Research on the Evolutionary Model of a Kind of Vertex-splitting Network

Narisa Zhao, Xiaoming Dai
Institute of Systems Engineering, Dalian University of Technology, Dalian, China
Email: {nmgnrs,ssec}@dlut.edu.cn

***Abstract*—A problem is presented about the evolutionary process of the vertex splitting complex network. The rule of the evolution is: every newly-added vertex is the copy of or is split from the existing vertex. The analytic equation set of this network evolutionary model and arithmetic of iteration are put forward. A series of simulation calculation prove that the complex network is Scale Free Network and the power-law increases along with the increment of the splitting similarity degree of $\lambda(t)$ and even approaches to $+\infty$. When the initial degree of each new vertex is constant, the evolutionary process of the network is similar to that of the BA model.**

***Index Terms*—Scale-free Network, Power-Law, Self-Similarity, Vertex-Splitting**

I. Introduction

Barabási and Albert discovered that many networks in reality have the highly self-organizable characteristic, whose vertex degree distribution follows the power-law distribution. Because the power function has the invariance of the scale, this kind of network is called scale- free network [1]. Barabási and Albert present us the evolutionary model (B-A model) to construct the scale- free network and calculate the power exponent of vertex degree distribution using the theory of mean-field [2]. B-A model contains the power-law formative mechanism of many networks in reality and starts the new phase of the research on the complex network [3-6].

In the real life, there are some kinds of networks which have the evolutionary function of simulating and copying vertex. For example, in the Friends network, the newcomer (the new vertex) will be introduced by someone (the existing vertex) to his associate (the adjacent point of the existing vertex). The computer network has the same development characteristic, which shows some kind of self-similarity between the vertexes. With related to this phenomenon, authors propose the evolutionary model of the complex network with vertex-splitting mechanism and put forward the analytic equation set of this network evolutionary model and arithmetic of iteration. A series of simulation calculation prove that the complex network is Scale Free Network and the power-law increases along with the increment of the splitting similarity degree of $\lambda(t)$ and even approaches to $+\infty$. When the initial degree of each new vertex is constant, the evolutionary process of the network is similar to that of the BA model.

II. Evolutionary Rule and Model

Let the initial state is a connected graph with m_0 vertexes, notes $G_0=(V,E)$, where $V=\{\upsilon_1,\upsilon_2,\cdots,\upsilon_{m_0}\}$ is the vertex set, $E=\{e_1,e_2,\cdots,e_{n_0}\}$ is the edge set. And the function $e(i,j)$ is introduced which represents whether vertex i is connected with vertex j. $e(i,j)=1$ means that they are connected, $e(i,j)=0$ means that they are not connected.

From the initial network graph G_0, the algorithm goes as follows:

1) Choose a vertex in the network decided randomly with some probability;

2) Construct a new vertex similar to the chosen vertex, that is, a new vertex is added to the network and connected to a part of (or the whole) adjacent vertices of the chosen vertex in step (1). Go on to step (1).

According to this algorithm, every added vertex is the copy of the existing vertex, that is to say is split from it. (Fig. 1)

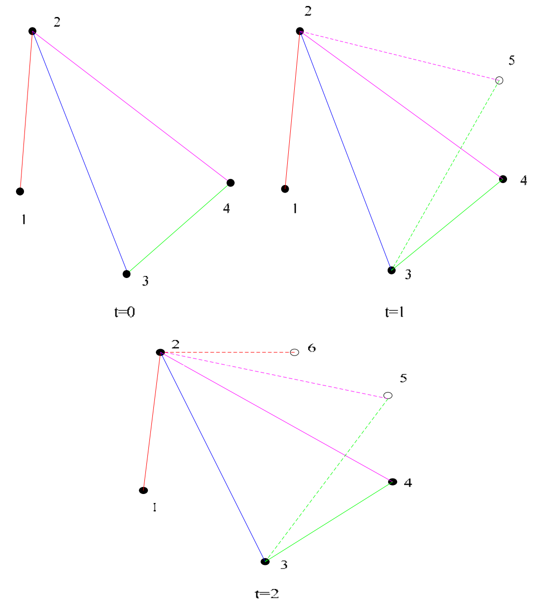

Fig. 1 the evolvement of vertex-splitting network (complete copy)

In Fig.1, when t=0, the system contains m_0=4 vertices. Add a vertex in each interval and connect this vertex with all of the adjacent vertices of the chosen vertex. So, when

doi:10.4304/jnw.6.6.904-906

t=1, the added vertex 5 is copied by the chosen vertex 4. When t=2, the added vertex 6 is copied by the chosen vertex 1.

