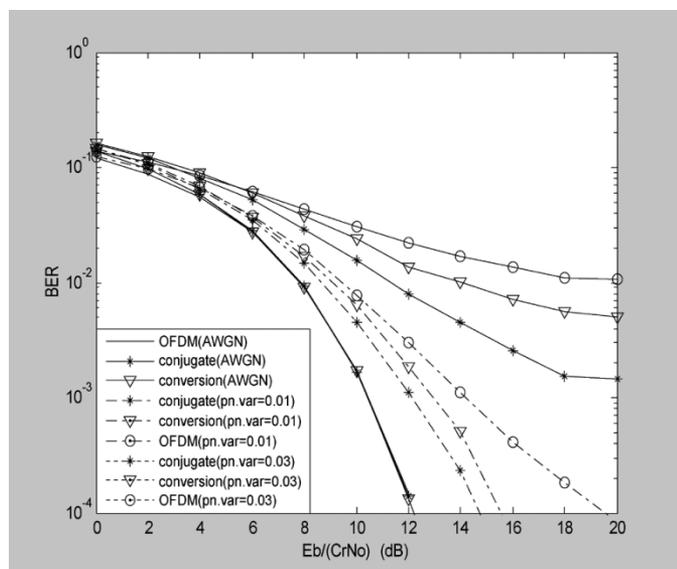
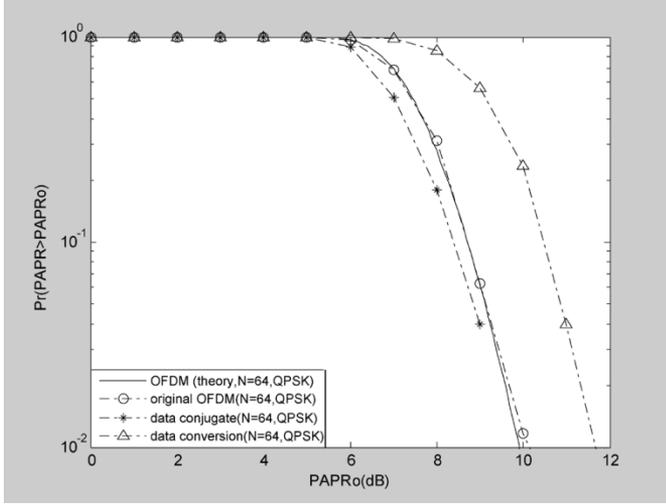


Fig. 4. BER comparison in AWGN channel (16-QAM, N = 64).

density (PSD) of the PLL phase noise to carrier ratio in a certain offset frequency  $f$ . Also,  $(N_{op}/C)_f$  can be found by the spectrum analyzer or by the analytical method such as using PLL mathematical phase equivalent model [10]–[12].

Figs. 2 and 3 show the BER performances of the four OFDM systems in the AWGN channel and quasistatic channel when QPSK modulation is used and phase noise variances ( $\sigma_\phi^2$ ) are supposed to 0.03 and 0.06, respectively. As seen in the figures, the data conjugate method has the best BER performance, compared with data conversion method, original OFDM and OFDM with convolution coding in the AWGN channel or fading channel. Specially, the difference becomes higher as the phase noise variance is large. It means that the data conjugate method outperforms data conversion method in the OFDM with phase noise although data conversion method has better CIR in OFDM only with frequency offset as mentioned in [5].

Fig. 4 and Fig. 5 show the BER performances of the three OFDM systems in the AWGN channel and quasistatic channel when 16QAM modulation is used and phase noise variances ( $\sigma_\phi^2$ ) are supposed to 0.01 and 0.03, respectively. As shown from


 Fig. 7. PAPR comparison (QPSK,  $N = 64$ ).

the figures, data conjugate method shows the best performance among all discussed methods.

Fig. 6 shows the BER performances in terms of phase noise variance ( $\sigma_\phi^2$ ) under the condition of 10 dB SNR. As seen in Fig. 6, data-conversion method and data-conjugate method can make significant performance improvement than the original OFDM. As expected, the data-conjugate method brings best performance among the three methods.