Next the analytical form of this network evolutionary model is deduced.

Let there are $N(t)$ vertices at t, apparently, $N(0)= m_0$. $k_i(t)$ represents the connectivity of vertex i at t. $p_i(t)$ represents the probability of the vertex i chosen as the splitting vertex at t. If the degree of the added vertex at $t+\Delta t$ is noted as $\Delta k(t)$, then we have

$$\Delta k(t)=\lambda(t)\sum_{i=1}^{N(t)}k_i(t)p_i(t) \quad (1)$$

Where $0<\lambda(t)\le 1$ is used to control the degree of connection between the added vertex and the adjacent vertices of the copied (splitting) vertex. When $\lambda(t)=1$, it means that the connectivity of the newly-added vertex is the same as that of the copied vertex.

Treating $k_i(t)$ as continuous, the equation for the degree of the vertex i at time t can be written in the following form:

$$\frac{\partial k_i(t)}{\partial t}=\Delta k(t)\pi(k_i(t)),\ i=1,2,\cdots,N(t) \quad (2)$$

Where $\pi(k_i(t))$ represents the probability of the newly-added vertex connected to vertex i at t. According to the algorithm proposed in the text, the increase of $k_i(t)$ is only related to its adjacent vertices, so

$$\pi(k_i(t))=\sum_{\ell\in Local(i)}p_\ell(t)\Big/\sum_{j=1}^{N(t)}k_j(t)p_j(t) \quad (3)$$

From Eq. 1-3, we arrive at

$$\frac{\partial k_i(t)}{\partial t}=\lambda(t)\sum_{\ell\in Local(i)}p_\ell(t) \quad (4)$$

Let vertex i is added to system at t_i with the connectivity $k_i(t_i)=\Delta k(t_i)$, which is the initial condition of the differential equation in Eq. 2.

Eq. 2 constitutes a non-linear differential equation set and the amount of the equation set increases along with t. It is commonly hard to get the concrete expression of the solution to this kind of non-linear differential equation set. Fortunately, we can get its numerical value solution using the difference iteration method.

III. THE DIFFERENCE ITERATION METHOD OF THE REVOLUTIONARY EQUATION SET

Starting from the initial network $G_0=(V,E)$, if we can get the adjacent relation between any existing vertex i and other vertices, then the numerical value solution to Eq. 2 $k_i(t)$, i=1,2 ,…, $N(t)$ can be calculated using the difference iteration method.

Therefore, $e(i,j)\in[0,1]$ is defined as the adjacent relation between vertex i and j, $e(i,j)=e(j,i)$. The initial $e(i,j)$ can be easily acknowledged. Let a vertex is added to the system at every Δt 's internal, vertex i created at t_i, vertex j created at t_j, $t_i<t_j$, and make

$$e(i,j)=\left.\frac{\partial k_i(t)}{\partial t}\right|_{t=t_j} \quad (5)$$

Then we can write

$$\sum_{\ell\in Local(i)}p_\ell(t)=\sum_{j=1}^{N(t)}e(i,j)p_j(t) \quad (6)$$

In this way, the process of difference iteration method of Eq. 2 can be written as follows:

A Calculate

$$\Delta k_i(t)=\frac{\partial k_i(t)}{\partial t}=\lambda(t)\sum_{\ell\in Local(i)}p_\ell(t)=\lambda(t)\sum_{j=1}^{N(t)}e(i,j)p_j(t)$$

$$k_i(t+\Delta t)=k_i(t)+\Delta k_i(t)\ \ i=1,2,\cdots,N(t)\,;\quad t=0,\Delta t,2\Delta t,\cdots \quad (7)$$

B. Calculate the degree of the added vertices

$$K_{N(t)+1}(t)=\Delta k(t)=\lambda(t)\sum_{i=1}^{N(t)}k_i(t)p_i(t),\quad t=0,\Delta t,2\Delta t,\cdots \quad (8)$$

C. Calculate the adjacent relation between the added vertex and the existing one

$$e(i,N(t)+1)=\Delta k_i(t),\ i=1,2,\cdots,N(t)\,;\quad t=0,\Delta t,2\Delta t,\cdots \quad (9)$$

Starting from the initial network graph $G_0=(V,E)$, if we have the chosen probability $p_i(t)$ of the split vertex and the splitting “similarity degree” $\lambda(t)$, the network evolutionary solution can be obtained using the difference iteration method above. For example, given the rule of the weak preferential splitting, we define the probability of the copied vertex that

$$p_i(t)=(1-\frac{k_i(t)}{kw(t)})\Big/(N(t)-1) \quad (10)$$

Where $kw(t)=\sum_{j=1}^{N(t)}k_j(t)$ represents the sum of all vertices degree at t.

The weak preferential splitting represented by Eq. 10 corresponds with preferential attachment in B-A model.

It is a simple way to take $\lambda(t)$ as constant, that is $\lambda(t)=c\in(0,1]$; or it can be taken as a normal limitary function $(0<\lambda(t)\le 1)$. If we take

$$\lambda(t)=m\Big/\sum_{i=1}^{N(t)}k_i(t)p_i(t) \quad (11)$$

where m is a positive integer, the initial degree of the newly-added vertex will keep constant, that is $\Delta k(t)=m$.

IV. ANALYSIS OF SIMULATION CALCULATION

Given a initial graph $G_0=(V,E):m_0$ =5, $e(0,0)$=0.0; $e(0,1)$=0.0; $e(0,2)$=1.0; $e(0,3)$=0.0; $e(0,4)$=0.0; $e(1,1)$=0.0; $e(1,2)$=1.0; $e(1,3)$=0.0; $e(1,4)$=1.0; $e(2,2)$=0.0; $e(2,3)$=1.0;

$e(2,4)$=1.0; $e(3,3)$=0.0; $e(3,4)$=1.0; $e(4,4)$=0.0 (Shown in the Fig. 2).

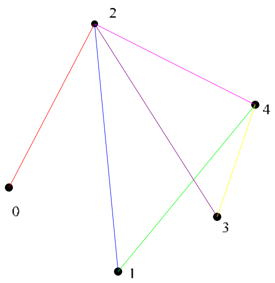

Fig.2 The initial graph.

The evolution complies with the weak preferential splitting rule, which is

$$p_i(t) = (1 - \frac{k_i(t)}{kw(t)}) \Big/ (N(t) - 1) \qquad (12)$$

We analyze some characteristics of the vertex-splitting network model in several situations as follows:

A. $\lambda(t)=c(0<c\leq 1)$ is the constant.

Take $\lambda(t)$=0.2, 0.5, 0.8, 1.0 respectively, 10,000 added vertices are produced in the iteration calculation with the arithmetic in the part 3. Calculate lg $p(k)$ with the vertex degree at the last moment, where $p(k)$ represents the probability density distribution of the vertex degree with its value is k. The relation of lg $p(k)$ and lg k with different $\lambda(t)$ is shown in Fig. 3.

Fig. 3, represents the relation among the newly-added 10,000 vertices distribution where asterisk represents the distribution relation when $\lambda(t)$=0.2, diamonds represent the distribution relation when $\lambda(t)$=0.5, squares represent the distribution relation when $\lambda(t)$=0.8, dots represent the distribution relation when λ=1.