Fig. 7 shows comparison of the peak-to average power ratio (PAPR) of OFDM signal in the three methods. High PAPR of OFDM signal results in the nonlinear distortion in the high power amplifier (HPA). As shown in the figure, data-conjugate method can considerably mitigate the high PAPR problem that is very serious problem in the data-conversion method.

## V. CONCLUSION

In this paper, we have analyzed the system performance of the OFDM system when phase noise exists. The data conjugate method is studied to compare with the original OFDM, OFDM with convolution coding and the data-conversion method.

- 1) CPE, ICI and CIR caused by phase noise are analyzed in the data-conjugate method and compared with other methods. From the analysis, the CPE of the data-conjugate method is shown to be zero when channel has flat characteristic.
- 2) BERs are found by the computer simulation to compare the system performance affected by phase noise in the four type systems. As results, the performance penalty can be reduced when two kinds of ICI self-cancellation method are used. Especially, the data-conjugate method brings the more significant improvement than the data-conversion method.

Overall, with respect to the PAPR and BER, the OFDM system of the data-conjugate method shows the best performances compared with the original OFDM, OFDM with

convolution coding and the data-conversion method. So, data conjugate ICI self-cancellation method may be very useful to the multi-carrier system of the high transmission quality.

## APPENDIX

- a) In the data-conjugate method, the parallel branch data are mapped in the form of  $X'_{2k} = X_k$ ,  $X'_{2k+1} = -X_k^*$ . Therefore, in the receiver, the  $2k$ th sub-carrier data after FFT is arranged as

$$\begin{aligned}
 Y_{2k} &= \frac{1}{N} \sum_{m=0}^{N-1} r[m] \cdot e^{-j\frac{2\pi}{N}2km} \\
 &= \sum_{m=0}^{N-1} \left\{ \frac{1}{N} \sum_{l=0}^{\frac{N}{2}-1} \left[ X_l \cdot H_{2l} \cdot e^{j\frac{2\pi}{N}2lm} \right. \right. \\
 &\quad \left. \left. - X_l^* \cdot H_{2l+1} \cdot e^{j\frac{2\pi}{N}(2l+1)m} \right] \right. \\
 &\quad \left. \cdot e^{j\phi_{tot}(m)} + n(m) \cdot e^{j\phi_{RX}(m)} \right\} \cdot e^{-j\frac{2\pi}{N}2km} \\
 &= \sum_{l=0}^{\frac{N}{2}-1} \left[ \frac{1}{N} \cdot X_l \cdot H_{2l} \cdot \sum_{m=0}^{N-1} e^{j(\frac{2\pi}{N}(2l-2k)m + \phi_{tot}(m))} \right. \\
 &\quad \left. - \frac{1}{N} \cdot X_l^* \cdot H_{2l+1} \right. \\
 &\quad \left. \cdot \sum_{m=0}^{N-1} e^{j(\frac{2\pi}{N}(2l+1-2k)m + \phi_{tot}(m))} \right] + N_{2k} \\
 &= \sum_{l=0}^{\frac{N}{2}-1} [X_l H_{2l} Q_{2l-2k} - X_l^* H_{2l+1} Q_{2l+1-2k}] + N_{2k}. \quad (a1) \\
 Q_L &= \frac{1}{N} \sum_{m=0}^{N-1} e^{j(\frac{2\pi}{N}Lm + \phi_{tot}(m))}. \quad (a2)
 \end{aligned}$$

In order to separate the signal and noise terms, using the phase noise linear approximation method, let us suppose  $\phi_{tot}[m]$  is so small that  $e^{j\phi_{tot}[m]}$  can be approximated into  $1 + j\phi_{tot}[m]$ . So,

$$Q_L = \frac{1}{N} \sum_{m=0}^{N-1} e^{j\frac{2\pi}{N}Lm} \cdot (1 + j\phi_{tot}(m)) \quad (a3)$$

In the (a3), in the case of  $2l - 2k = 0$ :

$$Q_0 = 1 + \frac{j}{N} \sum_{m=0}^{N-1} \phi_{tot}(m) \quad (a4)$$

$$Q_1 = \frac{j}{N} \sum_{m=0}^{N-1} e^{j\frac{2\pi}{N}m} \cdot \phi_{tot}(m) \quad (a5)$$

$$Q_{-1} = \frac{j}{N} \sum_{m=0}^{N-1} e^{-j\frac{2\pi}{N}m} \cdot \phi_{tot}(m) \quad (a6)$$