In this condition, lg $p(k)$ and lg k has the linear relation. That is, vertex splitting network obeys the distribution of power-law $p(k) \sim k^{-r_{\lambda(t)}}$ .It shows that the evolutionary network is scale-free and the slope of the lineation increases along with the growing of $\lambda(t)$. When $\lambda(t)$ =0.2, 0.5, 0.8, the slope $r_{\lambda(t)}$ is 2.11, 2.76, 6.00. When $\lambda(t)$ =1, the lineation tends to be perpendicular and the slope is close to ∞. This indicates that after a period of time's evolution, except a part of the initial vertices, the degrees of the majority of vertices are very close. The whole network takes on a kind of self-similarity.

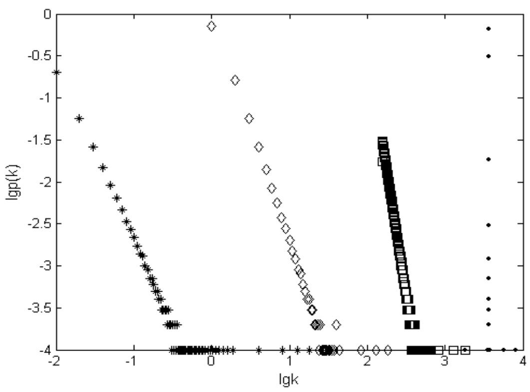

Fig.3 The degree distribution of the vertex in the network

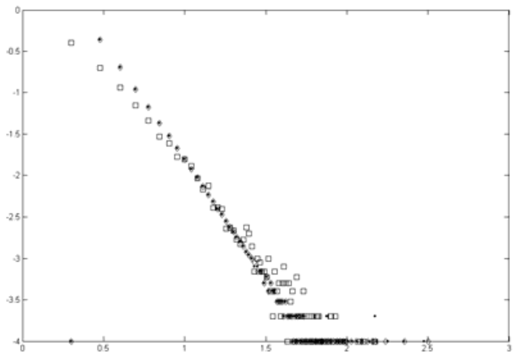

Fig.4 The comparison between the vertex splitting network and B-A model

B. The comparison between the vertex splitting network and B-A model.

If $\lambda(t)$ is taken as Eq. 11, then the initial degree of the added vertices will keep invariable, that is $\Delta k(t)=m$. This situation is similar to B-A model. But according to the algorithm in the text, the m edges of every newly-added vertex only connect with m adjacent vertices of some chosen (splitting) vertex. The solution in this situation shows that the evolution of this model is very close to that of B-A model (as Fig. 4).

Fig. 4 represents the relation among the newly-added 10,000 vertices distribution when m=3.Here, dot represents B-A model: square represents the realistic random simulation result of B-A model: diamond represents the analytical result of vertex-splitting network model when $\Delta k(t)=m$.

The max value of vertex's degree obtained by splitting network is 319.22, while the max value of vertex's degree in B-A model is 300.852. This also shows that splitting network has the higher connectivity.

V. CONCLUSION

A series of simulation calculation prove that the complex network presented in this paper is Scale Free Network and the power-law increases along with the increment of the splitting similarity degree of $\lambda(t)$ and even approaches to $+\infty$. When the initial degree of each new vertex is constant, the evolutionary process of the network is similar to that of the BA model. The larger the splitting similarity degree is, the more self-similarity exits among the vertices with the evolvement of the network.

REFERENCES

[1] A.L. Barabasi, R. Albert. Emergence of scaling in random networks [J]. Nature, Vol. 286(1998), p.509-512
[2] A.L.Barabasi, R.Albert, H.Jeong. Mean-field theory for scale-free random networks. Physica A, Vol. 272(1999), p.173-187
[3] R.Albert, A.L.Barabasi. Statistical mechanics of complex networks [J]. Reviews of Modern Physics, Vol. 1(2002), p.47-97
[4] S.N.Dorogovtsev, J.F.F.Mendes. Evolution of Networks [J]. Advances in Physics, Vol. 4 (2002), p.1079-1187
[5] J.S.Wu, Z.R. Di. Complex Networks in Statistical Physics [J]. Progress in Physics, Vol.1 (2004), p.19-46
[6] M.E.J.Newman. The structure and function of complex networks [J]. SIAM Review, Vol. 2 (2003), p.167-256