In the case of  $2l - 2k \neq 0$ :

$$Q_{2l-2k} = \frac{j}{N} \sum_{m=0}^{N-1} e^{j\frac{2\pi}{N}(2l-2k)m} \cdot \phi_{tot}(m). \quad (a7)$$

- b) At the receiver, the decision variable  $Z'_k$  of the  $k$ -th symbol can be found from the difference of adjacent sub-carrier signals affected by phase noise. That is,

$$\begin{aligned}
Z'_k &= \frac{(Y_{2k} - Y_{2k+1}^*)}{2} \\
&= \frac{1}{2} \left\{ \sum_{l=0}^{\frac{N}{2}-1} [X_l H_{2l} Q_{2l-2k} - X_l^* H_{2l+1} Q_{2l+1-2k}] + N_{2k} \right. \\
&\quad \left. - \left[ \sum_{l=0}^{\frac{N}{2}-1} [X_l H_{2l} Q_{2l-2k-1} - X_l^* H_{2l+1} Q_{2l-2k}] \right. \right. \\
&\quad \left. \left. + N_{2k+1} \right]^* \right\} \\
&= \frac{1}{2} \sum_{l=0}^{\frac{N}{2}-1} \{ X_l [H_{2l} Q_{2l-2k} + H_{2l+1}^* Q_{2l-2k}^*] \\
&\quad - X_l^* [H_{2l+1} Q_{2l+1-2k} + H_{2l}^* Q_{2l-2k-1}^*] \} + N_k \\
&= \frac{1}{2} X_k (H_{2k} Q_0 + H_{2k+1}^* Q_0^*) \\
&\quad - \frac{1}{2} X_k^* (H_{2k+1} Q_1 + H_{2l}^* Q_{-1}^*) \\
&\quad + \frac{1}{2} \sum_{\substack{l=0 \\ l \neq k}}^{\frac{N}{2}-1} \{ X_l [H_{2l} Q_{2l-2k} + H_{2l+1}^* Q_{2l-2k}^*] \\
&\quad - X_l^* [H_{2l+1} Q_{2l+1-2k} + H_{2l}^* Q_{2l-2k-1}^*] \} \\
&\quad + N_k \tag{b1}
\end{aligned}$$

where  $N_k = (1/2)(N_{2k} - N_{2k+1}^*)$  is the AWGN of the  $k$ -th branch parallel data in the receiver.

When  $H(k) = 1$ ,  $Z'_k$  is as follows.

$$\begin{aligned}
Z'_k &= \frac{1}{2} X_k (Q_0 + Q_0^*) - \frac{1}{2} X_k^* (Q_1 + Q_{-1}^*) \\
&\quad + \frac{1}{2} \sum_{\substack{l=0 \\ l \neq k}}^{\frac{N}{2}-1} \{ X_l [Q_{2l-2k} + Q_{2l-2k}^*] \\
&\quad - X_l^* [Q_{2l+1-2k} + Q_{2l-2k-1}^*] \} + N_k
\end{aligned}$$

$$\begin{aligned}
&= X_k - \frac{1}{2} X_k^* (Q_1 + Q_{-1}^*) \\
&\quad + \frac{1}{2} \sum_{\substack{l=0 \\ l \neq k}}^{\frac{N}{2}-1} \{ X_l [Q_{2l-2k} + Q_{2l-2k}^*] \\
&\quad - X_l^* [Q_{2l+1-2k} + Q_{2l-2k-1}^*] \} + N_k \\
&= X_k + \frac{1}{2} \sum_{\substack{l=0 \\ l \neq k}}^{\frac{N}{2}-1} \{ X_l [Q_{2l-2k} + Q_{2l-2k}^*] \\
&\quad - X_l^* [Q_{2l+1-2k} + Q_{2l-2k-1}^*] \} + N_k \\
&= X_k + CPE + ICI + N_k \tag{b2}
\end{aligned}$$

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